Relativistic Newton law solved cosmological problems

(Draft)

In this work we found a correction of special relativity and verified the relation between inertial mass, gravitational mass and rest mass. Also we found a generalization of Newton's law of gravitation and the Logarithmic gravitational potential responsible for the flat tendency of the rotation curves in spiral galaxies. We found the cosmological constant and the vacuum energy density in term of gravitational radius .we found spatial mass distribution of dark matter haloes and the condition appears in the circular velocity curve predicted from Navarro–Frenk–White, and the planets mass defect (sun-earth). We found new equation to Hawking radiation in term of curvature. We found the geodetic drift of the planets and the value of $\theta = 6.67528$ arc sec for the earth. Finally we explained the notion of Hypermass and Hyperscale and how it can be used to find MOND.

The relativistic kinetic energy is

$$K = mc^2 - m_0 c^2 \qquad (1)$$

Where m_0 the rest mass and m the relativistic mass.

Multiplying equation (1) by $\frac{1}{2}\frac{v^2}{c^2}$ and substitute the parameter, $\Xi = \frac{1}{2} m_0 v^2$ (Ξ has dimension of energy) one finds

$$\Xi = \frac{1}{2}\frac{v^2}{c^2}(mc^2 - K) \qquad (2)$$

For gravitating circular motion we have $v = \sqrt{\frac{GM}{r}}$ and the parameter $\Xi = \frac{Gm_0 M}{2r}$.

By rearranging equation (2) and substituting the values of $v = \sqrt{\frac{GM}{r}}$ and $\Xi = \frac{Gm_0 M}{2r}$, one finds the relativistic Newton's law of gravitation

$$F_g = \frac{mv^2}{r} = \frac{Gm_0 M}{r^2} + \frac{K}{r}\frac{v^2}{c^2} = \frac{Gm_0 M}{r^2} + \frac{m}{2r}\frac{v^4}{c^2} \qquad (3)$$

This can be solved to find the velocity of the object in terms of its orbital distance

$$v^2 = c^2\left[1 \pm \sqrt{1 - \frac{m_0}{m}\frac{R_s}{r}}\right] \qquad (4)$$

Where $R_s = \frac{2GM}{c^2}$ (Schwarzschild radius).

Equation (4) gives the relativistic equation for mass as

$$m = \gamma m_0 \qquad (5)$$

Where, $\gamma = \frac{R_S}{r\left(2\frac{v^2}{c^2}-\frac{v^4}{c^4}\right)} = \frac{g_0}{g\left(1-\frac{v^2}{2c^2}\right)} = \frac{1}{\left(1-\frac{v^2}{2c^2}\right)} = \frac{1}{1-\frac{R_S}{4r}}$ (6)

And $g_0 = \frac{GM}{r^2}$, $g = \frac{v^2}{r}$. One finds

$$mg = \frac{m_0 g_0}{\left(1-\frac{v^2}{2c^2}\right)} \quad (7)$$

In non-relativistic limit, $mg = m_0 g_0$.

Equation (7) represents the relation between inertial mass and gravitational mass. Gravitational mass is greater than the rest mass m_0.Also the factor γ is less than γ of special relativity $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$ with factor of $\left(\frac{R_S}{4r}\right)^2$ or $\frac{v^4}{4c^4}$.

The correction term $\frac{v^4}{4c^4}$ should be added to special relativity

$$\gamma_c = \frac{1}{\sqrt{1-\frac{v^2}{c^2}+\frac{v^4}{4c^4}}} \quad (8)$$

Therefore, gravitational mass m_g is given by

$$m_g = \frac{m_0}{1-\frac{R_S}{4r}} \quad (9)$$

We can expand the right hand side term to find

$$m_g = m_0\left(1 + \frac{R_s}{4r} + \left(\frac{R_s}{4r}\right)^2 + \left(\frac{R_s}{4r}\right)^3 + \cdots\right) \quad (10)$$

Tolman(R.C. Tolman, Relativity Thermodynamics and Cosmology, Oxford University Press, Oxford (1934).),define the gravitational mass density within the spherical body required both the energy density ε and the pressure P according to

$$\rho_m = \frac{\varepsilon_0+3P}{c^2} = \rho_{m0} + \frac{3P}{c^2} \quad (11)$$

Where ρ_m is the matter density.

