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Escape Velocities for the Flux Model of Gravity

Abstract

This conjecture supports the notion of a ***temporal-inertial (TI)*** field, a field that is subject to gravity and, in response to the acceleration of gravity, transmits its own acceleration to massive particles and objects comprising massive particles. The relation between the Higgs field or the Higgs mechanism and what I designate as the TI field is undefined. I may attribute properties to the TI field (such as the particles of the field being subject to gravity) that do not obtain for the Higgs field. The ***flux model*** posits that in a gravitational field, the velocity of the TI field combines with that of the gravitons emitted by the gravitational body to increase the flux of gravitons relative to the TI field. Thus the response of the TI field to gravity adds to the force of gravity. Equations defining the acceleration and velocity profiles about a gravitational body are developed for the flux model. The flux model describes a system in which particles of the TI field fall radially toward the central gravitational mass. The flux model may offer an explanation for the unexpected changes in velocity seen by the so-called flyby spacecraft in their gravity assist maneuvers past Earth.

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^ Author's Note

The relation of the Higgs field or the Higgs mechanism [1] and what I designate as the TI field is undefined. I may attribute properties to the TI field (such as the particles of the field being subject to gravity) that do not obtain for the Higgs field.

This paper comprises an extensive rewrite of *Conjecture on a Flux Model of Gravity* [2], the posting of which was removed on September 14, 2014. The effects of the infall velocity on the augmentation of gravity have been extended in this paper. Descriptions of the behavior of the model of gravity in the referenced paper have been substantially revised. The effects of relativity on the behavior of the model have been added.

^ The Temporal-Inertial (TI) Field

The flux model of gravity posits the following:

1. The TI field is a field of particles that participates in the inertial and gravitational interactions.
2. When a matter particle or an object composed of matter particles is accelerated by an external force, its motion is resisted by its acceleration relative to the TI field. This reactive force of the TI field of space is the familiar inertial force.
3. The TI field is subject to gravity.
4. Particles of the TI field are accelerated by gravity directly toward the center of each gravitational body just as a test particle would be and reaches the escape velocity of such a particle at the distance of that particle from the barycenter of the gravitational body.
5. The gravitational acceleration of the TI field relative to a matter particle or an object composed of matter particles applies a force to that matter particle or object. This force is the familiar gravitational force applied indirectly through the intermediary of the acceleration of the TI field of space.
6. The TI field accelerates massive particles at the same rate as its own acceleration.
7. Acceleration of the TI field in its own response to gravity is the sole accelerator of massive particles in response to gravity. Accordingly, massive particles are not directly subject to the gravitational force.
8. Acceleration of the TI field is moderated by a second field termed the static field.
9. The gravitational force on particles of the TI field is balanced by the inertial reaction force caused by the acceleration of the particles of the TI field relative to the static field.
10. The TI field supports the propagation of photons.

The flux model posits that in the gravitational field of a massive body, the velocity of the TI field combines with that of the gravitons emitted by the body to increase the flux of gravitons relative to the TI field. Thus the response of the TI field to gravity adds to the force of gravity.

^ The Flux Model

Two variants of the flux model are introduced briefly; the ***circulation variant*** and the ***infall variant***. It is shown in the discussions below that motion of the TI field relative to a gravitational body augments the gravitational field of that body. The augmentation achieved by the infall variant can be an appreciable fraction of the Newtonian valuation for extremely large gravitational masses such as black holes, but is vanishingly small, but not zero, in the modest gravitational fields of the Sun and planets of the Solar System.

^ *The Circulation Variant of the Flux Model*

In this variant of the flux model, particles of the TI field are assumed to move in circular orbits (elliptical orbits with zero eccentricity) about a central mass, such as the planets in the Solar System.

Reference [1] indicates that the Global Positioning System will work properly only if the Earth does not move relative to the temporal-inertial (TI) field. Clearly the Earth moves, so the conclusion must be that particles of the TI field move in orbit with Earth (or vice versa). It is not clear whether this conclusion can be extrapolated for the entire solar system. The orbital velocity of the TI field about a gravitational body asserted by the circulation variant of the flux model does not augment the flux of gravitons.

^ *The Infall Variant of the Flux Model*

In this variant of the flux model, particles of the TI field are assumed to fall radially into the central mass. To support the notion that such a model exists, I cite the following description of the space surrounding a black hole.

Referring to the geometry of a black hole, Andrew Hamilton [3] states “Free-fall coordinates reveal that the Schwarzschild geometry looks like ordinary flat space, with the distinctive feature that space itself is flowing radially inwards at the Newtonian escape velocity, $v = (2GM / r)^{1/2}$. The infall velocity v passes the speed of light c at the horizon.”

I interpret this statement by Hamilton to mean that there exists ‘a field of space’ that is subject to gravity and, at least in the case of black holes, that this field flows inward toward the center of a black hole. Furthermore this field supports the propagation of photons. At the event horizon of a black hole this field flows radially inward at the speed of light, thus restricting the escape of photons from inside the radius of the event horizon. (I shall argue later that this interpretation is incomplete.) If the TI field is subject to the gravity of a black hole, it is subject to the gravity of any gravitational body, however small.

^ Graviton Flux is Increased by the Infall Velocity of the TI Field Toward a Gravitational Body

As particles of the TI field fall toward the gravitational body the flux of gravitons ‘seen’ by particles of the TI field increases in proportion with the velocity of the TI field particles. As the flux of gravitons increases, the acceleration of the TI field increases in direct proportion to the increase in graviton flux. This behavior is described in Appendix A.

^ Determination of the Acceleration Profiles and Escape Velocities from Gravitational Bodies as Asserted by the Flux Model of Gravity

The flux model of gravity posits the following:

1. The infall velocity of particles of the TI field falling toward a gravitational body is the same magnitude (but opposite in sign) as the escape velocity at any given distance from the body.
2. The velocity of the particles of the TI field toward a gravitational body increases the flux of gravitons 'seen' by these particles and thus increases the gravitational force acting on the particles. ***The magnitude of the infall velocity of the TI field is increased beyond the value expressed in the classic formula:***

   $v_{Infall} = v_{Escape} = (2GM / r)^{1/2}$.
3. We must evaluate the effect of this change. In other words, the increase of the infall velocity increases the gravitational force which then increases the infall velocity and so on.
4. The process to determine the outcome of this feedback problem is to step through the following:
   a. Calculate the acceleration at a given distance from a gravitational body for the current value of the infall velocity.
   b. Integrate the acceleration to yield a new expression for the infall velocity.
   c. Continue back to step a until the contribution of the last iteration is negligible.

