SMALL JUMP WITH NEGATION-UTM TRAMPOLINE

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1. Introduction

This paper divide some complexity class by using fixpoint of Decidable Universal Turing Machine (UTM). Decidable Deterministic Turing Machine (DTM) have fixpointless combinator that add no extra resources (like Negation), but UTM makes some fixpoint in the combinator if UTM Target DTM set close under the combinator. This means that we can jump out of the fixpointless combinator system by making more complex problem with UTM and diagonal method.

We proof that L is not P as concrete example. We can make Polynomial time UTM that emulate all Logarithm space DTM (LDTM). LDTM set close under Negation, therefore UTM does not close under LDTM set. We can proof this theorem like halting problem and time/space hierarchy theorem. We can extend this proof to divide time/space limited DTM set. These are new hierarchy that use UTM and Negation.

As appendix, We proof P is not NP by using P is not L.

2. P is not L

Definition 1. "DTM" is defined as Decidable Deterministic Turing Machine set. "$LDTM$" is defined as logarithmic space DTM. "$pDTM$" is defined as polynomial time space DTM. "$\bigcirc DTM$" is defined as DTM that some resource (time, space) limited.

"UTM" is defined as Universal Turing Machine set that emulate all $M \in DTM$. "$UTM\left(C\right)$" is defined as UTM for $C \subset DTM$. $\left\langle M\right\rangle$ is defined as code number of a $M \in DTM$ that $U \in UTM$ emulate. That is, $\forall w\left[U\left(\left\langle M\right\rangle ,w\right)=M\left(w\right)\right]$ and $U\left(\left\langle M\right\rangle \right)=M$.

"$Negate\left(C\right)$" is defined as minimum Negation system that include C. That is, $\forall C\left[\left(C\subset Negate\left(C\right)\right)\wedge\left(\forall c\in Negate\left(C\right)\left[\neg c\in Negate\left(C\right)\right]\right)\right]$.

Theorem 2. $\forall r\in\bigcirc DTM\left(\neg r\in\bigcirc DTM\right)$

Proof. It is trivial from DTM structure.

If DTM is

$M=\left(Q,\Sigma,\Gamma,\delta,q_{0},q_{1},q_{2}\right)$

then this dual machine

$\overline{M}=\left(Q,\Sigma,\Gamma,\delta,q_{0},q_{2},q_{1}\right)$

compute $\neg M$ without extra resources.

Therefore negation of $\bigcirc DTM$ is also in $\bigcirc DTM$. □

Theorem 3. $\exists U\in UTM\left(LDTM\right)\left[U\in pDTM\right]$

Proof. It is trivial because some $U' \in UTM$ can emulate all $LDTM$ in polynomial time. Therefore, we can make $U \in pDTM$ by limiting at polynomial time (if U' compute over polynomial time, U reject these input). □

Theorem 4. $L \subsetneq P$

Proof. We can proof this theorem like halting problem and time/space hierarchy theorem.

Because of
$\forall U \in UTM\left(LDTM\right), M \in LDTM\left[U\left(\langle M\rangle\right) = M\right]$
all $M \in LDTM$ have index $\langle M\rangle$. Mentioned above 2,
$\forall r \in LDTM\left(\neg r \in LDTM\right)$
Therefore we can make G that is Negation of diagonalization.
$G\left(\langle M\rangle\right) = \neg U\left(\langle M\rangle, \langle M\rangle\right)$

$$\begin{array}{lllllll} & & \langle M_0\rangle & \langle M_1\rangle & \langle M_2\rangle & \langle M_3\rangle & \cdots \\ M_0 & =\{ & \underline{\top} & \top & \bot & \top & \cdots \\ M_1 & =\{ & \bot & \underline{\bot} & \top & \bot & \cdots \\ M_2 & =\{ & \bot & \bot & \underline{\bot} & \top & \cdots \\ M_3 & =\{ & \top & \top & \bot & \underline{\bot} & \cdots \\ \vdots & & \vdots & \vdots & \vdots & \vdots & \vdots \\ G & =\{ & \bot & \top & \top & \top & \cdots \end{array}$$

$G \notin LDTM$ because $\forall M \in LDTM\left[G\left(\langle M\rangle\right) \neq M\left(\langle M\rangle\right)\right]$. On the other hand, $G \in pDTM$ because $\neg U \in pDTM$ 2 and G input size is at least half of U.