Comparing equation (10) and (11)

$$PV = \frac{m_0 c^2}{3}\left[\frac{R_s}{4r} + \left(\frac{R_s}{4r}\right)^2 + \left(\frac{R_s}{4r}\right)^3 + \cdots\right] = \rho_\Lambda V \quad (12)$$

Where the vacuum energy density

$$\rho_\Lambda = \frac{m_0 c^2}{3V}\left[\frac{R_s}{4r} + \left(\frac{R_s}{4r}\right)^2 + \left(\frac{R_s}{4r}\right)^3 + \cdots\right] \quad (13)$$

On the other hand, the contribution of the cosmological constant in Einstein equation is

$$T_{\mu\nu}^{vac} = \frac{\Lambda c^4}{8\pi G} g_{\mu\nu} \qquad (14)$$

The quantity $\frac{\Lambda c^4}{8\pi G}$ has dimension of pressure. This leads to

$$\frac{\Lambda c^4}{8\pi G} = \frac{m_0 c^2}{3V}\left[\frac{R_s}{4r} + \left(\frac{R_s}{4r}\right)^2 + \left(\frac{R_s}{4r}\right)^3 + \cdots\right]$$

$$\Lambda = \frac{8\pi G m_0}{3c^2 V}\left[\frac{R_s}{4r} + \left(\frac{R_s}{4r}\right)^2 + \left(\frac{R_s}{4r}\right)^3 + \cdots\right] \approx \frac{2\pi G m_0}{3c^2 V}\frac{R_s}{r} = \frac{2\pi G \rho_0}{3c^4}\frac{R_s}{r} \qquad (15)$$

When $r \gg R_s$. ρ_0 the rest energy density.

Equation (15) gives the acceptable value of the cosmological constant ,with ($m_0 \sim 10^{52} kg, V \sim 10^{78} m^3, R_s \sim r$) that $\Lambda \sim 10^{-52} m^{-2}$.This value agrees with observations and measurements obtained by the High-Z Supernova Team and the Supernova Cosmological Project.(

Garnavich, P.M., *et al.*, *Astrophys. J.* **493**, L53 (1998). (astro-ph/9710123)

Schmidt, B.P., *et al.*, *Astrophys. J.* **507**, 46 (1998). (astro-ph/9805200)

Riess, A.G., *et al.*, *Astronom. J.* **116**, 1009 (1998). (astro-ph/9805201)

Garnavich, P.M., *et al.*, *Astrophys. J.* **509**, 74 (1998). (astro-ph/9806396)

Perlmutter, S., *et al.*, *Astrophys. J.* **483**, 565 (1997). (astro-ph/9608192)

Perlmutter, S., *et al.*, *Nature* **391**, 51 (1998). (astro-ph/9712212)

Perlmutter, S., *et al.*, *Astrophys. J.* **517**, 565 (1999). (astro-ph/9812133))

We can estimate the mass defect ratio as

$$\frac{\Delta m}{m_0} = \frac{R_s}{4r - R_s} \qquad (16)$$

This ratio in a good agreement with Z.Chylinski,1993(by Z.Chylinski,gravity and non-extensive nature of mass,Acta physica polonica B, Vol.24,1993) for sun-earth mass defect.

Employing equations (4) and (3), the force is given by

$$F_g = \frac{G m_0 M}{r^2} + \frac{K}{r}\left[1 - \sqrt{1 - \frac{m_0}{m}\frac{R_s}{r}}\right] \qquad (17)$$

Finally, the gravitational force becomes

$$F_g = \frac{G m_0 M}{r^2} + \frac{mc^2}{2r}\left[1 - \sqrt{1 - \frac{m_0}{m}\frac{R_s}{r}}\right]^2 = \frac{mc^2}{r}\left[1 - \sqrt{1 - \frac{m_0}{m}\frac{R_s}{r}}\right] \qquad (18)$$

Expanding the square root in (18) one gets

$$F_g = \frac{mc^2}{r}\left[1 - \left(1 - \frac{m_0}{2m}\frac{R_s}{r} + \eta\right)\right] = \frac{G m_0 M}{r^2} - \eta\frac{mc^2}{r} \qquad (19)$$

The perturbation term $\eta = -\frac{1}{8}\left(\frac{m_0}{m}\frac{R_s}{r}\right)^2 + \cdots + \frac{-\frac{1}{4}\cdots\left(\frac{3}{2}-n\right)}{n!}\left(-\frac{m_0}{m}\frac{R_s}{r}\right)^n + \cdots$ is always negative, which provides additional force to Newtonian force.

Equation (19) is a generalization of Newton's law of gravitation that should be used in studying any gravitational interaction of gravitating bodies.