This process is developed in Appendix B. The process and results are summarized in Table 1.

^ Table 1. Summary of the Iterative Evaluation of the Acceleration and Infall Velocity Profiles About a Weak Gravitational Body

Step	Expression **
Determine escape velocity v	$v = (2\,GM / r)^{1/2}$
Determine acceleration $a = (GM / r^2)\,(1 + v/c)$	$a = (GM / r^2) * (1 + (r_S / r)^{1/2})$
Integrate acceleration to get next iteration of escape velocity v.	$v = (2\,GM / r)^{1/2} * [1 + (1/3)\,(r_S / r)^{1/2}]$
Determine acceleration $a = (GM / r^2)\,(1 + v/c)$	$a = (GM / r^2) * [1 + (r_S / r)^{1/2} + (1/3)\,(r_S / r)]$
Integrate acceleration to get next iteration of escape velocity v.	$v = (2\,GM / r)^{1/2} * [1 + (1/3)\,(r_S / r)^{1/2} + (1/12)\,(r_S / r)]$
Determine acceleration $a = (GM / r^2)\,(1 + v/c)$	$a = (GM / r^2) * [1 + (r_S / r)^{1/2} + (1/3)\,(r_S / r) + (1/12)\,(r_S / r)^{3/2}]$
Integrate acceleration to get next iteration of escape velocity v.	$v = (2\,GM / r)^{1/2} * [1 + (1/3)\,(r_S / r)^{1/2} + (1/12)\,(r_S / r) + (1/60)\,(r_S / r)^{3/2}]$
Determine acceleration $a = (GM / r^2)\,(1 + v/c)$	$a = (GM / r^2) * [1 + (r_S / r)^{1/2} + (1/3)\,(r_S / r) + (1/12)\,(r_S / r)^{3/2} + (1/60)\,(r_S / r)^2]$
The general form of the series for the infall velocity can now be written	$v = (2\,GM / r)^{1/2} * [1 + \sum$ (from n= 1 to ∞) $[(2 / (n+2)! * (r_S / r)^{n/2}]$
The general form of the series for the infall acceleration can now be written	$a = (GM / r^2) * [1 + \sum$ (from n= 1 to ∞) $[(2 / (n+1)! * (r_S / r)^{n/2}]$

** I use the abbreviation $r_S = (2\,GM / c^2)$ even though the development here is not valid for black holes. The term r_S is the Schwarzschild radius [4].

^ *Comparisons of the Acceleration Profiles and Escape Velocities from Gravitational Bodies as Asserted by the Flux Model of Gravity*

The expressions for the escape velocity and acceleration from a gravitational body derived for the infall variant of the flux model of gravity comprise an infinite series with declining values of the elements in the series. This series converges so rapidly that the term $1/360 (r_S / r)^{5/2}$ is negligible and is not included in evaluating the acceleration profiles about the modest gravitational bodies of the Solar System and that of a white dwarf. Tables 2 and 3 list these comparisons.

^ Table 2. Comparison of the Escape Velocity from a Gravitational Body for the Flux and Newtonian Models of Gravity

Body	Escape Velocity, Flux Model, km/sec	Difference Between the Escape Velocity of the Flux Model and that of the Newtonian Model, m/sec
Sun	618.00	424.0
Mercury	4.25	0.020
Venus	10.36	0.119
Earth	11.19	0.139
Mars	5.03	0.028
Jupiter	60.21	4.030
Saturn	36.10	1.450
Uranus	21.38	0.508
Neptune	23.57	0.617
White Dwarf (1 solar mass)	7,345.00	59.02 km/sec

** Governing Equations for Table 2:

Escape Velocity of the Flux Model: $v_{Escape} = (2 GM / r^2) * [1 + (1/3) (r_S / r)^{1/2} + (1/12) (r_S / r) + (1/60) (r_S / r)^{3/2}]$

Velocity Difference = $(2 GM / r^2) * [(1/3) (r_S / r)^{1/2} + (1/12) (r_S / r) + (1/60) (r_S / r)^{3/2}]$

^ Table 3. Comparison of the Gravitational Acceleration of Various Gravitational Bodies for the Flux and Newtonian Models of Gravity (Third Iteration) **

Body	Gravitational Acceleration at the Object's Surface, Flux Model, m/sec^2	Ratio of the Gravitational Acceleration of the Flux Model to that of the Newtonian Model
Sun	27.45	1.0020613
Mercury	3.70	1.0000142
Venus	8.87	1.0000346
Earth	9.82	1.0000373
Mars	3.73	1.0000168
Jupiter	25.93	1.0002008
Saturn	11.19	1.0002040
Uranus	9.01	1.0000713
Neptune	11.28	1.0000786
White Dwarf (1 solar mass, radius = 5000 km)	5,439.00	1.0245014

** Governing Equation: $\text{Ratio} = [\, 1 + (r_S / r)^{1/2} + (1/3)\,(r_S / r) + (1/12)\,(r_S / r)^{3/2} + (1/60)\,(r_S / r)^2 \,]$

On the surface of the Earth, for example, the increase in acceleration caused by the infall of the TI field is about one part in 26,800. This value would contribute about 1/8 ounce to the weight of a 200 pound person, 2 ounces to a 3500 pound vehicle and 3.7 tons to the weight of a 100,000 ton container ship.

Why wouldn't the difference in the assessment of the gravitational force asserted by this conjecture have been noticed before? Most probably because the difference between the valuation made by the conjecture is a factor of 3 less than the uncertainty in the established value of the gravitational constant itself.

“The gravitational constant is perhaps the most difficult physical constant to measure to high accuracy. In SI units, the 2010 CODATA recommended value of the gravitational constant (with standard uncertainty in parentheses) is:

$G = 6.67384(80) \times 10^{-11}\ m^3\ kg^{-1}\ s^{-2}$

with relative standard uncertainty 1.2×10^{-4} ” [5] .

