A Proof of the ABC Conjecture

Zhang Tianshu

Zhanjiang city, Guangdong province, China

Email: chinazhangtianshu@126.com

Introduction: The ABC conjecture was proposed by Joseoh Oesterle in 1988 and David Masser in 1985 respectively independently. Its general formulation is that for any infinitesimal quantity ε > 0, there exists a constant $C_\varepsilon > 0$, such that for any three relatively prime integers a, b and c satisfying a+b=c, and the inequality $\max(|a|, |b|, |c|) \le C_\varepsilon \prod_{p|abc} p^{1+\epsilon}$ holds, where p/abc indicates that the product is over prime p which divide the product abc. This is an unsolved problem hitherto although Shinichi Mochizuki published four papers on the internet claiming prove it.

Abstract

We first get rid of three kinds from A+B=C according to their respective odevity and gcf (A, B, C) =1. Next expound relations between C and paf (ABC) by the symmetric law of odd numbers. Finally we have proven $C \le C_\varepsilon$ [paf (ABC)] $^{1+\varepsilon}$ such being the case A+B=C, and gcf (A, B, C) =1.

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A + B = C on gcf (A, B, C) =1

For any natural number N, let paf (N) denotes the product of all distinct prime factors of N, e.g. when $N=2^5\times11^2\times13^4$, paf (N) =2×11×13 =286.

In addition, let gcf (A, B, C) denotes greatest common factor of A, B and C.

Well then, the ABC conjecture can thus state that given a real number $\varepsilon > 0$, there exists a constant $C_\varepsilon > 0$, such that every triple of positive integers A, B and C satisfying A + B = C, and gcf (A, B, C) =1, so we have $C \le C_\varepsilon$ [paf (ABC)]$^{1+\varepsilon}$.

Let us first get rid of three kinds from A+B=C according to their respective odevity and gcf (A, B, C) =1, as listed below.

1. If A, B and C all are positive odd numbers, then A+B is an even number, yet C is an odd number, evidently there is only A+B≠C according to an odd number ≠ an even number.

2. If any two in A, B and C are positive even numbers, and another is a positive odd number, then when A+B is an even number, C is an odd number, yet when A+B is an odd number, C is an even number, so there is only A+B≠C according to an odd number ≠ an even number.

3. If A, B and C all are positive even numbers, then they have at least a common prime factor 2, manifestly this and the given prerequisite gcf (A, B, C) =1 are inconsistent, so A, B and C can not be three positive even numbers together.

Therefore, we can only continue to have a kind of A+B=C, namely A, B and

C are two positive odd numbers and one positive even number.

So let following two equalities add together to replace A+B=C such being the case A, B and C are two positive odd numbers and one positive even number. Undoubtedly, that is feasible completely.

1. $A+B=2^XS$, where A, B and S all are positive odd numbers without any common prime factor >1, and X is an integer ≥ 1, similarly hereinafter.

2. Since A and B in A+B=C have only a positive even number, so let B is positive even number 2^YV, then we get $A+2^YV = C$, where A, V and C all are positive odd numbers without any common prime factor >1, and Y is an integer ≥ 1, similarly hereinafter.

Or rather, the proof for ABC conjecture, by now, it is exactly to prove that the following two inequalities hold water.

(1). $2^XS \le C_\varepsilon [\text{paf}(A, B, 2^XS)]^{1+\varepsilon}$ such being the case $A+B=2^XS$;

(2). $C \le C_\varepsilon [\text{paf}(A, 2^YV, C)]^{1+\varepsilon}$ such being the case $A+2^YV=C$.

Deducing at Sequence of Natural Numbers

We use 4n as modulus to divide all positive odd numbers, so obtain two congruence classes of odd numbers, i.e. A whose remainder is 1, and B whose remainder is 3. Well then, the form of A is 1+4n, and the form of B is 3+4n, where n≥0. Two congruence classes of odd numbers are all positive odd numbers. They are arranged as follows.

A: 1, 5, 9, 13, 17, 21, 25, 29, 33, 37, 41, 45, 49, 53, 57, 61, 65, 69…1+4n …

B: 3, 7, 11, 15, 19, 23, 27, 31, 35, 39, 43, 47, 51, 55, 59, 63, 67…3+4n …

We list from small to great positive integers, well then you would discover that Permutations of seriate positive integers show up a certain law.