This force can be associated with a central potential energy U_g of the form

$$\overrightarrow{F_g} = -\vec{\nabla} U_g \qquad (20)$$

This yields a gravitational potential energy

$$U_g = mc^2 \ln\left[\frac{e^2}{2}\left(1 - \frac{m_0}{2m}\frac{R_s}{r} + \sqrt{1 - \frac{m_0}{m}\frac{R_s}{r}}\right)\left(e^{-2\sqrt{1-\frac{m_0 R_s}{m\ r}}}\right)\right] = mc^2 \ \ln \xi \qquad (21)$$

Logarithmic gravitational potential responsible for the flat tendency of the rotation curves in spiral galaxies.

Equation (21) satisfies the condition that the potential energy vanished when $r \to \infty$ or $m_0 \to 0$.

On the other hand, when $r \to R_s$ the potential depends on relativistic ratio of mass only.

The mass can be found by ordinary relation to mass density ρ_m

$$M = \frac{4}{3}\pi r^3 \rho_m \qquad (22)$$

From above discussion the mass can be related to Schwarzschild's radius R_s

$$M(R) = \frac{c^2}{2G} R_s \qquad (23)$$

It is clear that $M \to M(R)$ at the limit $r \to R_s$ (V.E. Kuzmichev, V.V. Kuzmichev, Low-velocity cosmic strings in accelerating universe, arXiv:1111.0172v2 [astro-ph.CO] 15 Mar 2012) and references there in. This implies $\frac{4}{3}\pi r^2 \rho_m = \frac{c^2}{2G}$ which can be reduced to

$$\frac{16\pi G\rho}{c^4} = \frac{6}{r^2} = \mathcal{R} \qquad (24)$$

Where $\mathcal{R}$ is the scalar curvature and $\rho = \rho_m c^2$ is the energy density.

Equation (24) agrees with the observational constraints on the parameter K in the model of K-matter obtained by Kolb [E.W. Kolb, Astrophys. J. 344, 543 (1989).] and Gott and Rees [J.R. Gott and M.J. Rees, MNRAS 227, 453 (1987).].It gives the scalar curvature of 3-sphere (sphere in 4-dimensional Euclidean space).It can be compared with Einstein field equation with cosmological constant

$-\mathcal{R} + 4\Lambda = \frac{8\pi G\rho}{c^4}$, finding the cosmological constant as

$$\Lambda = \frac{6\pi G\rho_m}{c^2} \qquad (25)$$

Equation (25) gives the exact value of the cosmological constant in agreement with equation (15).

Equation (25) implies that

$\rho_\Lambda = \frac{3}{4}\rho_m$ (26)

Equation (26) shows the coincidence between matter and vacuum energies. The nearly equal contribution of matter and dark energy to the total energy of the universe at the present era is required to solve the coincidence problem.

Spatial mass distribution of dark matter haloes:

We can write

$$\Lambda = \frac{6\pi G\rho_m}{c^2} = \frac{2\pi G\rho_{m0}}{3c^2}\frac{R_s}{r}\left[1 + \left(\frac{R_s}{4r}\right) + \left(\frac{R_s}{4r}\right)^2 + \left(\frac{R_s}{4r}\right)^3 + \cdots\right] \qquad (27)$$

And mass density as

$$\rho_m = \frac{\rho_{m0}}{9}\frac{R_s}{r}\left[1 + \left(\frac{R_s}{4r}\right) + \left(\frac{R_s}{4r}\right)^2 + \left(\frac{R_s}{4r}\right)^3 + \cdots\right] \qquad (28)$$

Compare equation (28) with the density of dark matter of Navarro–Frenk–White profile ,

$$\rho(r) = \frac{r_s}{r}\frac{\rho_0}{\left(1+\frac{r}{r_s}\right)^2} \qquad (29)$$

one finds,

$$r \le 2r_s \qquad (30)$$

This condition appears in the circular velocity curve predicted from(Navarro–Frenk–White) (astro-ph/9508025 7 Aug 1995),the circular velocity peaks at $r_{max} \approx 2r_s$.

Cosmic microwave background temperature:

Hawking radiation [S.W. Hawking, \Particle Creation by Black Holes ", Comm. Math. Phys. 43, 199 (1975)] is the thermal radiation emitted by a black hole of mass, M. It occurs as a consequence of placing quantum fields in the gravitational background of a black hole. An observer who stays at a fixed distance, R, from a black hole of mass, M, will measure a temperature given by(D. Singleton and S. Wilburn, \Hawking Radiation, Unruh Radiation, and the Equivalence Principle", Phys. Rev. Lett. 107, 081102 (2011));

$$T_H = \left(\frac{\hbar c}{GM}\right)\left(\frac{c^2}{k_B}\right)\left(\frac{1}{8\pi\sqrt{1-\frac{R_s}{r}}}\right) \qquad (31)$$