^ The Heresy Confirmed

The heresy I refer to in this section is the conjecture that the force of gravity is mediated indirectly by the acceleration of particles of the Temporal-Inertial field which then accelerate massive particles. The TI field is subject to gravity and, in response to the acceleration of gravity, transmits its own acceleration to massive particles and objects comprising massive particles. Massive particles do not themselves respond directly to the gravitational force.

Revert now to the model in which there is no TI field and gravity is mediated directly by the exchange of gravitons between masses. Would a flux model operating directly on massive particles be operative in this situation? Consider that a small test mass falling in the gravitational field of a massive body would experience a greater flux of gravitons due to its velocity toward the large body. The acceleration of the small mass would thus be increased relative to its value if the flux phenomenon were not active. A problem with this model arises when you consider a second test mass at the same distance from the gravitational body, but falling faster than the first mass. The acceleration of the second mass, at the same distance from the large body, would be greater than that of the slower moving first mass. The acceleration of objects in the vicinity of a gravitational body would thus depend both on their distance from the body and on their velocity relative to the body. This behavior is chaotic, and while the effect is feeble in the moderate gravitational fields of the Solar System, it would be considerable in the field of a black hole. The gravitational field of such a model could not be characterized solely as a function of the mass of the gravitational body.

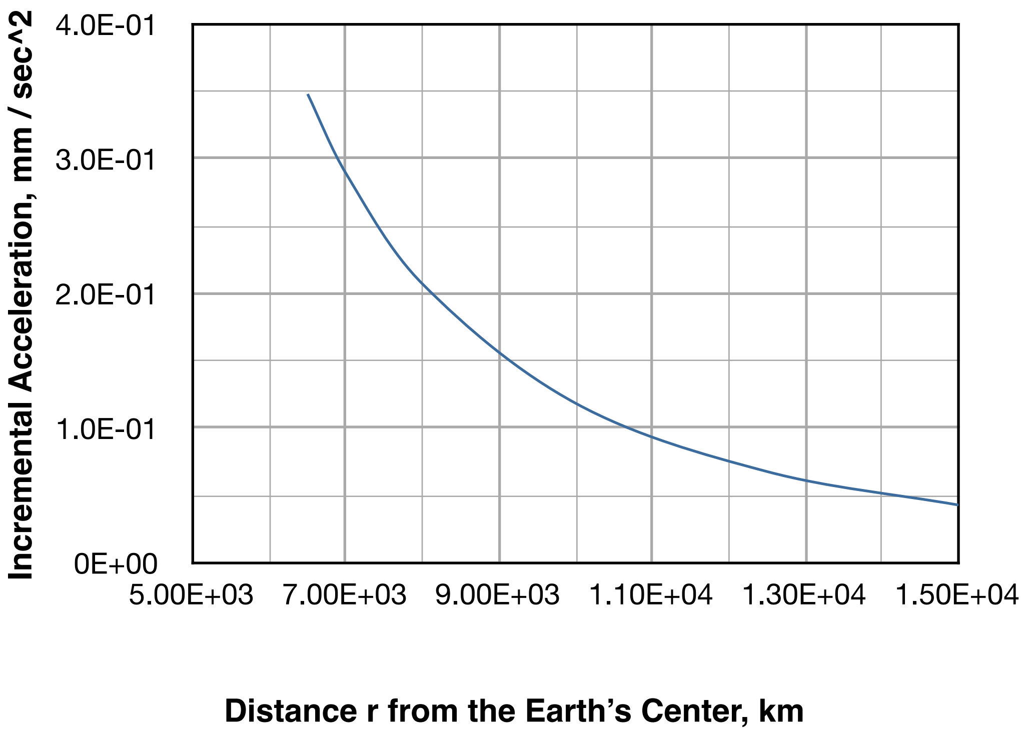
The idea of a flux model seems irrevocable, but only in concert with the existence of the TI model in its role in mediating the gravitational force between masses.

^ Possible Test of the Flux Model

The flux model offers an explanation for the anomalous gains in velocity of spacecraft flown by the Earth in gravity-assist maneuvers. A more detailed discussion of these observations will be found in reference [6].

A number of spacecraft, using Earth for gravity assist maneuvers, have experienced increases in energy unaccounted for by conventional physics. These increases in energy are measured by increases in velocity of the spacecraft of a few mm/sec [7]. The flux model may offer an explanation for these unexpected changes in velocity.

The incremental acceleration profile about the Earth produced by the flux model of gravity is graphed in Figure 1. To determine whether the flux model can account for a particular flyby anomaly, the acceleration at the radius from the Earth of the spacecraft would have to be integrated over the path of the spacecraft.



^ Figure 1. Incremental Acceleration Toward the Earth Predicted by the Flux Model, mm/sec^2

Governing equation: $a_{Incremental} = 2^{1/2}\,(GM)^{3/2} / (\, r^{5/2}\, c)$

^ The Effect of Relativity on the Acceleration of Particles of the TI Field

The iterative process delineated in Appendix B of calculating the infall velocity by integrating the acceleration and then using that value of velocity to calculate a new value of acceleration and so on works well until the velocity approaches a relativistic value. Then we must consider the growing relativistic mass of particles of the TI field. The mass of the particles of the TI field is a measure of their resistance to acceleration. It must be accounted for in our calculations.

If we simply divide the equation for acceleration by the Lorentz factor we get an equation that cannot be readily integrated to give the infall velocity at a given radius from the gravitational body. This dilemma is discussed in Appendix C.

^ Numerical Integration of the Flux Model of Gravity

The intractability of integrating the equation for acceleration derived in Appendix B when the effect of relativistic mass is included forces the calculation of the infall velocity to be done numerically. The procedure for this calculation is shown in Appendix D.

The procedure is not needed for the modest gravitational bodies of the solar system or even for white dwarfs. However, this procedure is needed to calculate the acceleration and velocity profiles about condensed gravitational bodies such as neutron stars and black holes [8].

^ Parameters Used in the Calculations

The parameters that are used in the calculations for this paper of various gravitational bodies of the Solar System and beyond are listed in Appendix E. These parameters were obtained from references [9] and [10].