Therefore, $G \in pDTM\left(G \notin LDTM\right)$ and $L \subsetneq P$. □

We can expand above result to general DTM.

Theorem 5. $\forall CC \subset DTM\left[Negate\left(UTM\left(Negate\left(CC\right)\right)\right) \nsubseteq Negate\left(CC\right)\right]$

Proof. We omit the proof because this proof is same as previous. □

Corollary 6. $Negate\left(UTM\left(\bigcirc DTM\right)\right) \nsubseteq \bigcirc DTM$

3. Trampoline Hierarchy between Negation and UTM

This result shows that we can jump over border of asymptotic analysis by using Negation (fixpointless combinator) and UTM (fixpoint creator). Therefore, combination of UTM and Negation make new complexity class. That is, there are some Hierarchy to apply UTM and Negation.

4. Appendix: Divide other complexity class

By using result $L \subsetneq P$, we can divide some another complexity class.

Definition 7. "$R\left(M\right)$" is defined as Reversible Deterministic Turing Machine that can compute M. "$R^{-1}\left(M\right)$" is defined as Inverse Turing Machine of $R\left(M\right)$. If $R^{-1}\left(M\right)\left(t\right)$ do not define then $R^{-1}\left(M\right)\left(t\right)$ value is $\bot$. "$\{1\}$" is defined as finite automata like projection functions.

Lemma 8. $TIME\left(R\left(M\right)\right) = TIME\left(M\right)$

Proof. 3 tape $R\left(M\right)$ can emulate M using same time [1] and single tape Turing Machine can emulate multi tape Turing Machine at most $O\left(t^2\left(n\right)\right) \mid t\left(n\right) \geq n$ time [2]. Therefore $TIME\left(R\left(M\right)\right) = TIME\left(M\right)$. □

Lemma 9. $\forall M \in NP - Complete,\ t \in P \left[M \circ R^{-1}(t) \in NP - Complete \right]$

Proof. It is trivial that $M \circ R^{-1}(t) \in NP$ because $M \in NP - Complete$ and $R^{-1}(t) \in P$.

It is also trivial that we can reduce $M \circ R^{-1}(t) \to M$ by using $R(t) \in P$.

$M \circ R^{-1}(t) \circ R(t) = M$

Therefore $M \circ R^{-1}(t) \in NP - Complete$ □

Theorem 10. $P \subsetneq NP$

Proof. (Proof by contradiction.) Assume to the contrary that $P = NP$.

This means that

$\forall p \in NP,\ q \in P - Complete\ \exists r \in L\, [p = q \circ r]$

Let p be $M \circ R^{-1}(t) \in NP - Complete \mid M \in NP - Complete,\ t \in P$

$\forall M \in NP - Complete,\ t \in P,\ q \in P - Complete\ \exists r \in L \left[M \circ R^{-1}(t) = q \circ r \right]$

Let M be $\{1\} \circ R(M)$

$\forall M \in NP{-}Complete,\ t \in P,\ q \in P{-}Complete\ \exists r \in L \left[\{1\} \circ R(M) \circ R^{-1}(t) = q \circ r \right]$

Let t be M under $P = NP$

$\forall M \in NP{-}Complete,\ q \in P{-}Complete\ \exists r \in L \left[\{1\} \circ R(M) \circ R^{-1}(M) = \{1\} = q \circ r \right]$

This means $L = P$, but this result contradicting previously result 4. □

References

[1] Kenichi MORITA, "Reversible Computing", 2012, p.15

[2] Michael Sipser, "Introduction to the Theory of COMPUTATION Second Edition (Japanese)", 2008, pp.302-303