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# Why TGD and What TGD is?

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# January 27, 2014

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#### Abstract

This piece of text was written as an attempt to provide a popular summary about TGD. This is of course mission impossible since TGD is something at the top of centuries of evolution which has led from Newton to standard model. This means that there is a background of highly refined conceptual thinking about Universe so that even the best computer graphics and animations fail to help. One can still try to create some inspiring impressions at least. This chapter approaches the challenge by answering the most frequently asked questions. Why TGD? How TGD could help to solve the problems of recent day theoretical physics? What are the basic princples of TGD? What are the basic guidelines in the construction of TGD?

These are examples of this kind of questions which I try to answer in using the only language that I can talk. This language is a dialect of the language used by elementary particle physicists, quantum field theorists, and other people applying modern physics. At the level of practice involves technically heavy mathematics but since it relies on very beautiful and simple basic concepts, one can do with a minimum of formulas, and reader can always to to Wikipedia if it seems that more details are needed. I hope that reader could catch the basic principles and concepts: technical details are not important. And I almost forgot: problems! TGD itself and almost every new idea in the development of TGD has been inspired by a problem.

# 1 Introduction

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#### 1.1 Why TGD?

The first question is "Why TGD?". The attempt to answer this question requires overall view about the recent state of theoretical physics.

Obviously standard physics plagued by some problems. These problems are deeply rooted in basic philosophical - one might even say ideological - assumptions which boil down to -isms like reductionism, materialism, determinism, and locality.

Thermodynamics, special relativity, and general relativity involve also postulates, which can be questioned. In thermodynamics second law in its recent form and the assumption about fixed arrow of thermodynamical time can be questions since it is hard to understand biological evolution in this framework. Clearly, the relationship between the geometric time of physics and experienced time is poorly understood. In general relativity the beautiful symmetries of special relativity are in principle lost and by Noether's theorem this means also the loss of classical conservation laws, even the definitions of energy and momentum are in principle lost. In quantum physics the basic problem is that the non-determinism of quantum measurement theory is in conflict with the determinism of Schrödinger equation.

Standard model is believed to summarize the recent understanding of physics. The attempts to extrapolate physics beyond standard model are based on naive length scale reductionism and have products Grand Unified Theories (GUTs), supersymmetric gauge theories (SUSYs). The attempts to include gravitation under same theoretical umbrella with electroweak and strong interactions has led

to super-string models and M-theory. These programs have not been successful, and the recent dead end culminating in the landscape problem of super string theories and M-theory could have its origins in the basic ontological assumptions about the nature of space-time and quantum.

# 1.2 How could TGD help?

The second question is "Could TGD provide a way out of the dead alley and how?". The claim is that is the case. The new view about space-time as 4-D surface in certain fixed 8-D space-time is the starting point motivated by the energy problem of general relativity and means in certain sense fusion of the basic ideas of special and general relativities.

This basic idea has gradually led to several other ideas. Consider only the identification of dark matter as phases of ordinary matter characterized by non-standard value of Planck constant, extension of physics by including physics in p-adic number fields and assumed to describe correlates of cognition and intentionality, and zero energy ontology (ZEO) in which quantum states are identified as counterparts of physical events. These new elements generalize considerably the view about space-time and quantum and give good hopes about possibility to understand living systems and consciousness in the framework of physics.

#### 1.3 Two basic visions about TGD

There are two basic visions about TGD as a mathematical theory. The first vision is a generalization of Einstein's geometrization program from space-time level to the level of "world of classical worlds" identified as space of 4-surfaces. There are good reasons to expect that the mere mathematical existence of this infinite-dimensional geometry fixes it highly uniquely and therefore also physics. This hope inspired also string model enthusiasts before the landscape problem forcing to give up hopes about predictability.

Second vision corresponds to a vision about TGD as a generalized number theory having three separate threads.

- 1. The inspiration for the first thread came from the need to fuse various p-adic physics and real physics to single coherent whole in terms of principle that might be called number theoretical universality.
- 2. Second thread was based on the observation that classical number fields (reals, complex numbers, quaternions, and octonions) have dimensions which correspond to those appearing in TGD. This led to the vision that basic laws of both classical and quantum physics could reduce to the requirements of associativity and commutativity.
- 3. Third thread emerged from the observation that the notion of prime (and integer, rational, and algebraic number) can be generalized so that infinite primes are possible. One ends up to a construction principle allowing to construct infinite hierarchy of infinite primes using the primes of the previous level as building bricks at new level. Rather surprisingly, this procedure is structurally identical with a repeated second quantization of supersymmetric arithmetic quantum field theory for which elementary bosons and fermions are labelled by primes. Besides free many-particle states also the analogs of bound states are obtained and this means the situation really fascinating since it raises the hope that the really hard part of quantum field theories understanding of bound states could have number theoretical solution.

It is not yet clear whether both great visions are needed or whether either of them is in principle enough. In any case their combination has provided a lot of insights about what quantum TGD could be.

#### 1.4 Guidelines in the construction of TGD

The construction of new physical theory is slow and painful task but leads gradually to an identification of basic guiding principles helping to make quicker progress. There are many such guiding principles.

- "Physics is uniquely determined by the existence of WCW" is a conjecture but motivates highly interesting questions. For instance: "Why  $M^4 \times CP_2$  a unique choice for the imbedding space?", "Why space-time dimension must be 4?", etc...
- Number theoretical Universality is a guiding principle in attempts to realize number theoretical vision, in particular the fusion of real physics and various p-adic physics to single structure.
- The construction of physical theories is nowadays to a high degree guesses about the symmetries of the theory and deduction of consequences. The very notion of symmetry has been generalized in this process. Super-conformal symmetries play even more powerful role in TGD than in super-string models and gigantic symmetries of WCW in fact guarantee its existence.
- Quantum classical correspondence is of special importance in TGD. The reason is that where classical theory is not anymore an approximation but in well-defined sense exact part of quantum theory.

There are also more technical guidelines.