1, 2^1, 3, 2^2, 5, $2^1\times3$, 7, 2^3, 9, $2^1\times5$, 11, $2^2\times3$, 13, $2^1\times7$, 15, 2^4, 17, $2^1\times9$, 19, $2^2\times5$, 21, $2^1\times11$, 23, $2^3\times3$, 25, $2^1\times13$, 27, $2^2\times7$, 29, $2^1\times15$, 31, 2^5, 33, $2^1\times17$, 35, $2^2\times9$, 37, $2^1\times19$, 39, $2^3\times5$, 41, $2^1\times21$, 43, $2^2\times11$, 45, $2^1\times23$, 47, $2^4\times3$, 49, $2^1\times25$, 51, $2^2\times13$, 53, $2^1\times27$, 55, $2^3\times7$, 57, $2^1\times29$, 59, $2^2\times15$, 61, $2^1\times31$, 63, 2^6, 65, $2^1\times33$, 67, $2^2\times17$, 69, $2^1\times35$, 71, $2^3\times9$, 73, $2^1\times37$, 75, $2^2\times19$, 77, $2^1\times39$, 79, $2^4\times5$, 81, $2^1\times41$, 83, $2^2\times21$, 85, $2^1\times43$, 87, $2^3\times11$, 89, $2^1\times45$, 91, $2^2\times23$, 93, $2^1\times47$, 95, $2^5\times3$, 97, $2^1\times49$, 99, $2^2\times25$, 101, $2^1\times51$, 103 …→

Integers which indicated an exponent of 2 are all even numbers, yet others are odd numbers, in the above-listed sequence of natural numbers.

After the above-listed each odd number is replaced by a congruence class of itself, the sequence of natural numbers is changed into the above-listed form.

A 2^1 B 2^2 A $2^1\times3$ B 2^3 A $2^1\times5$ B $2^2\times3$ A $2^1\times7$ B 2^4 A $2^1\times9$ B $2^2\times5$ A $2^1\times11$ B $2^3\times3$ A $2^1\times13$ B $2^2\times7$ A $2^1\times15$ B 2^5 A $2^1\times17$ B $2^2\times9$ A $2^1\times19$ B $2^3\times5$ A $2^1\times21$ B $2^2\times11$ A $2^1\times23$ B $2^4\times3$ A $2^1\times25$ B $2^2\times13$ A $2^1\times27$ B $2^3\times7$ A $2^1\times29$ B $2^2\times15$ A $2^1\times31$ B 2^6 A $2^1\times33$ B $2^2\times17$ A $2^1\times35$ B $2^3\times9$ A $2^1\times37$ B $2^2\times19$ A $2^1\times39$ B $2^4\times5$ A $2^1\times41$ B $2^2\times21$ A $2^1\times43$ B $2^3\times11$ A $2^1\times45$ B $2^2\times23$ A $2^1\times47$ B $2^5\times3$ A $2^1\times49$ B $2^2\times25$ A $2^1\times51$ B …→

Thus it can seen, leave from any given even number >2, there are finitely cycles of BA leftwards until 3(B) 1(A), and there are infinitely many cycles of AB rightwards up to infinite.

If we regard a positive even number as a symmetric center, then two odd numbers of every bilateral symmetry are A and B, and a sum of bilateral symmetric A and B is the double of the even number. For example, odd numbers 23(B) and 25(A), 21(A) and 27(B), 19(B) and 29(A) etc are respectively bilateral symmetry whereby even number $2^3\times3$ to act as the center of the symmetry, then there are $23+25=2^4\times3$, $21+27=2^4\times3$, $19+29=2^4\times3$ etc. In addition, odd numbers 49(A) and 51(B), 47(B) and 53(A), 45(A) and 55(B) etc are respectively bilateral symmetry whereby even number $2^4\times3+2$ to act as the center of the symmetry, then there are $49+51=2^5\times3+2^2=2^25^2$, $21+27=2^25^2$, $19+29=2^25^2$ etc.

Again give an example, 63(B) and 65(A), 61(A) and 67(B), 59(B) and 69(A) etc are respectively bilateral symmetry whereby even number 2^6 to act as the center of the symmetry, then there are $63+65=2^7$, $61+67=2^7$, $59+69=2^7$ etc.

Overall, there is $A+B=2^{X+1}S$ such being the case A and B are bilateral symmetry whereby 2^XS to act as the center of the symmetry.

One number of A and B on the left of 2^XS is the very number of pairs of A and B wherewith to express the sum as $2^{X+1}S$, thus for any finite-great even number $2^{X+1}S$, a number of pairs of A and B wherewith to express their sum is still finite. This combines with above-mentioned three examples, we can count and get that when $A+B=2^{X+1}S$, a number of pairs of bilateral symmetric A and B for symmetric center 2^XS is a half of 2^XS , i.e. $2^{X-1}S$; when $A+B=2^{X+1}S+2^2n$, a number of pairs of bilateral symmetric A and B for symmetric center $2^X S+2n$ is a half of $2^X S+2n$, i.e. $2^{X-1}s+n$; when $A+B=2^{X+2}$, a number of pairs of bilateral symmetric A and B for symmetric center 2^{X+1} is a half of 2^{X+1}, i.e. 2^X.