^ Conclusions

1. The flux model describes a system in which particles of the TI field fall radially toward the central gravitational mass.
2. The flux model conjecture asserts that in a gravitational field, the velocity of particles of the TI field combines with the velocity of outgoing gravitons to increase the flux of gravitons relative to the TI field. Thus the response of the TI field to gravity adds to the force of gravity.
3. Equations defining the acceleration and velocity profiles about a gravitational body are developed.
4. The increased flux of gravitons 'seen' by particles of the TI field causes an increase in acceleration of particles of the TI field in direct proportion with the increased flux.
5. The acceleration profile of particles of the TI field in the vicinity of each planet in the Solar System follows the flux model.
6. The incremental acceleration toward a gravitational body (over and above the Newtonian value) is $2^{1/2}$ $(GM)^{3/2}$ / ($r^{5/2}$ c).
7. Determination of whether the unexpected velocity changes of the flyby anomalies could be caused by the incremental acceleration toward the Earth predicted by the flux model may offer an explanation for these unexpected changes in velocity and validation of the flux model.

^ Appendix A

Graviton Flux is Increased by the Motion of the TI Field Toward a Gravitational Body

The acceleration profile about a gravitational body is given by Eq (1) repeated here as Eq (A-1).

$$a = GM / r^2 \tag{A-1}$$

Particle flux is defined in reference [11] as

> 'Particle flux, the rate of transfer of particles through a unit area ([number of particles] m^{-2} s^{-1})'

The value of acceleration in Eq (A-1) is only a first cut at establishing the acceleration profile about a gravitational body. We must account for the effect on graviton flux on the particles of the TI field as those particles fall into the gravitational body.

The graviton flux at a given distance from a gravitational body is proportional to the gravitational acceleration a at that distance. What graviton flux is 'seen' by particles of the TI field? If particles of the TI field are moving toward the gravitational body the flux of gravitons seen by particles of the TI field should increase in some proportion with the velocity of the TI field particles. Examine now the increase in graviton flux 'seen' by particles of the TI field.

Conduct a thought experiment in which an object moves at constant velocity v through space toward a second object that emits a signal with a frequency of f. Call this signal a marker. The marker signal (say a short pulse of photons) moves at the speed of light c. At the start of our experiment, the emitter object is located at a distance d from the moving object. The question is: how many markers does the moving or receiving object encounter from the start of its trip until it reaches the emitter?

The answer is this: The moving or receiving object encounters all the markers that are propagating toward the object at the start of the trip plus all the markers that are emitted during the trip.

$$\text{Markers encountered} = d / \lambda + d * f / v \tag{A-2}$$

where

d is the distance between the start and the end of the trip.

λ is the wavelength of the marker signal or the distance between markers.

Now, the distance λ between markers is their speed of propagation divided by the frequency of emission:

$$\lambda = c / f \tag{A-3}$$

Substitute Eq (A-3) into Eq (A-2) to give

$$\text{Markers encountered} = d * f / c + d * f / v \tag{A-4}$$

Rewrite Eq (A-4) as

Markers encountered = d f (v + c) / vc (A-5)

If the receiving object had stayed at home for the time of its trip instead of moving toward the emitting object it would have received the number of markers equal to the duration of the trip (not taken) times the frequency of the emitter.

Markers seen by the stay-at-home receiver = d f / v (A-6)

The ratio of markers seen by the moving receiver vs the stationary receiver is given by the value in Eq (A-5) divided by that in Eq (A-6).

Ratio of markers seen by the moving receiver relative to the stationary receiver is

Ratio = (v + c) / c (A-7)

Relativistic effects notwithstanding, the increase in marker flux (if you will) is v / c.

The duration of the trip is

Trip duration T = d / v (A-8)

The frequency at which markers were encountered by the moving object is given by dividing the number of markers encountered during the trip by the duration of the trip.

Marker frequency = (d f (v + c) / vc) / (d / v) (A-9)

Marker frequency = f (v + c) / c (A-10)

Now, think of the emitter object as a gravitational body emitting gravitons at the speed of light and the moving object as a particle or group of particles of the TI field. The marker frequency of Eq (A-10) is analogous to the flux of gravitons encountered by the TI field. The flux of gravitons is increased by the velocity of particles of the TI field toward the gravitational body. The increase in graviton flux over the flux at a stationary point at a given distance from the gravitational body is proportional to v / c.

FluxMoving / FluxStationary = 1 + v / c (A-11)

The increased flux of gravitons ‘seen’ by particles of the TI field causes an increase in acceleration of particles of the TI field in direct proportion with the increased flux. This behavior is the crux of the flux model. (Pardon the rhyme.)

The expression for the acceleration a of particles of the TI field at a distance r from a gravitational mass M, given in Eq (A-1) must now be augmented by the increase in graviton flux caused by the infall velocity of the TI field.

$a = GM / r^2 * (1 + v / c)$ (A-12)

The infall velocity of the TI field is the same magnitude as the escape velocity at a given radius from the gravitational body.

$v_{Infall} = v_{Escape} = (2 GM / r)^{1/2}$ (A-13)

Substitute the value of v_{infall} in Eq (A-13) for v in Eq (A-12) to give

$a = GM / r^2 * [1 + (2 GM / r c^2)^{1/2}]$ (A-14)

The gravitational acceleration expressed in Eq (A-14) is only the second cut at determining the effect of the infall velocity on the graviton flux (and hence the acceleration) on particles of the TI field.

^ Appendix B

Iterative Development of the Acceleration Profile About a Weak Gravitational Body

The flux model of gravity posits the following:

1. The infall velocity of particles of the TI field falling toward a gravitational body is the same magnitude (but opposite in sign) as the escape velocity at any given distance from the body.
2. The velocity of the particles of the TI field toward a gravitational body increases the flux of gravitons 'seen' by these particles and thus increases the gravitational force acting on the particles. The magnitude of the infall velocity of the TI field is increased beyond the value expressed in the classic formula:

   $v_{Infall} = v_{Escape} = (2GM / r)^{1/2}$.
3. We must evaluate the effect of this change. In other words, the increase of the infall velocity increases the gravitational force which then increases the infall velocity and so on.
4. The process to determine the outcome of this feedback problem is to step through the following:
   a. Calculate the acceleration at a given distance from a gravitational body for the current value of the infall velocity.
   b. Integrate the acceleration to yield a new expression for the infall velocity.
   c. Continue back to step a until the contribution of the last iteration is negligible.