- Strong form of General Coordinate invariance (GCI) is very strong assumption. Already GCI leads to the assumption that Kähler function is Kähler action for a preferred extremal defining the counterpart of Bohr orbit. Even in a form allowing the failure of strict determinism this assumption is very powerful. Strong form of general coordinate invariance requires that the light-like 3-surfaces representing partonic orbits and space-like 3-surfaces at the ends of causal diamonds are physically equivalent. This implies effective 2-dimensionality: the intersections of these two kinds of 3-surfaces and 4-D tangent space data at them should code for quantum states.
- Quantum criticality states that Universe is analogous to a critical system meaning that it has maximal structural richness. One could also say that Universe is at the boundary line between chaos and order. The original motivation was that quantum criticality fixes the basic coupling constant dictating quantum dynamics essentially uniquely.
- The notion of finite measurement resolution has also become an important guide-line. Usually this notion is regarded as ugly duckling of theoretical physics which must be tolerated but the mathematics of von Neumann algebras seems to raise its status to that of beautiful swan.
- What I have used to call weak form of electric-magnetic duality is a TGD version of electric-magnetic duality discovered by Olive and Monton [B1]. It makes it possible to realize strong form of holography implied actually by strong for of General Coordinate Invariance. Weak form of electric magnetic duality in turn encourages the conjecture that TGD reduces to almost topological QFT. This would mean enormous mathematical simplification.
- TGD leads to a realization of counterparts of Feynman diagrams at the level of space-time geometry and topology: I talk about generalized Feynman diagrams. The highly non-trivial challenge is to give them precise mathematical content. Twistor revolution has made possible a considerable progress in this respect and led to a vision about twistor Grassmannian description of stringy variants of Feynman diagrams. In TGD context string like objects are not something emerging in Planck length scale but already in scales of elementary particle physics. The irony is that although TGD is not string theory, string like objects and genuine string world sheets emerge naturally from TGD in all length scales. Even TGD view about nuclear physics predicts string like objects.

# 2 The great narrative of standard physics

Narratives allow a simplified understanding of very complex situations. This is why they are so powerful and this is why we love narratives. Unfortunately, narrative can also lead to the wrong track when one forgets that only a rough simplification of something very complex is in question.

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## 2.1 Philosophy

In the basic philosophy of physics reductionism, materialism, determinism, and locality are four basic dogmas forming to which the great narrative relies.

#### 2.1.1 Reductionism

Reductionism can be understood in many manners. One can imagine reduction of physics to few very general principles, which is of course just the very idea of science as an attempt to understand rather than only measure. This reductionism is naive length scale reductionism. Physical systems consist of smaller building bricks which consist of even smaller building bricks... The entire physics would reduce to the dance of quarks and this would reduce to the dynamics of super strings in the scale of Planck length. The brief summary about the reductionistic story would describe physics as a march from macroscopic to increasingly microscopic length scales involving a series of invasions:

Biology  $\rightarrow$  biochemistry  $\rightarrow$  chemistry  $\rightarrow$  atomic physics as electrodynamics for nuclei and electrons. Nuclear physics for nuclei  $\rightarrow$  hadronic physics for nuclei and their excitations  $\rightarrow$  strong and weak interactions for quarks and and leptons.

One can of course be skeptic about the first steps in the sequence of conquests. Is biology really in possession? Physicists cannot give definition of life and can say even less about consciousness. Even the physics based definition of the notion of information central for living systems is lacking and only entropy has physics based definition. Do we really understand the extreme effectiveness of bio-catalysts and miracle like replication of DNA, transcription of DNA to mRNA, and translation of mRNA to aminoacids. It is yet impossible to test numerically whether phenomenological notions like chemical bond really emerge from Schrödinger equation.

The reduction step from nuclear physics to hadron physics is purely understand as is the reduction step from hadron physics to the physics of quarks and gluons. Here one can blame mathematics: the perturbative approach to quantum chromodynamics fails at low energies and one cannot realize deduce hadrons from basic principle by analytical calculations and must resort to non-perturbative approaches like QCD involving dramatic approximations.

The standard model is regarded as the recent form of reductionism. The generalization of standard model: Grand Unified Theories (GUTs), Supersymmetric gauge theories (SUSYs), and super string models and M-theory are attempts to continue reductionistic program beyond standard model making an enormous step in terms of length scales directly to GUT scale or Planck scale. These approaches have been followed during last forty years and one must admit that they have not been very successful. This point will be discussed in detail later.

Therefore reductionistic dogma involves many bridges assumed to exist but about whose existence we do not really know. Further, reductionistic dogma cannot be tested. This untestability might be the secret of its success besides the natural human laziness and temptations of groupthink, which could quite generally explain the amazing success of great narratives even when they have been obviously wrong.

#### 2.1.2 Materialism

Materialism is another big chunk in the great narrative of physics. What it states is that only the physically measurable properties matter. One cannot measure the weight of the soul, so that there is no such thing as soul. The physical state of the brain at given moment determines completely the contents of conscious experience. In principle all sensory qualia, say experience of redness, must have precise correlates at the level of brain state.

At what level does life and consciousness appear. What makes matter conscious and behaving as if would have goals and intentions and need to survive? This is difficult question for the materialistic approach one postulates the fuzzy notion of emergence. When the system becomes complex enough, something genuinely new - be it consciousness or life - emerges. The notion of emergence seems to be in obvious conflict with that of naive length scale reductionism and a lot of handwaving is needed to get rid of unpleasant questions. What this something new really is is very difficult or even impossible to define in in the framework reductionistic physics.

The problems culminate in neuroscience and consciousness theory which has become a legitimate field of science during last decade. The hard problem is the coding of the properties of the physical state of the brain to conscious experience. Recent day physics does not provide a slightest clue

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regarding this correspondence. One has of course a lot of correlations. Light with certain wavelength creates the sensation of red but a blow in the head can produce the same sensation. EEG and nerve pulse activity correlate with the contents of conscious experience and EEG seems to even code for contents of conscious experience. Only correlates are however in question. It is also temporal patterns of EEG rather than EEG at given moment of time which matters from the point of view of conscious experience. This relates closely to another dogma of standard quantum physics stating that time=constant slice of time evolution contains all information about the state of the system.

#### 2.1.3 Determinism

The successes of Newtonian mechanism were probable the main reason for why determinism became a basic dogma of physics. Determinism implies a romantic vision: theoretician working with mere paper and pencil can predict the future. This leads also to the idea that Nature can be governed: this idea has dominated western thinking for centuries and led to the various crises that human kind is suffering. Ironically, this idea is actually in conflict with the belief in strict determinism! Also the narrative provided by Darwinism assumes survival as a goal, which means that organisms behave like intentional agents: something in conflict with strict determinism predicting clockwork Universe. On the other hand, genetic determinism assumes that genes determine everything. The great narrative is by no means free of contradictions. They are present and one must simply put them under the rug in order to keep the faith. The situation is same as in religions: everyone realizes that Bible is full of internal contradictions and one must just forget them in to not lose the great narrative provided by it.