On the supposition that A and B are bilateral symmetric odd numbers whereby $2^X S$ to act as the center of the symmetry, then $A+B=2^{X+1}S$, needless to explain, this is known. Now let A added to $2^{X+1}S$, then B and $A+2^{X+1}S$ are still bilateral symmetry whereby $2^{X+1}S$ to act as the center of the symmetry, and $B+ (A+2^{X+1}S) = 2^{X+2}S$.

Provided substitute B for A, let B added to $2^{X+1}S$, then A and $B+ 2^{X+1}S$ are too bilateral symmetry whereby $2^{X+1}S$ to act as the center of the symmetry, and $A+ (B+2^{X+1}S) =2^{X+2}S$.

If both let A added to $2^{X+1}S$, and let B added to $2^{X+1}S$, then $A+2^{X+1}S$ and $B+2^{X+1}S$ are likewise bilateral symmetry whereby $3\times 2^X S$ to act as the center of the symmetry, and $(A+2^{X+1}S) + (B+2^{X+1}S) = 3\times 2^{X+1}S$.

Since there are merely A and B at two odd places of every bilateral symmetry on two sides of each even number, then aforementioned $B+(A+2^{X+1}S)=2^{X+2}S$ or $A+(B+2^{X+1}S)=2^{X+2}S$ is exactly $A+B=2^{X+2}S$, and write $(A+2^{X+1}S)+(B+2^{X+1}S)=3\times 2^{X+1}S$ into $A+B=3\times 2^{X+1}S=2^{X+1}S_1$, where S_1 is an odd number >1.

Do it like this, enable one by one equality like as $A+B=2^{X+1}S$ is proven to continue the existence, along with which the values of X plus S are getting greater and greater, equalities like as $A+B=2^{X+1}S$ are getting more and more, up to there are infinitely more $A+B=2^{X+1}S$.

Pro tanto, we have expounded that regard each positive even number $2^X S$ as a symmetric center, then there are infinitely more $A+B=2^{X+1}S$, including finite $A+B=2^{X+1}S$ for each value of X, at the sequence of natural numbers.

Thereinafter, we need to regard each positive odd number as a symmetric center, prove that there are infinitely more $A+2^Y V=C$ by the aid of the infinitude of $A+B=2^{X+1}S$, at the sequence of natural numbers.

Since equalities like as $A+2^Y V=C$ are essentially such equalities that two sides of $A+B=2^{X+1}S$ either added to a positive odd number or subtracted a positive odd number to get, but when use the subtraction, the subtractive positive odd number must be not greater than C-4, so we adopt the addition hereinafter, that is enough.

Now that there is equality $A+2^Y V=C$, i.e. equality $A+ (2^Y V+p) =C+ p$, where p is a positive odd number, so let odd number $2^Y V+p=B$, and even number $C+ p =2^{X+1}S$, well then $A+ (2^Y V+p) = C+ p$ is exactly $A +B =2^{X+1}S$.

Since there are infinitely more $A+B=2^{X+1}S$, i.e. $A+ (2^Y V+p) =C+ p$ at the sequence of natural numbers, then for each and every $A+ (2^Y V+p) = C+ p$, its two sides subtracted odd number p together, so we get that there are infinitely more $A+2^Y V=C$, including finite $A+2^Y V=C$ i.e. a number of pair of A and $2^Y V$ wherewith to express sum C, at the sequence of natural numbers.

After factorizations of A, B, C, S and V in $A+B=2^{X+1}S$ and $A+2^Y V=C$, if part prime factors of terms of each equality have greater exponents, then there are

$2^{X+1}S \geq$ paf (A, B, $2^{X+1}S$) and $C \geq$ paf (A, $2^{Y}V$, C), for example, $2^7 >$ paf (3, 5^3, 2^7) $=3\times5\times2=30$, due to $3+5^3=2^7$, i.e. 128; and $3^{10} >$ paf (5^6, $2^5\times23\times59$, 3^{10}) $=5\times2\times23\times59\times3=40710$, due to $5^6+2^5\times23\times59=3^{10}$, i.e. 59049.

On the contrary, there are $2^{X+1}S \leq$ paf (A, B, $2^{X+1}S$) and $C \leq$ paf (A, $2^{Y}V$, C), for example, $2^2\times7 <$ paf (13, 3×5, $2^2\times7$) $= 13\times3\times5\times2\times7=2730$, due to $13+3\times5=2^2\times7$, i.e. 28; and $3^4 <$ paf (2^2, 11×7, 3^4) $= 2\times11\times7\times3=462$, due to $11\times7+2^2=3^4$, i.e. 81.