The First Iteration of the Development of the Acceleration Profile About a Gravitational Body

Calculate the acceleration at a given distance from a gravitational body for the current value of the infall velocity (1st iteration)

The acceleration a of gravity by a central mass M at a distance r is given by Newtonian mechanics as

$$a = GM / r^2 \tag{B-1}$$

If gravity is mediated by gravitons, then the acceleration in Eq (B-1) is proportional to the flux of gravitons at a distance r from a gravitational body of mass M. The graviton flux seen by particles of the TI field is augmented by the velocity of particles of the TI field at a given radius from the gravitational body. As described in Appendix A the augmentation in graviton flux relative to the value given implicitly in Eq (B-1) is proportional to v / c, where v is the velocity of particles of the TI field toward the gravitational body and c is the velocity of light. Particles of the TI field are thus accelerated as shown in Eq (B-2).

$$a = (GM / r^2) * (v + c) / c \qquad \text{(B-2)}$$

The escape velocity at a distance r from a gravitational body of mass M is given [12] by

$$v_{Escape} = (2\ GM / r)^{1/2} \qquad \text{(B-3)}$$

The magnitude of the infall velocity of particles of the TI field at a distance r is the same as the escape velocity of a mass particle falling from infinity radially toward the gravitational mass at that radius from the barycenter of the gravitational mass.

$$v_{Infall} = v_{Escape} = (2\ GM / r)^{1/2} \qquad \text{(B-4)}$$

The value of v in Eq (B-2) is the value given for v_{Infall} in Eq (B-4). The expression for the acceleration of particles of the TI field in Eq (B-2) becomes Eq (B-5).

$$a_{Total} = (GM / r^2) * (v + c) / c$$

$$a_{Total} = (GM / r^2) * ((\ 2\ GM / r\ )^{1/2} + c\ ) / c$$

$$a_{Total} = (GM / r^2) * (\ 1 + (2\ GM / r\ c^2\ )^{1/2}\ ) \qquad \text{(B-5)}$$

An object in free fall at a distance r from the gravitational mass would experience this same acceleration. [13]

Substitute the expression for the Schwarzschild radius r_S into Eq (B-5) and drop the subscript Total.

$$r_S = 2\ GM / c^2 \qquad \text{(B-6)}$$

$$a = (GM / r^2) * (\ 1 + (r_S / r\ )^{1/2}\ ) \qquad \text{(B-7)}$$

Integrate the acceleration to yield a new expression for the infall velocity (1st iteration continued)

I adapt the procedure of reference [12] for the derivation of the escape velocity for the flux model:

Let a = dv / dt and apply the chain rule:

$$dv / dt = (dv / dr) * (dr / dt) \qquad \text{(B-8)}$$

Now v = dr / dt, so

$$v * dv / dr = GM\ [\ (1 / r^2) + (r_S / r^5)^{1/2}\ ] \qquad \text{(B-9)}$$

$$v * dv = GM\ [\ (1 / r^2) + (r_S / r^5)^{1/2}\ ]\ dr \qquad \text{(B-10)}$$

Now integrate v over the range from v_0 to v(t) where v_0 is the escape velocity and v(t) is the velocity at infinity (it's zero).

The right side of Eq (B-10) is integrated from r_0 to r(t) where r_0 is the radius from the barycenter of the gravitational body and r(t) is infinity.

To simplify the integration, delete the terms that evaluate to zero, namely v(t) and terms with r(t) in the denominator. These are both the upper limits of the integration. The lower limits with the terms v_0 and r_0 are retained.

$$- v_0^2 / 2 = GM [1 / r_0 - r_S^{1/2} / r_0^{3/2} / (- 3/2) \qquad \text{(B-11)}$$

$$- v_0^2 / 2 = GM [1 / r_0 + r_S^{1/2} / r_0^{3/2} / (3/2) \qquad \text{(B-12)}$$

$$- v_0^2 = 2 GM [1 / r_0 + 2/3 (r_S^{1/2} / r_0^{3/2})] \qquad \text{(B-13)}$$

Drop the subscript 0 and move the factor of r outside the brackets. Note that the infall velocity v is the negative of the escape velocity. We want the infall velocity, so drop the minus sign before v in Eq (B-13).

$$v^2 = (2 GM / r) * [1 + 2/3 (r_S / r)^{1/2}] \qquad \text{(B-14)}$$

Take the square root of Eq (B-14)

$$v = \{ (2 GM / r) * [1 + 2/3 (r_S / r)^{1/2}] \}^{1/2} \qquad \text{(B-15)}$$

For $[2/3 (r_S / r)^{1/2}] << 1$, the square root of the bracketed term in Eq (B-15) can be approximated by $[1 + 1/3 (r_S / r)^{1/2}]$. Equation (B-15) becomes:

$$v = (2 GM / r)^{1/2} * [1 + 1/3 (r_S / r)^{1/2}] \qquad \text{(B-16)}$$

The Second Iteration of the Development of the Acceleration Profile About a Gravitational Body

Calculate the acceleration at a given distance from a gravitational body for the current value of the infall velocity (2nd iteration)

Equation (B-16) expresses the infall velocity of particles of the TI field in our first iteration of this value. Previous argument has asserted that the velocity of the TI field toward a gravitational body increases the flux of gravitons on particles of the TI field and accelerates these particles further. The increase in acceleration of particles of the TI field is expressed in Eq (B-17).

$$a = (GM / r^2) * (v + c) / c \qquad \text{(B-17)}$$

or

$$a = (GM / r^2) * (1 + v / c) \qquad \text{(B-18)}$$

Substituting the value of v in Eq (B-16) for v in Eq (B-18) gives

$$a = (GM / r^2) * \{1 + (2\ GM / r)^{1/2} * [\ 1 + 1/3\ (\ r_S / r\ )^{1/2}\ ]\ \ / c\ \} \qquad \text{(B-19)}$$

Move the divisor c inside the second term inside the curly brackets.

$$a = (GM / r^2) * \{1 + (2\ GM / r\ c^2)^{1/2} * [\ 1 + 1/3\ (\ r_S / r\ )^{1/2}\ ]\ \} \qquad \text{(B-20)}$$

Substitute the expression for the Schwarzschild radius r_S into Eq (B-20).

$$r_S = 2\ GM / c^2$$

$$a = (GM / r^2) * \{1 + (r_S / r)^{1/2} * [\ 1 + 1/3\ (r_S / r)^{1/2}\ )\ ]\ \} \qquad \text{(B-21)}$$