In quantum theory one is forced to give up the notion of strict determinism at the level of individual systems. The outcome of state function reduction occurring in quantum measurement is not predictable at the level of individual systems. For ensembles one can predict probabilities of various outcomes so that classical determinism is replaced with statistical determinism, which of course involves the idealized notion of ensemble consisting of large number of identical copies of the system under consideration.

In consciousness theory strict determinism means denial of free will. One could ask whether the non-determinism of state function reduction could be interpreted in terms of free will so that even elementary particles would be conscious systems. It seems that this identification cannot explain intentional goal directed free will. State function reductions produce entropy and this provides deeper justification for the second law and quantum mechanism makes it possible to calculate various parameters like viscosity and diffusion constants needed in the phenomenological description of macroscopic systems. Living systems however produce and store information and experience it consciously. Quantum theory in its recent form does not have the descriptive power to describe this. Something more is needed: one should bring the notion of information to physics.

#### 2.1.4 Locality

Locality is fourth basic piece of great narrative. What locality says that physical systems can be split into basic units and that understanding the behavior of this units and the interaction between them is enough to understand the system. This is very much akin to naive length scale reductionism stating that everything can be reduced to the level of elementary particles or even to the level of superstrings.

Already in quantum theory one must give up the notion of locality although Schrödinger equation is still local. Standard quantum theory tells that in macroscopic scales entanglement has no implications. Quantum entanglement is now experimentally demonstrated to be possible between systems with macroscopic distance and even between macroscopic and microscopic systems. What does this mean: is the standard quantum theory really all that is needed or should we try to generalize it?

Locality dogma becomes especially problematic in living systems. Living systems behave as coherent units behaving very "quantally" and it is very difficult to understand how sacks of water containing some chemicals could climb in trees and even compose symphonies. The attempts to produce something which would look like living from a soup of chemicals have not been successful.

The proposed cure is macroscopic quantum coherence and macroscopic entanglement. There exist macroscopically quantum coherent systems such as suprafluids and super-conductors but these systems are very simple all particles are in same state- Bose Einstein condensate and quite different from

living matter. Standard quantum theory is also unable to explain macroscopic quantum coherence and preservation of entanglement at physical temperatures.

Evidence for quantum coherence in cell scales and at physiological temperatures is however accumulating. Photosynthesis, navigation behavior of some birds and fishes, and olfaction represent examples of this kind. The recent finding that microtubules carry quantum waves should be also mentioned. Does this mean that something is missing from standard quantum theory. The small value of Planck constant characterizes the sizes of quantum effects and tells that spatial and temporal scales of quantum coherence are typically rather short. Is Planck constant really constant. One can of course ask whether this problem could relate to another mystery of recent day physics: the dark matter. We know that it exists but there is no generally accepted idea about what it is. Could living systems involve dark matter in an essential manner and could it be that Planck constant does not have only its standard value?

Locality postulate has far reaching implications for science policy. There is a lot of anecdotal evidence for various remote mental interactions such as telepathy, clairvoyance, psychokinesis of various kinds, remote healing, etc... The common feature of these phenomena is non-locality so that standard science denies them as impossible. Fr this reason people trying to study these phenomena have automatically earned the label of crackpot. Therefore experimental demonstration of these phenomena is very difficult since we do not have any theory of consciousness. Situation is not helped by the fact that skeptics deny in reflex like manner all evidence.

## 2.2 Classical physics

Classical physics began with the advent of Newton's mechanics and brought the dogma of determinism to physics. In the following only thermodynamics and special and general relativities are discussed as examples about classical physics because they are most relevant from the TGD viewpoint.

#### 2.2.1 Thermodynamics

Second law is the basic pillar of thermodynamics. It states that the entropy of a closed system tends to increase and achieve maximum in thermodynamical equilibrium. This law does not tell about the detailed evolution but only poses the eventual goal of evolution. This means irreversibility: one cannot reverse the arrow of thermodynamical time. For instance, one one can live life in the reverse direction of time.

The physical justification for the second law comes from quantum theory. Again one must however make clear that the basic assumption that that time characteristic time scale for interactions involved is short as compared to the time scale one monitors the system. In time scales shorter the quantum coherence time the situation changes. If quantum coherence is possible in macroscopic time scales, one cannot apply thermodynamics.

The thermodynamical time has a definite arrow and is believe to be the same always. Living matter might form exception to this belief and Fantappie has proposed that this is indeed the case and proposed the notion of syntropy to characterize systems which seem to have non-standard arrow of time. Also phase conjugate laser rays seem to dissipate in wrong direction of time so that entropy seems to decrease from them when they are viewed in standard time direction.

The basic equations of physics are not believed posses arrow of time. Therefore the relationship between thermodynamical time and the geometric time of Einstein is problematic. Thermodynamical arrow of time relates closely to that of experienced/psychological arrow of time. Is the identification of experienced time and geometric time really acceptable? They certainly look different notions: experienced time has not future unlike geometric time, and experienced time is irreversible unlike geometric time. Certainly the notion of geometric time is well-understood. The notion of experienced time is not. Are we hiding ourselves behind the back of Einstein when we identify these two times. Should we bravely face the reality and ask what experienced time really is? Is it something different from geometric time and why these two times have also many common aspects - so many that we have identified them.

Second law provides a rather pessimistic view about future: Universe is unavoidably approaching heat death as it approaches thermodynamical equilibrium. Thermodynamics provides a measure for entropy but not for information. Is biological evolution really a mere thermodynamical fluctuation in which entropy in some space-time volume is reduced? Can one really understand information created

and stored by living matter as a mere thermodynamical fluctuation? The attempt to achieve this has been formulated as non-equilibrium thermodynamics for open systems. One can however wonder whether could go wrong in the basic premises of thermodynamics?

#### 2.2.2 Special Relativity

Relativity principle is the basic pillar of special relativity. It states that all system with respect to each other with constant relativity are physically equivalent: in other worlds the physics looks the same in these systems. Light velocity is absolute upper limit for signal velocity.