Equation (10) gives the temperature as

$$T = \left(\frac{m_0 R_s}{r}\right)\left(\frac{c^2}{k_B}\right)\left(\frac{1}{12}\left(1 + \frac{R_s}{4r} + \left(\frac{R_s}{4r}\right)^2 + \left(\frac{R_s}{4r}\right)^3 + \cdots\right)\right) \qquad (32)$$

By the comparison equation (31) and (32), one finds $\frac{\hbar c}{GM} = \frac{m_0 R_s}{r}$,which gives the mass

$$m_0 = \frac{2\hbar r}{{R_s}^2 c} = \frac{\hbar\kappa}{c} \qquad (33)$$

And the background temperature is

$$T_v = \frac{\hbar c\kappa}{3k_B} \qquad (34)$$

And the curvature κ is

$$\kappa = \frac{2r}{{R_s}^2} \qquad (35)$$

The term $\frac{R_s{}^2}{2r}$ represents the departure from flat space.
And the background radiation frequency is

$$\nu = \frac{2cr}{R_s{}^2} = c\kappa \qquad (36)$$

The temperature of background due to sun according to equation (34)is
$T_\nu \approx 2.7\ k$ when $\kappa \approx 3535\ m^{-1}$, $r \approx 1.5 \times 10^{10}\ m$.

Geodetic drift:

We can calculate the geodetic drift by

$$\theta = \tan^{-1}\left(\frac{180R_s}{2\pi[2n+1]r}\right)\{radian\} \qquad (37)$$

Where $n = \pm 1$ from earth.

Planet	N	r(m)	θ(arc sec)	
Mercury	1	5.7909227×10^{10}	5.74848	
Venus	-1	$1.08209475 \times 10^{11}$	-9.22849	5.74848-9.22849+6.67528 =3.19527
Earth	0	$1.49598262 \times 10^{11}$	6.67528	
Mars	1	$2.27943824 \times 10^{11}$	1.46031	5.74848-9.22849+6.67528+ 1.46031 =4.65527
Jupiter	-1	$7.78340821 \times 10^{11}$	-1.283 , 6.68064*	-1.283+5.74848-9.22849 +6.6752+1.46031=3.37227
Saturn	1	$1.426666422 \times 10^{12}$	0.2333	5.74848-3.19527+6.67528+17.0793-6.8525+0.2333=19.45529
Uranus	-1	$2.870658186 \times 10^{12}$	-0.347868 0.85452*	
Neptune	1	$4.498396441 \times 10^{12}$	0.074	
Pluto	-1	5.914×10^{12}	-0.168867	

* theta from own mass.

This leads to relativistic mass relation

$$m = m_0(1 + \tan\theta) \qquad (38)$$

For earth one finds $\tan\theta = 3.236 \times 10^{-5}$ .that means the mass of earth is heavier by 3.236×10^{-3}% of the estimation of the International Astronomical Union.
If the normal matter in universe is 4.9% ,we find $R_s = 0.71R$, R the radius of universe.

The notion of Hypermass and Hyperscale

Now, farther step can be moved by comparing Planck mass with the Hypermass

$$M(R) = \frac{c^2}{2G}\frac{4}{3}\pi\, R \qquad (39)$$

Without identified it with Schwarzschild's radius.

Planck mass is a threshold mass connecting microscopic scale, where $m < m_P$, with macroscopic scale, where $m > m_P$. In the same manner Hypermass, where the value of the mass, $m > \frac{2\pi c^2}{3G}$, can be treated as a threshold mass connecting macroscopic scale with Hyperscale. Therefore, the transformation that should be constructed to interpret the equations in macroscopic scale to Hyperscale according to equations (22) and (23) is

$$M = \frac{4}{3}\pi r^3 \rho_m \rightarrow \frac{c^2}{2G}\frac{4}{3}\pi R \qquad (40)$$

The same result was proposed by Z.Chylinski,1993(by Z.Chylinski,gravity and non-extensive nature of mass,Acta physica polonica B, Vol.24,1993),he showed that for large objects, the non-extensive character of mass in gravitational interaction affecting coupling constant due the mass defect.

To make sense to the transformation (40), in Newton law the acceleration is given by $a = \frac{GM}{r^2}$ .by using the transformation, one gets

$$a = \frac{2\pi c^2}{3r^2} R \qquad (41)$$

Squaring equation (41) yields

$$a^2 = \frac{4\pi^2 c^2}{9r} \frac{c^2}{r^3} R^2 \qquad (42)$$

Equation (39) construes to MOND acceleration $a^2 = a_0 a_N$, in case of $r = R \to R_s$. Where $a_0 = \frac{8\pi^2 c^2}{9R_s}$ and a_N is Newtonian acceleration.