Multiply the terms inside the square brackets.

$$a = (GM / r^2) * \{1 + (\ r_S / r\ )^{1/2} + 1/3\ (\ r_S / r\ )\ \} \qquad \text{(B-22)}$$

Multiply the terms inside the curly brackets.

$$a = GM * \{1 / r^2 + (\ r_S / r^5\ )^{1/2} + 1/3\ (\ r_S / r^3\ )\ \} \qquad \text{(B-23)}$$

Integrate the acceleration to yield a new value for the escape velocity (infall velocity) (2nd iteration continued)

Once again I adapt the procedure of reference [12] for the derivation of the escape velocity for the flux model:

B-29

Let a = dv / dt and apply the chain rule:

$$dv / dt = (dv / dr) * (dr / dt) \qquad \text{(B-24)}$$

Now v = dr / dt, so

$$v * dv / dr = GM * \{1 / r^2 + (\ r_S / r^5\ )^{1/2} + 1/3\ (\ r_S / r^3\ )\ \} \qquad \text{(B-25)}$$

or

$$v * dv = GM * [1 / r^2 + (r_S / r^5)^{1/2} + 1/3 (r_S / r^3)] dr \qquad \text{(B-26)}$$

Now integrate v over the range from v_0 to v(t) where v_0 is the escape velocity and v(t) is the velocity at infinity (it's zero).

The right side of Eq (B-26) is integrated from r_0 to r(t) where r_0 is the radius from the barycenter of the gravitational body and r(t) is infinity.

Again, to simplify the integration, delete the terms that evaluate to zero, namely v(t) and terms with r(t) in the denominator. These are both the upper limits of the integration.

The lower limits with the terms v_0 and r_0 are retained.

$$- v_0^2 / 2 = GM * [1 / r_0 - r_S^{1/2} / (- 3/2 r_0^{3/2}) - 1/3 r_S / (-2 r_0^2)] \qquad \text{(B-27)}$$

We want the infall velocity, so drop the minus sign before v in Eq (B-27).

$$v_0^2 / 2 = GM * [1 / r_0 + (2/3 r_S^{1/2} / r_0^{3/2}) + 1/6 r_S / r_0^2] \qquad \text{(B-28)}$$

Drop the subscript 0 and move the 1 / r term outside the brackets.

$$v^2 / 2 = GM / r * [1 + (2/3 (r_S / r)^{1/2} + 1/6 r_S / r] \qquad \text{(B-29)}$$

Multiply by 2 and take the square root.

$$v = (2 GM / r)^{1/2} * [1 + 2/3 (r_S / r)^{1/2} + 1/6 r_S / r]^{1/2} \qquad \text{(B-30)}$$

For those values of r_S / r that allow $2/3 (r_S / r)^{1/2} + 1/6 r_S / r << 1$, Eq (B-30) can be approximated by Eq (31).

$$v = (2 GM / r)^{1/2} * [1 + (1/3) (r_S / r)^{1/2} + (1/12) (r_S / r)] \qquad \text{(B-31)}$$

The Third Iteration of the Development of the Acceleration Profile About a Gravitational Body

Calculate the acceleration at a given distance from a gravitational body for the current value of the infall velocity (3rd iteration)

Equation (B-31) expresses the infall velocity of particles of the TI field in our second iteration of this value. Previous argument has asserted that the velocity of the TI field toward a gravitational body increases the flux of gravitons on particles of the TI field and accelerates these particles further. The increase in acceleration of particles of the TI field is expressed in Eq (B-32).

$$a = (GM / r^2) * (v + c) / c \qquad \text{(B-32)}$$

or

$$a = (GM / r^2) * (1 + v / c) \qquad \text{(B-33)}$$

Substituting the value of v in Eq (B-31) for v in Eq (B-33) gives

$$a = (GM / r^2) * \{1 + (2\ GM / r)^{1/2} * [\ 1 + (1/3)\ (\ r_S / r\ )^{1/2} + (1/12)\ (\ r_S / r\ )\ ] / c\ \} \qquad \text{(B-34)}$$

Move the divisor c inside the second term inside the curly brackets.

$$a = (GM / r^2) * \{1 + (2\ GM / r\ c^2)^{1/2} * [\ 1 + (1/3)\ (\ r_S / r\ )^{1/2} + (1/12)\ (\ r_S / r\ )\ ]\ \} \qquad \text{(B-35)}$$

Substitute the expression for the Schwarzschild radius r_S into Eq (B-35).

$$r_S = 2\ GM / c^2$$

$$a = (GM / r^2) * \{1 + (\ r_S / r\ )^{1/2} * [\ 1 + (1/3)\ (\ r_S / r\ )^{1/2} + (1/12)\ (\ r_S / r\ )\ ]\ \} \qquad \text{(B-36)}$$

Collect terms.

$$a = (GM / r^2) * [\ 1 + (\ r_S / r\ )^{1/2} + (1/3)\ (\ r_S / r\ ) + (1/12)\ (\ r_S / r\ )^{3/2}\ ] \qquad \text{(B-37)}$$

Integrate the acceleration to yield a new value for the escape velocity (infall velocity) (3rd iteration continued)

Move the r^2 term of Eq (B-37) inside the brackets.

$$a = (\ GM\ ) * [\ 1 / r^2 + (r_S^{1/2} / r^{5/2}\ ) + 1/3\ (r_S / r^3\ ) + 1/12\ (r_S^{\ 3/2} / r^{7/2}\ )\ ] \qquad \text{(B-38)}$$

Integrate acceleration over time to obtain the infall velocity.

I again adapt the procedure of reference [12] for the derivation of the escape velocity for the flux model:

Let a = dv / dt and apply the chain rule:

$$dv / dt = (dv / dr) * (dr / dt)$$

Now v = dr / dt, so

$$v * dv / dr = GM * [\ 1 / r^2 + (r_S^{1/2} / r^{5/2}\ ) + 1/3\ (r_S / r^3\ ) + 1/12\ (r_S^{\ 3/2} / r^{7/2}\ )\ ] \qquad \text{(B-39)}$$

$$v * dv = GM * [\ 1 / r^2 + (r_S^{1/2} / r^{5/2}\ ) + 1/3\ (r_S / r^3\ ) + 1/12\ (r_S^{\ 3/2} / r^{7/2}\ )\ ]\ dr \qquad \text{(B-40)}$$

Now integrate v over the range from v_0 to v(t) where v_0 is the escape velocity and v(t) is the velocity at infinity (it's zero).