This kind of principle holds true also in Newton's mechanics and is known as Galilean relativity. Now there is however not upper bound for signal velocity. The difference between these principles follows from different meaning for what it is to move with constant relativity velocity. In special relativity time is not absolute anymore but the time shown by the clocks of two systems are different: time and spatial coordinates are mixed by the transformation between the systems.

Maxwell's electrodynamics satisfied the Relavity Principle and in modern terminology Poincare group generated by rotations, Lorentz transformations (between systems moving with respect to each other with constant velocity), translations in spatial and time drections act as symmetries of Maxwell's equations. In particle physics and quantum theory the formulation of relativity principle in terms of symmetries has become indispensable.

The essence of Special theory of relativity is geometric. Minkowski space is four-dimensional analog of Riemannian geometry with metric which characterizes what length and angle measurement mean mathematically. The metric is characterized in terms of generalization of the law of Pythagoras stating  $ds^2 = dt^2 - dx^2 - dy^2 - dz^2$  in Minkowski coordinates. What is special is that time and space are in different positions in this infinitesimal expression for line element telling the length of the diameter of 4-dimensional infinitesimal cube.

Time dilation and Lorentz contraction are two effects predicted by special relativity. Time dilation day-to-day phenomenon in particle physics: particles moving with high velocity live longer in the laboratory system. Lorentz contraction must be also taken into account. Lorentz himself believed for long that Lorentz contraction is a physical rather than purely geometric effect but finally admitted that Einstein was right.

There are some pseudo paradoxes associated with Special Relativity and regularly some-one comes and claims that is some horrible logical error in the formulation of the theory. One paradox is twin paradox. One consider twins. Second goes for a long space-time travel moving very near to light-velocity and experiences time dilation. When he arrives at home he finds that his twin brother is very old. One can however argue that by relativity principle it is the second twin who has made the travel and should look older. The solution of the paradox is trivial. The situation is not symmetric since the second brother is not entire time in motion with constant velocity since he must turn around during the travel and spend this period in accelerated motion.

#### 2.2.3 General Relativity

Einstein based his theories on general principles and maybe this is why they have survived all the tests. The theoretical physics has become very technical since the time of Einstein and the formulation of theories in terms of principles has not been in fashion. Instead, concrete equations and detailed models have replaced this approach. Super string models provide a good example. Maybe this explains why the modest success.

In general relativity there are two basic principles. General Coordinate Invariance and Equivalence Principle.

General Coordinate Invariance (GCI) states that the formulation of physics must be such that the basic equations are same in all coordinate systems. This is very powerful principle when formulated in terms of space-time geometry which is assumed to be generalization of Riemannian geometry from that for the Minkowski space of special relativity. Now line element is expressed as  $ds^2 = g_{ij}dx^idx^j$  and it can be reduced to Minkowskian form only in vacuum regions far enough from massive bodies. Another new element is curvature of space-time which can be concretized in terms of spherical geometry. For triangles at the surface of sphere having as sides pieces of big circles (geodesic lines, which now represent the analog of free rectilinear motion) the sum of angles is larger than 180 degrees. For

geodesic triangles at the surface of saddle like surface the sum is smaller than 180 degrees. This holds for arbitrarily small geodesic triangles and is therefore a local property of Riemann geometry.

Quite often one encounters the belief that GCI is generalization of Relativity Principle. This is not the case. Relativity Principle states that the isometries of Minkowski space consisting of Poincare transformations leave the physics invariant. General Coordinate transformations are not in general isometries of space-time and in the case of general space-time there are not isometries. Therefore GCI is only a constraint on the form of field equations: they just remain invariant under general coordinate transformations. Tensor analysis is the mathematical tool making it possible to express this universality. Tensor analysis allows to express the space-time geometry algebraically in terms of metric tensor, curvature tensor, Ricci tensor and Einstein tensor, and Ricci scalar associated with it. In particular, the notion of angle defect can be expressed in terms of curvature tensor.

In the case of Equivalence Principle (EP) the starting point is the famous thought experiment involving lift. In stationary elevator material objects fall down with accelerated velocity. One can however study the situation in freely falling life and in this case the material objects remain stationary as if there were not gravitational force. The idea is therefore that gravitational force is not a genuine force but only apparent coordinate forces which vanishes locally in suitable coordinates known as geodesic coordinates for which coordinate lines are geodesic lines. Gravitational force would be analogous to apparent forces like centripetal forces and Coriolis force appearing in rotating coordinate systems already in Newton's mechanics. The characteristic signature is that the associated acceleration does not depend on the mass of the particle. This leads to the postulate that the motion of particles occurs along geodesic lines in absence of other than gravitational interactions. Equivalence Principle is already present in Newton's theory of gravitation and states that inertial masses appearing in F = ma can be chosen to be same as the gravitational mass appearing in the expression of gravitational forces  $F_{qr} = GmM/r^2$  between bodies with gravitational masses m and M. Equivalence Principle looks rather innocent and almost trivial but its formulation in competing theories is surprisingly difficult and the situation is not made easier by the fact that the mathematics involved is highly non-linear.

Tensor analysis allows the tools to deduce the implications of EP. The starting point is the equality of inertial and gravitational masses but made a local statement for the corresponding mass densities or more generally corresponding tensors. For inertial mass energy momentum tensor characterizing the density and currents of four-momentum components is the notion needed. For gravitational energy the only tensor quantities to be considered are Einstein tensor and metric tensor because they satisfy the conservation of energy and momentum locally in the sense that their covariant divergence is vanishing. Also energy momentum tensor should be conserved and thus have vanishing divergence. The manner to achieve this is to assume that the two tensor are proportional to each other. This identification actually realizes EP and gives Einstein's equations. Cosmological term proportional to the metric tensor can be present and Einstein consider also this possibility since otherwise cosmology was predicted to be expanding and this did not fit with the prevailing wisdom. The cosmological expansion was observed and Einstein regarded his proposal as the worst blunder of this professional life. Ironically, the recently observed acceleration of cosmic expansion might be understood if cosmological term is present after all albeit with sign different than in Einstein's proposal. Einstein's equations state that matter serves as a source of gravitational fields and gravitational fields tell for matter how to move in presence of gravitational interaction. These equations have been amazingly successful.