In $A+B=2^{X+1}S$, either A or B plus an even number is still an odd number, and $2^{X+1}S$ plus an even number is still an even number, thereby we can still use equality $A+B=2^{X+1}S$ to express every such equality after the addition. Of course, can too continue to use equality $A+2^{Y}V=C$, namely two sides of $A+B=2^{X+1}S$ to wit $A+(2^{Y}V+p)=C+p$ subtracted p, to express every such equality after the addition. Therefore there are infinitely more $2^{X+1}S \geq$ paf (A, B, $2^{X+1}S$) plus $2^{X+1}S \leq$ paf (A, B, $2^{X+1}S$) such being the case $A+B=2^{X+1}S$, as well there are infinitely more $C \geq$ paf (A, $2^{Y}V$, C) plus $C \leq$ paf (A, $2^{Y}V$, C) such being the case $A+2^{Y}V=C$, at sequence of natural numbers. But if part $2^{X+1}S \geq$ paf (A, B, $2^{X+1}S$) and $2^{X+1}S \leq$ paf (A, B, $2^{X+1}S$), and part $C \geq$ paf (A, $2^{Y}V$, C) and $C \leq$ paf (A, $2^{Y}V$, C), then inequalities like as each of them, all in all, we conclude not whether they are still infinitely more or are finitely.

Proving $C \leq C_{\varepsilon}$ [paf (A, B, C)]$^{1+\varepsilon}$

Hereinbefore we have deduced that there are both $2^{X+1}S \leq$ paf (A, B, $2^{X+1}S$), $2^{X+1}S \geq$ paf (A, B, $2^{X+1}S$) such being the case $A+B=2^{X}S$, and $C \leq$ paf (A, $2^{Y}V$, C), $C \geq$ paf (A, $2^{Y}V$, C) such being the case $A+2^{Y}V = C$, at sequence of natural numbers.

If a positive even number on the right side of each of above-mentioned four inequalities added to a smaller integer ≥1, then the result is both equivalent to multiply the positive even number by a smaller fraction, and equivalent to add a tiny real number >0 to the exponent of the positive even number. Actually three such ways of doing all are in order to increase the value on the base of an identical positive even number.

Judging from this, on the one hand, there are both $2^{X+1}S \leq$ paf (A, B, $2^{X+1}S$) and $C \leq$ paf (A, $2^{Y}V$, C) at sequence of natural numbers, then a positive even number on the right of every $2^{X+1}S \leq$ paf (A, B, $2^{X+1}S$) plus every $C \leq$ paf (A, $2^{Y}V$, C) added to a smaller positive integer to turn themselves into one $2^{X+1}S \leq$ [paf (A, B, $2^{X+1}S$)]$^{1+\varepsilon}$ or one $C \leq$ [paf (A, $2^{Y}V$, C)]$^{1+\varepsilon}$, naturally there are both $2^{X+1}S \leq$ [paf (A, B, $2^{X+1}S$)]$^{1+\varepsilon}$ and $C \leq$ [paf (A, $2^{Y}V$, C)]$^{1+\varepsilon}$. Even need not to multiply the right of each of them by C_{ε}, as well enable two such inequalities hold water successively, thus we need not to prove again these circumstances.

On the other hand, there are both $2^{X+1}S \geq$ paf (A, B, $2^{X+1}S$) and $C \geq$ paf (A, $2^{Y}V$, C) at sequence of natural numbers, then a positive even number on the right of every $2^{X+1}S \geq$ paf (A, B, $2^{X+1}S$) plus every $C \geq$ paf (A, $2^{Y}V$, C) added to a smaller positive integer to turn themselves into one $2^{X+1}S \geq C_{\varepsilon}$ [paf (A, B, $2^{X+1}S$)]$^{1+\varepsilon}$ and one $C \geq C_{\varepsilon}$ [paf (A, $2^{Y}V$, C)]$^{1+\varepsilon}$, naturally there are both $2^{X+1}S \geq$ [paf (A, B, $2^{X+1}S$)]$^{1+\varepsilon}$ and $C \geq$ [paf (A, $2^{Y}V$, C)]$^{1+\varepsilon}$, after that, must multiply an even number on the right of each inequality by C_ε, just can get one $2^{X+1}S \le C_\varepsilon [paf (A, B, 2^{X+1}S)]^{1+\varepsilon}$ and one $C \le C_\varepsilon [paf (A, 2^Y V, C)]^{1+\varepsilon}$.

Taken one with another, there are infinitely more both $2^{X+1}S \le [paf (A, B, 2^{X+1}S)]^{1+\varepsilon}$ plus $2^{X+1}S \le C_\varepsilon [paf (A, B, 2^{X+1}S)]^{1+\varepsilon}$, and $C \le [paf (A, 2^Y V, C)]^{1+\varepsilon}$ plus $C \le C_\varepsilon [paf (A, 2^Y V, C)]^{1+\varepsilon}$.