The right side of Eq (B-40) is integrated from r_0 to r(t) where r_0 is the radius from the barycenter of the gravitational body and r(t) is infinity.

Again, to simplify the integration, delete the terms that evaluate to zero, namely v(t) and terms with r(t) in the denominator. These are both the upper limits of the integration.

The lower limits with the terms v_0 and r_0 are retained.

$$- v_0^2 / 2 = GM * [1 / r_0 + 2/3\, r_S^{1/2} / r_0^{3/2} + 1/6\, r_S / r_0^2 + 1/30\, r_S^{3/2} / r_0^{5/2}] \qquad \text{(B-41)}$$

We want the infall velocity, so drop the minus sign before v in Eq (B-41). Drop the subscript 0 and move the factor 1 / r outside the brackets.

$$v^2 = (2\, GM / r) * [1 + 2/3\, r_S^{1/2} / r^{1/2} + 1/6\, r_S / r + 1/30\, r_S^{3/2} / r^{3/2})] \qquad \text{(B-42)}$$

Collect terms.

$$v^2 = (2\, GM / r) * [1 + 2/3 (r_S / r)^{1/2} + 1/6 (r_S / r) + 1/30 (r_S / r)^{3/2}] \qquad \text{(B-43)}$$

Take the square root.

$$v = (2\, GM / r)^{1/2} * [1 + 2/3 (r_S / r)^{1/2} + 1/6 (r_S / r) + 1/30 (r_S / r)^{3/2}]^{1/2} \qquad \text{(B-44)}$$

For modest gravitational systems in which

$$2/3 (r_S / r)^{1/2} + 1/6 (r_S / r) + 1/30 (r_S / r)^{3/2} << 1,$$

the square root inside the brackets of Eq (B-44) can be approximated.

$$v = (2\, GM / r)^{1/2} * [1 + 1/3 (r_S / r)^{1/2} + 1/12 (r_S / r) + 1/60 (r_S / r)^{3/2}] \qquad \text{(B-45)}$$

Augment the acceleration through the infall velocity.

$$a = (GM / r^2) * (v + c) / c \qquad \text{(B-46)}$$

$$a = (GM / r^2) * (1 + v / c) \qquad \text{(B-47)}$$

$$a = (GM / r^2) * \{ 1 + (2\, GM / r)^{1/2} * [1 + 1/3 (r_S / r)^{1/2} + 1/12 (r_S / r) + 1/60 (r_S / r)^{3/2}] / c \} \qquad \text{(B-48)}$$

Move the divisor c inside the second term inside the curly brackets.

$$a = (GM / r^2) * \{ 1 + (2 GM / r c^2)^{1/2} * [1 + 1/3 (r_S / r)^{1/2} + 1/12 (r_S / r) + 1/60 (r_S / r)^{3/2}] \} \tag{B-49}$$

Substitute the expression r_S / r for $(2 GM / r c^2)^{1/2}$ and multiply by the terms inside the square brackets.

$$a = (GM / r^2) * [1 + (r_S / r)^{1/2} + 1/3 (r_S / r) + 1/12 (r_S / r)^{3/2} + 1/60 (r_S / r)^2] \tag{B-50}$$

The progression of terms is clear and the next term in the sequence can be included.

$$a = (GM / r^2) * [1 + (r_S / r)^{1/2} + 1/3 (r_S / r) + 1/12 (r_S / r)^{3/2} + 1/60 (r_S / r)^2 + 1/360 (r_S / r)^{5/2}] \tag{B-51}$$

This series converges so rapidly that the term $1/360 (r_S / r)^{5/2}$ is negligible and is not included in evaluating the acceleration.

The expressions for the escape velocity and acceleration from a gravitational body derived for the flux model of gravity comprise an infinite series with declining values of the elements in the series. The expressions for the acceleration and infall velocity of the TI field for a gravitational body can be written in series form as follows:

$$a = (GM / r^2) * [1 + \textstyle\sum \text{(from } n = 1 \text{ to } \infty) [(2 / (n+1)! * (r_S / r)^{n/2}] \tag{B-52}$$

$$v = (2 GM / r)^{1/2} * [1 + \textstyle\sum \text{(from } n = 1 \text{ to } \infty) [(2 / (n+2)! * (r_S / r)^{n/2}] \tag{B-53}$$

The sequence and results of these iterations are listed in Table 1 in the main section of this paper.

Without the augmentation of the gravitational force by the infall velocity of the TI field, the acceleration profile about a gravitational body is expressed by Eq (B-1).

The effect of the infall velocity on the acceleration of particles of the TI field toward a gravitational body is given by the term inside the square brackets of Eq (B-51). I call this term the acceleration multiplier. The value of this factor is quite small at the surfaces of the modest gravitational bodies in the solar system.

Determine the Incremental Acceleration Produced by the Flux Model of Gravity

The Newtonian acceleration profile about a mass M is given in Eq (B-1) and by the first term in Eq (B-51) that is repeated below as Eq (B-54).

$$a_{Newtonian} = GM / r^2 \qquad \text{(B-54)}$$

For the modest gravitational bodies of the Solar System, the terms in Eq (B-51) beyond the first two contribute only negligible acceleration. Accordingly, the incremental acceleration produced by the flux model is obtained from the second term in the parenthesis of Eq (B-51) where $r_S / r = (2GM / r\, c^2)$.