There is however a problem relating to the difference between GCI and Principle of Relativity already mentioned. Noether's theorem states that symmetries and conservation laws correspond to each other. In quantum theory this theorem has become the guiding principle and construction of new theories is to high degree postulation of various kinds of symmetries and deducing the consequences. In generic curved space-time the presence of massive bodies makes space-time curved and Poincare symmetries of empty Minkowski space are lost. This does not imply not only non-conservation of otherwise conserved quantities. These quantities do not even exist mathematically. This is a very serious conceptual drawback and the only manner to circumvent the problem is to make an appeal to the extreme weakness of gravitational interaction and say that gravitational four-momentum can be assigned to a system in regions very far from it because gravitational field is very weak.

This difficulty might explain why the quantization of gravitation by starting from Einstein's equations has been so difficult. It must be however noticed that the perturbative quantization of supersymmetric variant of Einstein's equation works amazingly well in flat Minkowski background and it has been even conjectured that divergences which plague practically every quantum field theory might

be absent. Here the twistor Grassmann approach has allowed to overcome the formidable technical difficulties due to the extreme non-linearity the action principle involved. Still the question remains: could it be possible to modify general relativity in such a manner that the symmetries of special relativity would not be lost?

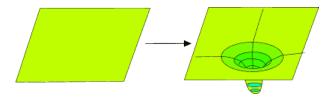


Figure 1: Matter makes space-time curved and leads to the loss of Poincare invariance so that momentum and energy are not well-defined notions in GRT.

# 2.3 Quantum physics

Quantum physics forces to change both the ontology and epistemology of classical physics dramatically.

#### 2.3.1 Quantum theory

In the following I just list the basic aspects of quantum theory which distinguish it from classical physics.

- 1. Point like particle is replaced in quantum physics by wave function. This is rather radical abstraction in ontology. For mathematician this looks almost trivial transition from space to function space: the 3-D configuration for particle is replaced by the space of complex valued functions in this space Schrödinger amplitudes. From the point of view of physical interpretation this is big step since wave function means abstraction which cannot be visualized in terms of sensory experience. This transition is repeated in second quantization whether the function space is replaced with functional space consisting of functions defined classical fields. Also the proper interpretation of Schrödinger amplitudes is found to be in terms of classical fields. The new exotic elements are spinor fields, which are anti-commuting already at the classical level. They are introduced to describe fermions: this element is however not absolutely necessary.
  - The interpretation is as probability amplitudes square roots of probability densities familiar from probability theory applied in kinetic theory.
- 2. Schrödinger amplitude is mathematically analogous to a classical field, say classical electromagnetic fields fields appearing in Maxwell's theory. Interference for probability amplitudes leads to completely analogous effects such as interference and diffraction. The classical experiment demonstrating diffraction is double slit experiment in which electron bream travels along double slit system and is made visible at screen behind it. What one observes a distribution reflecting interference pattern for Schrödinger waves from the two slits just as for classical electromagnetic fields. The modulus square for probability amplitude inhibits the interference pattern. As the other slit is closed, interference pattern disappears. One cannot explain the interference pattern using ordinary probability theory: in this case electrons of the beam would not "know" which slits are open and destructive interference would be impossible. In quantum world they "know" and behave accordingly. Physics is not anymore completely local.
- 3. The model of electrons in atoms relies on Schrödinger amplitude and this might suggests that Schrödinger amplitude is classical field. This is however not the case. To understand what is involved one must introduce the notion of state function reduction and Uncertainty Principle. It was learned basically by doing experiments that quantum measurements differ from classical ones. First of all, even ideal quantum measurement typically changes the system, which does not

happen in ideal classical measurement. The outcome of the measurement is non-deterministic and there are several outcomes, whose number is typically finite. One can predict only the probability of particular outcome and it is dictated by the state of the system and the measured observables.

Uncertainty Principle is a further new element and dramatic restriction to ontology. For instance, one cannot measure momentum and position of the particle simultaneously in arbitrary accuracy. Ideal momentum measurement delocalizes the particle completely and vice versa. This is very difficult to understand in the framework of classical mechanics were particle is point of space. If one accepts the mathematician's view that particle states are elements of function space, Uncertainty Principle can be understood and is present already in Fourier analysis. One also can get rid of ontological un-easiness created by statements like "electron can exist simultaneously in many places". Also the construction of more complex systems using simpler ones as building bricks (second quantization) is easy to understand in this framework: in classical particle picture second quantization looks rather mysterious procedure. It is however not at all easy for even mathematical physicist to think that function space could be something completely real rather than only a figment of mathematical imagination.

4. What remains something irreducibly quantal is the occurrence of the non-deterministic state function reduction. This seems to be the core of quantum physics. The rest might reduce to deterministic physics in some function space characterizing physical states.

The real problem is that the non-determinism of state function is not consistent with the determinism of Schrödinger equation. It seems that the laws of physics cease to hold temporarily and this has motivated the statements about craziness of quantum theory. More plausible view is that something in our view about time - or more precisely, about the relation between the geometric time of physicist and experienced time is wrong. These times are identified but we know that they are different: geometric time as no intrinsic arrow whereas subjective time has and future does not exist for subjective time but for geometric time it exists.

There have been several attempts to reduce also state function reduction to deterministic classical physics or change the ontology so that it does not exist, but these attempts have not been successful. Ironically the core of quantum physics has remained also the taboo of quantum physics. The formulation is as "shut and calculate" paradigm which has dominated academic theoretical physics for century. One can only imagine where we could be without this professional taboo.

5. Quantum entanglement is a phenomenon without any classical counterpart. Schrödinger cat has become the standard manner to illustrate what is involved. One considers cat and bottle of poison which can be either open or closed. Classically one has two states: cat alive-bottle closed and cat dead-bottle open. Quantum mechanically also the superposition of these two states is possible and this obviously does not make sense in classical ontology. We cannot however observe quantum entanglement. When we want to know whether cat is dead or alive we induce state function reduction selecting either of these two states and the situation become completely classical. This suggests epistemological restriction: the character of conscious experience is that it produces always classical world as an outcome. One should of course not take this as dogma. The so called interaction free measurement allows to get information about system without destroying entanglement.