But then for an individual inequality $2^{X+1}S \le paf (A, B, 2^{X+1}S)$, $C \le paf (A, 2^Y V, C)$, $2^{X+1}S \ge paf (A, B, 2^{X+1}S)$ or $C \ge paf (A, 2^Y V, C)$, after an even number on the right of each of them added to a smaller positive integer to turn themselves into $2^{X+1}S \le [paf (A, B, 2^{X+1}S)]^{1+\varepsilon}$, $C \le [paf (A, 2^Y V, C)]^{1+\varepsilon}$, $2^{X+1}S \ge [paf (A, B, 2^{X+1}S)]^{1+\varepsilon}$ and $C \ge [paf (A, 2^Y V, C)]^{1+\varepsilon}$, there are only finite $2^{X+1}S \le [paf (A, B, 2^{X+1}S)]^{1+\varepsilon}$, $C \le [paf (A, 2^Y V, C)]^{1+\varepsilon}$, $2^{X+1}S \ge [paf (A, B, 2^{X+1}S)]^{1+\varepsilon}$ and $C \ge [paf (A, 2^Y V, C)]^{1+\varepsilon}$ at sequence of natural numbers. This is because that after paf (A, B, $2^{X+1}S$) added to a smaller positive integer to turn itself into $[paf (A, B, 2^{X+1}S)]^{1+\varepsilon}$, notwithstanding there are still $2^{X+1}S \le [paf (A, B, 2^{X+1}S)]^{1+\varepsilon}$ and $2^{X+1}S \ge [paf (A, B, 2^{X+1}S)]^{1+\varepsilon}$, but any even number like as $2^{X+1}S$ is not the symmetric center of odd numbers A and B already, thus there are only $A+B \ne 2^{X+1}S$, and $A+2^Y V \ne C$ due to $A+B \ne 2^{X+1}S$ to wit $A+(2^Y V+p) \ne C+ p$.

In order to obtain anew an equality under the prerequisite that continue to have the value which increased paf (A, B, $2^{X+1}S$), must adjust values among A, B and $2^{X+1}S$, but such an equality after the adjustment is not any of equalities like as $A+B=2^{X+1}S$ already. Thus after paf (A, B, $2^{X+1}S$) is added a value into [paf (A, B, $2^{X+1}S$)] $^{1+\varepsilon}$, in any case, it can not come into an equality like as $A+B=2^{X+1}S$ always.

Obviously, after paf (A, $2^Y V$, C) is added a value into [paf (A, $2^Y V$, C)] $^{1+\varepsilon}$, it can not come into an equality like as $A+2^Y V=C$ always either, along with the impossibility of equality like as $A+B=2^{X+1}S$ on paf (A, B, $2^{X+1}S$) $^{1+\varepsilon}$.

What deserve to mention is that if added to a smaller integer p on the two sides of $2^{X+1}S \le$ paf (A, B, $2^{X+1}S$) plus $2^{X+1}S \ge$ paf (A, B, $2^{X+1}S$), then their results $2^{X+1}S+ p \le$ [paf (A, B, $2^{X+1}S$)] $^{1+\varepsilon}$ and $2^{X+1}S+ p \ge$ [paf (A, B, $2^{X+1}S$)] $^{1+\varepsilon}$ would have complete differentia, even though this is a superfluous remark.

Since it is so, we shall continue to prove concretely $2^{X+1}S \le C_\varepsilon$ [paf (A, B, $2^{X+1}S$)] $^{1+\varepsilon}$ such being the case $2^{X+1}S \ge$ paf (A, B, $2^{X+1}S$), secondly prove concretely $C \le C_\varepsilon$ [paf (A, $2^Y V$, C)] $^{1+\varepsilon}$ such being the case $C\ge$ paf (A, $2^Y V$, C).

First let us factorize A as $a_1^a\, a_2^b \dots a_d^y$, B as $b_1^\alpha\, b_2^\beta \dots b_\lambda^\mu$, and S as $s_1^e\, s_2^f \dots s_\varphi^z$, where a, b…y, α, β…μ, e, f…z are positive integers; and d, λ and $\varphi \ge 1$, also a_1, $a_2 \dots a_d$, b_1, $b_2 \dots b_\lambda$, s_1, $s_2 \dots s_\varphi$ are one another's-disparate odd prime numbers.