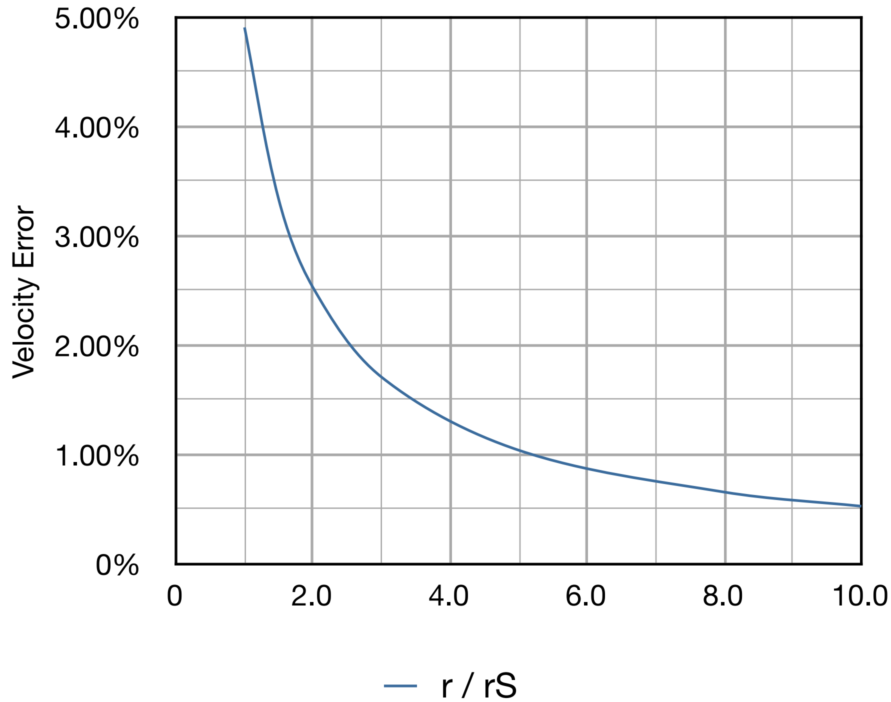
$$a_{Incremental} = (GM / r^2) * (2GM / r\, c^2)^{1/2} \qquad \text{(B-55)}$$

Writing Eq (B-55) more compactly:

$$a_{Incremental} = 2^{1/2}\, (GM)^{3/2} / (\, r^{5/2}\, c^2) \qquad \text{(B-56)}$$

Error Introduced by the Square Root Approximation

Figure B-1 graphs the error introduced by the square root approximation of Eqs (B-16), B-31) and (B-45) vs r / r_S. For values of r / r_S greater than 10, the error is less than 0.5%.



^ Figure B-1. Velocity Error Introduced by the Square Root Approximation of
Equations (B-16), (B-31) and (B-45)

^ Appendix C

The Effect of Relativity on the Acceleration of Particles of the TI Field

As the speed of particles falling toward the gravitational body increases, the relativistic mass of the particles increases by the factor of Eq (C-1). Relativistic mass is a measure of the resistance of particles of the TI field to acceleration at very high velocity [14].

$$m_{Rel} / m_{Rest} = 1 / (1 - v^2 / c^2)^{1/2} \qquad \text{(C-1)}$$

where

m_{Rel} is the relativistic mass.

m_{Rest} is the rest mass.

v is the velocity of the particle mass.

c is the velocity of light.

The infall acceleration of particles of the TI field is expressed in Eq (B-51), repeated below as Eq (C-2).

$$a = (GM / r^2) * [(1 + (r_S / r)^{1/2} + 1/3 (r_S / r) + 1/12 (r_S / r)^{3/2} + 1/60 (r_S / r)^2 + 1/360 (r_S / r)^{5/2}] \qquad \text{(C-2)}$$

The acceleration expressed in Eq (C-2) must be reduced by the factor of Eq (C-1).

$$a = (GM / r^2) * [(1 + (r_S / r)^{1/2} + 1/3 (r_S / r) + 1/12 (r_S / r)^{3/2} + 1/60 (r_S / r)^2 + 1/360 (r_S / r)^{5/2}] * (1 - v^2 / c^2)^{1/2} \qquad \text{(C-3)}$$

Integration of this equation to obtain the infall velocity is intractable. The infall velocity can be obtained from Eq (C-3) by numerical integration. This process is described in Appendix D.

^ Appendix D
Numerical Integration of the Flux Model of Gravity

^ Table D-1. Algorithm for the Numerical Integration

Step	Initialization **
1	Initialize r(0)
2	$a = (GM / r^2) * [(1 + (r_S / r)^{1/2} + (1/3) (r_S / r) + (1/12) (r_S / r)^{3/2} + (1/60) (r_S / r)^2 + (1/360) (r_S / r)^{5/2}]$
3	$v = (2\ GM / r)^{1/2} * [1 + (1/3) (r_S / r)^{1/2} + (1/12) (r_S / r) + (1/60) (r_S / r)^{3/2} + (1/360) (r_S / r)^2]$
4	deltaT = 0
Step	**Computation**
1	$a(i+1) = (GM / r(i)^2) * (1 + v(i)/c) * (1 - v(i)^2 / c^2)^{1/2}$
2	Enter a value for deltaT
3	deltaV(i+1) = a(i) * deltaT
4	v(i+1) = v(i) + deltaV(i+1)
5	$deltaR(i+1) = v(i) * deltaT + 0.5\ [a(i) * deltaT^2]$
6	r(i+1) = r(i) - deltaR(i+1)
7	Continue until the singularity is as close as intended for the analysis.
8	Go To Computation Step 1

** I use the notation r_S ($r_S = 2\ GM / c^2$) for brevity even though reference [8] asserts that a black hole has no event horizon.

^ Appendix E
Parameters Used in the Calculations

^ Table E-1. Parameters Used in the Calculations [9] [10]

Body	Parameter	Value	Units
Sun	GM	1.327E+11	km^3 / sec^2
Sun	Radius, r	6.960E+05	km
Mercury	GM	2.203E+04	km^3 / sec^2
Mercury	Radius, r	2.440E+03	km
Venus	GM	3.249E+05	km^3 / sec^2
Venus	Radius, r	6.052E+03	km
Earth	GM	3.986E+05	km^3 / sec^2
Earth	Radius, r	6.371E+03	km
Mars	GM	4.283E+04	km^3 / sec^2
Mars	Radius, r	3.390E+03	km
Jupiter	GM	126 686,534	km^3 / sec^2
Jupiter	Radius, r	6.991E+04	km
Saturn	GM	3.793E+07	km^3 / sec^2
Saturn	Radius, r	5.823E+04	km
Uranus	GM	5.794E+06	km^3 / sec^2
Uranus	Radius, r	2.536E+04	km
Neptune	GM	6.837E+06	km^3 / sec^2
Neptune	Radius, r	2.462E+04	km
White Dwarf (1 solar mass)	GM	1.327E+11	km^3 / sec^2

Body	Parameter	Value	Units
White Dwarf (1 solar mass)	Radius, r	5.000E+03	km

^ References

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