#### 2.3.2 Standard model

Standard model summarizes our recent official understanding about physics. The attribute "official" is important here: there exists a lot of claims for anomalies, which are simply denied by the mainstream as impossible. Reductionists believe standard model to summarize even physics accessible to us. Standard model has been extremely successful in elementary particle physics. Even Higgs particle was found at LCH with predicted properties.

There are however issues related to the Higgs mechanism. Higgs particle has mass that it should not have and SUSY particles are too heavy to help in the problem. Stabilization of Higgs mass by cancelling radiative corrections to Higgs mass from heavy particles was one of the basic motivations for

postulating SUSY in TeV energy scaled studied at LHC. Therefore one has what is called fine tuning problem for the parameters characterizing the interactions of Higgs and theory loses its predictivity.

Even worse, RHIC and LCH provide data telling that perturbative QCD does not seem to work at high energies where it should work. What was though to be quark gluon plasma - something behaving in very simple manner - was something different and one cannot exclude that there is some new physics there.

Neutrinos are the black sheep of the standard model. Each of the three leptons is accompanied by neutrino and in the most standard standard model they are massless. This has turned out to be not the case. Neutrinos also mix with each other as do also quarks. This phenomenon relates closely to the massivation. There are also indications that neutrinos could have several states with different mass values. The experimental neutrino physics is however extremely difficult since neutrinos are so weakly interaction so that the experimental progress is slow and plagued by uncertainties.

Therefore there are excellent reasons to be skeptical about standard model: one should continue to ask questions about the basics of the standard model. The attempt to answer this kind of fundamental questions concerning standard model could lead to re-awakening of particle physics from its recent stagnation. In particular, one could wonder what might be the origin of standard model quantum numbers and what is the origin of quark and gluon color. Standard model gauge group has very special and apparently un-elegant structure - something not suggested by GUT ideology. Why this Could this reflect some deeper principles?

This kind of questions were possible at sixties, and they led to the amazingly fast evolution of standard model. This hippie era in theoretical physics continued to the beginning of eighties but then the super string revolution around 1984 changed suddenly everything. Comparison with the revolution leading to birth of Soviet Union might be very rewarding. For me hippie era meant the possibility to make my thesis at Helsinki Technological University receiving even little salary: officially the goal was to make me a citizen able to take care of myself. Nowadays the idea about a person writing thesis about his own theory of everything is something totally unthinkable.

#### 2.3.3 Grand Unified Theories

According to the great narrative the next step was huge: something like 13 orders of magnitude from the length scale of electroweak bosons ( $10^{-17}$  meters) to the length scale of extremely have gauge bosons of GUTs. At the time when I was preparing my thesis, GUTs were the highest fashion and every graduate student in particle physics had the opportunity to become the new Einstein and pick up his/her own gauge group and build up the GUT. All the needed formulas could be found easily and there was even a thick article containing all the recipes ranging from formulas for tensor products of group representations to beta functions for given group.

Both leptons and quarks form single family belonging to same multiplet of the big GUT gauge symmetry. The new gauge interactions predicted that and lepton and baryon number are not separately conserved so that proton is not stable. The theory allowed to predict its lifetime. The disappointing fact has been that no decays of proton have been however observed and this has led to a continual fine tuning of coupling parameters to keep proton alive for long time enough. This of course should put bells ringing since the stability of proton is extremely powerful guideline in theory building would suggest totally different track to follow based on question "Can one imagine any scenario in which B and L are separately conserved?"

The mass splittings between different fermions (quarks and leptons) believed to be related by gauge symmetries are huge: the mass ratio for top quark and neutrinos would be of the order  $10^{12}$ , which is a huge number. Quite generally, the mass scales between symmetry related particles would be huge, which suggests that the notion of mass scale is part of physics. Also could serve as extremely powerful hint for a theory builder who is not afraid for becoming kicked out from the academic community.

GUT approach predicts a huge desert without any new physics ranging from electroweak scale to GUT length scale! So many orders of magnitude without any new physics looks like an incredible prediction when one recalls that 2 orders of magnitude separating electron and nuclei is the record hitherto. This assumption is of course just a scaled up variant of the child's assumption that the world ends at the backyard, and its basic virtue is that it makes theorist's life simple. There is nothing bad in this kind of assumption when taken as simplifying working hypothesis. The problem is that people have forgot that GUT hypothesis is only a pragmatic working hypothesis and believe that it represent an established piece of physics. Nothing could be farther from truth.

#### 2.3.4 Super Symmetric Yang Mills theories

GUTs were followed by supersymmetric Yang-Mills theories - briefly SUSYs. The ambitious idea was to extend the unification program even further. Also fermions and bosons - particles with different statistics - would belong to same multiplet of some big symmetry group replaced with something even more general- super symmetry group. This required generalization of the very notion of symmetry by extending the notion of infinitesimal symmetry. One manner to achieve this is to replace space-time with a more general structure - superspace - possessing fermionic dimensions. This is however not necessarily and many mathematicians would regard this structure highly artificial. As a mathematical idea the generalization of symmetry is however extremely beautiful and shows how powerful just the need to identify bigger patterns is. One can indeed generalize of the various GUTs to supersymmetric gauge theories.

The number  $\mathcal{N}$  of independent super-symmetries characterizes SUSY, and there are arguments suggesting that physically  $\mathcal{N}=1$  theories are the only possible ones. Certainly they are the simplest ones, and it is mostly these theories that particle phenomenologists have studied.  $\mathcal{N}=4$  SUSYs possesses in certain sense maximal SUSY in four-dimensions. It is unrealistic as a physical model but because of its exceptional simplicity has led to a mathematical breakthrough in theoretical physics. The twistor Grassmannian approach has been applied to these theories and led to a totally new view about how to calculate in quantum field theory. The earlier approach based on Feynman diagrams suffered from combinatorial explosion so that only few lowest orders could be calculated numerically. The new approach strongly advocated by Nima Arkani Hamed and his coworkers allows to sum up huge numbers of Feynman diagrams and write the answer which took earlier ten pages with few lines. Also a lot of new mathematics developed by leading Russian mathematicians has been introduced.

 $\mathcal{N}=1$  SUSY, whose particles would have mass scale of order TeV, the energy scale studied at LHC, was motivated by several reasons. One reason was that in that ideal situation that all particles remain massless the contributions of ordinary and supersymmetric particles to many kinds of radiative corrections in particle reactions cancel each other. In the case of Higgs this would mean stability of the parameters characterizing the interactions of Higgs with other particles. In particular, Higgs vacuum expectation value determining the masses of leptons and quarks and gauge bosons would be stable. All this depends sensitively on precise values of particle masses and unfortunately it happens that the mechanism does not stabilize the parameters of Higgs.