Well then, $2^{X+1}S=2^{X+1}s_1^e s_2^f \dots s_\varphi^z$; paf (A, B, $2^{X+1}S$) $=2a_1a_2\dots a_d b_1 b_2 \dots b_\lambda s_1 s_2 \dots s_\varphi$; $2^{X+1}S \ge$ paf (A, B, $2^{X+1}S$) to wit $2^{X+1}s_1^e s_2^f \dots s_\varphi^z \ge 2a_1a_2\dots a_d b_1 b_2 \dots b_\lambda s_1\, s_2 \dots s_\varphi$ under these circumstances of $A+B=2^{X+1}S$. Judging from this, two sides of the inequality are two even numbers, and they have common prime factors 2, s_1, $s_2 \dots s_{\varphi-1}$ and s_φ.

If a positive integer P_1 is added a smaller value into another positive integer P_2, where $P_2 > P_1$, then either P_1 added to an appropriate positive integer to get P_2, or multiply P_1 by a certain fraction to get P_2, even increase exponent of P_1 by an appropriate real number to get P_2.

Thus, if paf (A, B, $2^{X+1}S$) added to a smaller positive integer q to get $2a_1a_2\ldots a_db_1b_2\ldots b_\lambda s_1s_2\ldots s_\varphi+ q$, undoubtedly such a way of doing is equivalent to increase exponent of $2a_1a_2\ldots a_db_1b_2\ldots b_\lambda s_1s_2\ldots s_\varphi$ by a corresponding tiny real number such as ε, where q=1, 2, 3 etc., similarly hereinafter. That is to say, there is the equality as listed below.

$2a_1a_2\ldots a_db_1b_2\ldots b_\lambda s_1s_2\ldots s_\varphi+ q = (2a_1a_2\ldots a_db_1b_2\ldots b_\lambda s_1s_2\ldots s_\varphi)^{1+\varepsilon}$, from this get $1+\varepsilon=\log_{2a_1a_2\ldots a_db_1b_2\ldots b_\lambda s_1s_2\ldots s_\varphi}(2a_1a_2\ldots a_db_1b_2\ldots b_\lambda s_1s_2\ldots s_\varphi+ q)$, so further get $\varepsilon=[\log_{2a_1a_2\ldots a_db_1b_2\ldots b_\lambda s_1s_2\ldots s_\varphi}(2a_1a_2\ldots a_db_1b_2\ldots b_\lambda s_1s_2\ldots s_\varphi+ q)]-1$.

From $(2a_1a_2\ldots a_db_1b_2\ldots b_\lambda s_1s_2\ldots s_\varphi)^{1+\varepsilon} = (2a_1a_2\ldots a_db_1b_2\ldots b_\lambda s_1s_2\ldots s_\varphi)(2a_1a_2\ldots a_db_1b_2\ldots b_\lambda s_1s_2\ldots s_\varphi)^\varepsilon$, we know that this $(2a_1a_2\ldots a_db_1b_2\ldots b_\lambda s_1s_2\ldots s_\varphi)^\varepsilon$ belongs to the incremental factor on the base of paf (A, B, $2^{X+1}S$). Actually the incremental part out of paf (A, B, $2^{X+1}S$) is equal to q. That is to say, there is paf (A, B, $2^{X+1}S$) + q = [paf (A, B, $2^{X+1}S$)]$^{1+\varepsilon}$.

For $2^{X+1}S \geq$ paf (A, B, $2^{X+1}S$) such being the case $A + B =2^{X+1}S$ due to part prime factors of terms of $A + B =2^{X+1}S$ have greater exponents, if paf (A, B, $2^{X+1}S$) added to a smaller positive integer q, but also there is successively $2^{X+1}S \geq$ [paf (A, B, $2^{X+1}S$)]$^{1+\varepsilon}$ after the addition, if enable the sign which expresses inequality of $2^{X+1}S \geq$ [paf (A, B, $2^{X+1}S$)]$^{1+\varepsilon}$ is changed into the reverse direction, then must multiply [paf (A, B, $2^{X+1}S$)]$^{1+\varepsilon}$ by a constant C_ε, after that, there justly is $2^{X+1}S \leq C_\varepsilon$ [paf (A, B, $2^{X+1}S$)]$^{1+\varepsilon}$. For the constant C_ε, we can determine its value as follows.

From $2^{X+1}S \geq$ paf (A, B, $2^{X+1}S$) + q, i.e. $2^{X+1}s_1^e s_2^f \ldots s_\varphi^z \geq 2a_1a_2\ldots a_db_1b_2 \ldots b_\lambda s_1s_2\ldots s_\varphi + q$, C_ε is more than or equal to the quotient which $2^{X+1}s_1^e s_2^f \ldots s_\varphi^z$ divided by $(2a_1a_2\ldots a_db_1b_2\ldots b_\lambda s_1s_2\ldots s_\varphi +q)$, to wit $C_\varepsilon \geq 2^{X+1}s_1^e s_2^f \ldots s_\varphi^z / (2a_1 a_2\ldots a_db_1 b_2 \ldots b_\lambda s_1s_2\ldots s_\varphi +q)$.