Second motivation was that SUSY might provide solution to the dark matter mystery. The called lightest super-symmetric particle is predicted to be stable by so called R-parity symmetry which naturally accompanies SUSY but can be also broken. This particle is fermion and super partner of photon or weak boson  $Z^0$  or mixture of these. This particle would provide an explanation for the mysterious dark matter about which we recently know only its existence. Dark matter would be a remnant from early cosmology - those lightest supersymmetric particles which failed annihilate with their antiparticles to bosons because cosmic expansion reduced their densities and made annihilation rate too small.

The results from LCH were however a catastrophic event in the life of SUSY phenomenologists. Not a slightest shred of evidence for SUSY has been found. There is still hope that some fine tuned SUSY scenarios might survive but if SUSY is there it cannot satisfy the basic hopes put on it. The results from LHC arriving during 2005 will be decisive for the fate of SUSY.

The results of LHC do not of course exclude the notion of supersymmetry. There are lots of variants of supersymmetry and  $\mathcal{N}=SUSYs$  represents only one particular, especially simple variant in some respects and involving ad hoc assumptions such as straightforward generalization of Higgs mechanism as origin of particle massivation, which can be questioned already in standard model context. Furthermore,  $\mathcal{N}=1$  SUSY forces to give up separate conservation of lepton and baryon numbers for which there is no experimental evidence. For higher values of  $\mathcal{N}$  this is not necessary.

#### 2.3.5 Superstrings and M-theory

Super-strings mean a further extension for the notion of symmetry and thus reductionism at conceptual level. Conformal symmetries define infinite-dimensional symmetries and were first discovered in attempts to understand 2-dimensional critical systems. Critical system is a system in phase transition. There are two phases present that and the regions of given phase can have arbitrary large sizes. This means scale invariance and long range fluctuations: system does not behave as if it would consist of

billiard balls having only contact interactions. The discovery was that the notion of scale invariance generalizes to local scale invariance. The transformations of plane (or sphere or any 2-D space) known as conformal transformations preserve the angle between two curves and introduce local scaling of distances. These transformations appear in complex analysis as holomorphic maps.

In string model which emerged first as hadronic string model, hadrons are identified as strings. Their orbits define 2-D surfaces and conformal transformations for these surfaces appear as symmetries of the theory. One could say that strings physics resembles that of 2-D critical systems. Hadronic string model did not evolve to a real theory of hadrons: for instance, the critical dimension in which worked was 26 for bosonic strings and 10 for their super counterparts. Therefore hadronic string model was largely given up as quantum chromodynamics trying to reduce hadronic physics to that of point-like quarks and gluons emerged. This approach worked nicely at high energies but at low energies the problem is that perturbative approach fails. The already mentioned unexpected behavior of what was expected to be quark gluon plasma challenges also QCD.

String model contained also graviton like states possessing spin 2 and the description for their interactions resemble that for the description of gravitons with matter according to the lowest order predictions of quantized general relativity. This eventually led to the idea that maybe super-symmetric variants of string might provide the long sough solution to the problem of quantizing gravitation. Perhaps even more: maybe they could allow to unify all known fundamental interactions with framework of single notion: super string.

In superstring approach the last step in the reductionistic sequence of conquests would be directly to the Planck length scale making about 16 orders of magnitudes. The first superstring revolution shook physics world around 1984. During the first years gurus believed that proton mass would be calculated within few years and first Nobels would be received within decade. Gradually the optimism began to fade as it turned out that superstring theory is not so unique as it was believed to be. Also the building or the bridge to the particle phenomenology was not at all so easy as was believed first.

Superstring exists in mathematically acceptable manner only in dimension D=10 and this was of course a big problem. The notion of spontaneous compactification was needed and brought in an ugly ad hoc trick to the otherwise so beautiful vision. This mechanism would compactify 6 large dimensions of the 10-D Minkowski space so that they would become very small - the scale would be of the order of Planck length. For all practical purposes the 10-D space would look 4-dimensional. The 6 large dimensions would curl up to so called Calabi-Yau space and the finding of the correct Calabi-Yau was thought to be a simple procedure.

This was not the case. It turned out that there are very many Calabi-Yau manifolds [?] to begin with: the number 10<sup>500</sup> was introduced to give some idea about how many of them are - the number could be quite well infinite. The simple Calabi-Yau spaces did not produce the standard model physics at low energies. This problem became known as landscape problem. Landscape inspired in cosmology to the notion of multiverse: universe would split to regions which can have practically any imaginable laws of physics. There is no empirical support for this vision but this has not bothered the gurus.

Gradually it became clear that landscape problem spoils the predictivity of the theory and eventually many leading gurus turned they coat. The original idea was that string models are so wonderful because they predict unique physics. Now they were so beautiful because they force us to give up completely the belief that physical theories can predict something. In this framework antropic principle remains the only guideline in attempts to relate theory to the real world. This means that we can deduce the properties of the particular physics we happen to live from our own existence and by scanning through this huge repertoire of possible physics.

Around 1995 so called second superstring revolution took place. Five very different looking super string models had emerged. The great vision advocated especially by Witten was that they are limiting cases of one theory christened as M-theory. The 10-D target space for superstrings was replaced with 11-dimensional one. Besides this higher dimensional objects - branes- of varying dimension entered the picture and made it even more complex. This gave of course and enormous flexibility. For instance, the 4-D observed space-time could be understood as brane rather than the effectively 4-D target space obtained by spontaneous compactification. This gave for particle phenomenologists wanting to reproduce standard model an endless number of alternatives and the theory degenerated to endless variety of attempts to reproduce standard model by suitable configurations of branes. Around 2005 the situation in M-theory began to become public and so called string wars began. At this moment the funding of super-strings has reduced dramatically and the talks in string conferences hardly mention superstrings.

One can conclude that the forty years of unification based on naive length scale reductionism was a failure. What was thought to become the brightest jewel in the crown of reductionistic vision was a complete failure. If history could teach something, it should teach us that we should perhaps follow Einstein and his co-temporaries and be asking questions about fundamentals. The shut-up and calculate approach forbidding all discussion about the basic assumptions has leads nowhere during these four decades.