Excepting smaller positive integer q, a number which every sign of C_ε expresses is a given number. Actually q is a relative- smaller positive integer, if $2a_1a_2\ldots a_db_1 b_2\ldots b_\lambda s_1s_2\ldots s_\varphi$ tends to infinity, then any concrete positive integer so long as we can write out is a smaller positive integer, yet once the value of p is determined, it exactly is a known constant, therefore C_ε is a constant.

To sum up, we have got $2^{X+1}s_1^e s_2^f \ldots s_\varphi^z \leq C_\varepsilon (2a_1a_2\ldots a_db_1b_2\ldots b_\lambda s_1s_2\ldots s_\varphi +q)$, in other words, we have proven $2^{X+1}S \leq C_\varepsilon$ [paf (A, B, $2^{X+1}S$)] $^{1+\varepsilon}$ such being the case A + B $=2^{X+1}S$ and gcf (A, B, $2^{X+1}S$) =1.

There is no harm to give again an aforementioned concrete instance to explain above- mentioned calculations. For equality $3+5^3=2^7$, odd numbers 3 and 5^3 are bilateral symmetry whereby 2^6 to act as the center of the symmetry at sequence of natural numbers. And there is $2^7 >$ paf (3, 5^3, 2^7) = 3×5×2 = 30. If paf (3, 5^3, 2^7) added to integer 1, then it is equivalent to increase exponent of 3×5×2 by a corresponding tiny real number such as ε. That is to say, there is $3\times5\times2+1= (3\times5\times2)^{1+\varepsilon}$, i.e. $31=30^{1+\varepsilon}$, then $1+\varepsilon =\log_{30} 31$, so $\varepsilon = (\log_{30} 31)-1$. Nevertheless there is still $2^7 >30^{1+\varepsilon}$, so must multiply $30^{1+\varepsilon}$ by a constant C_ε, after that, there justly is $2^7 \leq C_\varepsilon \times 30^{1+\varepsilon}$, well then $C_\varepsilon \geq 2^7/30^{1+\varepsilon}$, where $\varepsilon = (\log_{30} 31)-1$.

Hereinafter we set to prove concretely $C \leq C_\varepsilon$ [paf (A, 2^Y V, C)] $^{1+\varepsilon}$ such being the case $C \geq$ paf (A, 2^YV, C).

Like that, we factorize A as $a_1^a a_2^b \ldots a_d^y$, C as $c_1^\alpha c_2^\beta \ldots c_\lambda^\mu$, and V as $v_1^e v_2^f \ldots v_\varphi^z$, where a, b…y, α, β…μ, e, f…z are positive integers; and d, λ and $\varphi \geq 1$, also a_1, $a_2 \ldots a_d$, c_1, $c_2 \ldots c_\lambda$, v_1, $v_2 \ldots v_\varphi$ are one another's-disparate odd prime numbers.

Well then, $C \geq$ paf (A, 2^YV, C) is exactly $c_1^\alpha c_2^\beta \ldots c_\lambda^\mu \geq 2a_1 a_2 \ldots a_d c_1 c_2 \ldots c_\lambda v_1 v_2 \ldots v_\varphi$ under these circumstances of $A+2^YV=C$. Thus it can seen, the left side of the inequality is an odd number, and the right side is an even number, however they have common prime factors c_1, $c_2 \ldots c_{\lambda-1}$ and c_λ.

Let paf (A, 2^YV, C) added to a smaller positive integer k to get $2a_1a_2 \ldots a_dc_1c_2 \ldots c_\lambda v_1v_2 \ldots v_\varphi + k$, evidently the way of doing is equivalent to increase exponent of $2a_1a_2 \ldots a_dc_1c_2 \ldots c_\lambda v_1v_2 \ldots v_\varphi$ by a corresponding tiny real number such as ε, where k=1, 2, 3 etc., similarly hereinafter. That is to say, there is the equality as listed below.

$2a_1a_2 \ldots a_dc_1c_2 \ldots c_\lambda v_1v_2 \ldots v_\varphi + k = (2a_1a_2 \ldots a_dc_1c_2 \ldots c_\lambda v_1v_2 \ldots v_\varphi)^{1+\varepsilon}$, from this get $1+\varepsilon = \log_{2a_1a_2 \ldots a_dc_1c_2 \ldots c_\lambda v_1v_2 \ldots v_\varphi}(2a_1a_2 \ldots a_dc_1c_2 \ldots c_\lambda v_1v_2 \ldots v_\varphi + k)$, so get further $\varepsilon = [\log_{2a_1a_2 \ldots a_dc_1c_2 \ldots c_\lambda v_1v_2 \ldots v_\varphi}(2a_1a_2 \ldots a_dc_1c_2 \ldots c_\lambda v_1v_2 \ldots v_\varphi + k)]-1$.

From $(2a_1a_2 \ldots a_dc_1c_2 \ldots c_\lambda v_1v_2 \ldots v_\varphi)^{1+\varepsilon} = (2a_1a_2 \ldots a_dc_1c_2 \ldots c_\lambda v_1v_2 \ldots v_\varphi)(2a_1a_2 \ldots a_dc_1c_2 \ldots c_\lambda v_1v_2 \ldots v_\varphi)^\varepsilon$, we know that this $(2a_1a_2 \ldots a_dc_1c_2 \ldots c_\lambda v_1v_2 \ldots v_\varphi)^\varepsilon$ belongs to the incremental factor on the base of paf (A, 2^YV, C). Actually the incremental part out of paf (A, 2^YV, C) is equal to k. That is to say, there is paf (A, 2^YV, C) + k = [paf (A, 2^YV, C)] $^{1+\varepsilon}$.

For C ≥ paf (A, 2^YV, C) such being the case A+2^YV=C due to part prime factors of terms of A +2^YV=C have greater exponents, if paf (A, 2^YV, C) added to a smaller positive integer k, but also there is successively C ≥ [paf (A, 2^YV, C)]$^{1+\varepsilon}$ after the addition, if enable the sign which expresses inequality of C ≥ [paf (A, 2^YV, C)]$^{1+\varepsilon}$ is changed into the reverse direction, then must multiply [paf (A, 2^YV, C)]$^{1+\varepsilon}$ by a constant C_ε, after that, there justly is C ≤ C_ε [paf (A, 2^YV, C)]$^{1+\varepsilon}$. For the constant C_ε, we can determine its value as follows.

From C ≥ paf (A, 2^YV, C) + k, i.e. $c_1^\alpha c_2^\beta \ldots c_\lambda^\mu \geq 2a_1a_2\ldots a_dc_1c_2\ldots c_\lambda v_1v_2\ldots v_\varphi + k$, C_ε is more than or equal to the quotient which $c_1^\alpha c_2^\beta \ldots c_\lambda^\mu$ divided by $(2a_1a_2\ldots a_dc_1c_2\ldots c_\lambda v_1 v_2\ldots v_\varphi +k)$, to wit $C_\varepsilon \geq c_1^\alpha c_2^\beta \ldots c_\lambda^\mu / (2a_1a_2\ldots a_dc_1 c_2 \ldots c_\lambda v_1v_2\ldots v_\varphi +k)$.

Excepting smaller positive integer k, a number which every sign of C_ε expresses is a given number. Actually k is a relative- smaller positive integer, if $2a_1a_2\ldots a_dc_1c_2\ldots c_\lambda v_1v_2\ldots v_\varphi$ tends to infinity, then any concrete positive integer so long as we can write out is a smaller positive integer, yet once the value of k is determined, it exactly is a known constant, therefore C_ε is a constant.

To sum up, we have got $c_1^\alpha c_2^\beta \ldots c_\lambda^\mu \leq C_\varepsilon (2a_1a_2\ldots a_dc_1c_2\ldots c_\lambda v_1v_2\ldots v_\varphi + k)$, in other words, we have proven C ≤ C_ε [paf (A, 2^YV, C)] $^{1+\varepsilon}$ such being the case A +2^YV = C, and gcf (A, 2^YV, C) =1.

Like that, we give an aforementioned concrete instance to explain above-mentioned calculations. For equality $5^6+ 2^5\times23\times59 =3^{10}$, there is $3^{10} >$ paf (5^6, $2^5\times23\times59$, 3^{10}) = $5\times2\times23\times59\times3 =40710$. If paf ($5^6$, $2^5\times23\times59$, 3^{10}) added to integer 1, then it is equivalent to increase exponent of $5\times2\times23\times59\times3$ by a corresponding tiny real number such as ε. That is to say, there is $5\times2\times23\times59\times3+1 = (5\times2\times23\times59\times3)^{1+\varepsilon}$, i.e. $40711=40710^{1+\varepsilon}$, well then $1+\varepsilon =\log_{40710} 40711$, so $\varepsilon = (\log_{40710} 40711)-1$. Nevertheless there is successively $3^{10} >40710^{1+\varepsilon}$, so must multiply $40710^{1+\varepsilon}$ by a constant C_ε, after that, there is justly $3^{10} \le C_\varepsilon\times40710^{1+\varepsilon}$, so $C_\varepsilon\ge 3^{10}/40710^{1+\varepsilon}$, where $\varepsilon =(\log_{40710} 40711)-1$.

Heretofore, the ABC conjecture is proven by us as the true. The proof was thus brought to a close. As a consequence, the ABC conjecture does hold water.