As one looks this process in the light of after wisdom, one realizes that there are two kinds of reductionisms involved. The naive length scale reductionism has not been successful. Time might be ripe for its replacement with the notion of fractality which postulates that similar looking structures appear in all length scales. Fractality is also a central aspect of the renormalization group approach to quantum field theory.

A second kind of reductionistic sequence has been realized at conceptual level. The notion of symmetry has evolved from ordinary symmetry to supersymmetry to super-conformal symmetry and even created new mathematical notions. The size of the postulated symmetry groups has steadily increased: note that already Einstein initiated this trend by postulating general coordinate invariance as a symmetry analogous to gauge symmetry. In superstring type approaches one can ask whether one should put all particles to same symmetry multiplet in the ultimate theory.

Symmetry breaking is what remains poorly understood in gauge theories and GUTS. Conformal field theories however provide a very profound and deep mechanism involving now ad hoc elements as Higgs mechanism does. Maybe one should try to understand particle massivation in terms of breaking of superconformal symmetries rather than blindly following the reductionistic approach and trying to reproduce SUSY and GUT approaches and Higgs mechanism as intermediate steps in the imagined reductionistic ladder leading from standard model to the ultimate theory. Maybe we should try to understand symmetry breaking as reflecting the limitations of the observer. For instance, in thermodynamical systems we can observe only thermodynamical averages of the properties of particles, such as energy.

# 2.4 Summary of the problems in nutshell

New theory must solve the problems of the old theory. The old theory indeed has an impressive list of problems. The last 30 or 40 years have been an Odysseia in theoretical physics. When did this Odysseia begin?

Did the discovery of super strings initiate the misery for thirty years ago? Or can we blame SUSY approach? Was the SUSY perhaps too simple - or perhaps better to say, too simplistic? Did already the invention of GUTs lead to a side track: is it too simplistic to force quarks and leptons to multiplets of single symmetry group? This forcing of the right leg to the left hand shoe predicts proton decay, which has not been observed?

Or is there something badly wrong even with the cherished standard model: do particles really get their masses through Higgs mechanism: is the fact that Higgs is too light indication that something went wrong? Do we really understand quark and gluon color and neutrinos? What about family replication and standard model quantum numbers in general? What about dark matter and dark energy? The only thing we know is that they exist and naive identifications for dark matter have turned out to be wrong. There is also the energy problem of General Relativity. Did we go choose a wrong track already almost century ago?

And even at the level of the basic theory - quantum mechanics - taken usually as granted we have the same problem that we had almost century ago.

# 3 Could TGD provide a way out of the dead end?

The following gives a concise summary of the basic ontology and epistemology of TGD followed by a more detailed discussion of the basic ideas.

## 3.1 What new ontology and epistemology TGD brings in?

TGD based ontology and epistemology involves several elements, which might help to solve the listed problems.

- 1. The new view about space-time as 4-D surface in certain 8-D imbedding space leads to the notion of many-sheeted space-time and to geometrization and topological quantization of classical fields replacing the notion of superposition for fields with superposition for their effect.
- 2. Zero energy ontology means new view about quantum state. Quantum states as states with positive energy are replaced with zero energy states which are pairs of states with opposite quantum numbers and "live" at opposite boundaries of causal diamond (CD) which could be seen as spotlight of consciousness at the level of 8-D imbedding space.
- 3. Zero energy ontology leads to a new view about state function reduction identified as moment of consciousness. Consciousness is not anymore property of physical states but something between two physical states, in the moment of recreation. One ends up to ask difficult questions: how the experience flow of time experience in this picture, how the arrow of geometric time emerges from that of subjective time, is the arrow of geometric time same always, etc...
- 4. Hierarchy of Planck constants is also a new element in ontology and means extension of quantum theory. It is somewhat matter of taste whether one speaks about hierarchy of effective or real Planck constants and whether one introduces only coverings of space-time surface or also those of imbedding space to describe what is involved. What however seems clear that hierarchy of Planck constants follows from fundamental TGD naturally. The matter forms phases with different values of  $h_{eff}(h = n)$  and for large values of n this means macroscopic quantum coherence so that application to living matter is obvious challenge. The identification of these new phases as dark matter is the natural first working hypothesis.
- 5. p-Adic physics is a further new ontological and epistemological element. p-Adic numbers fields are completions of rational numbers in many respects analogous to reals and one can ask whether the notion of p-adic physics might make sense. The first success comes from elementary particle mass calculations based on p-adic thermodynamics combined with very general symmetry arguments. It turned out that the most natural interpretation of p-adic physics is as physics describing correlates of cognition and intentionality. This brings to the vocabulary p-adic spacetime sheets, p-adic counterparts of field equations, p-adic quantum theory, etc.. The need to fuse real and various p-adic physics to gain by number-theoretical universality becomes a powerful constraint on the theory.

The notion of negentropic entanglement is natural outcome of p-adic physics. This entanglement is very special: all entanglement probabilities are identical and unitary entanglement matrix gives rise to this kind of entanglement automatically. The U-matrix characterizing interactions indeed consists of unitary building blocks giving rise to negentropic entanglement. Negentropic entanglement tends to be respected by Negentropy Maximization Principle (NMP) which defines the basic variational principle of TGD inspired theory of consciousness and negentropic entanglement defines kind of Akaschic records which are approximate quantum invariants. They form kind of universal potentially conscious data basis, universal library. This obviously represents new epistemology.

#### 3.2 Space-time as 4-surface

#### 3.2.1 Energy problem of GRT as starting point

The physical motivation for TGD was what I have christened the energy problem of General Relativity, which has been already mentioned. The notion of energy is ill-defined because the basic symmetries of empty space-time are lost in the presence of gravity. The presence of matter curves empty Minkowski space  $M^4$  so that its rotational, translational and Lorentz symmetries realized as transformations leaving the distances between points and thus shapes of 4-D objects invariant. Noether's theorem states that symmetries and conservation laws correspond to each other so that conservations laws are lost: energy, momentum, and angular momentum are not only non-conserved but even ill-defined. The mathematical expression for this is that the energy momentum tensor is 2-tensor so that it is impossible to assign with it any conserved energy and momentum mathematically except in empty Minkowski space. Usually it is argued that this is not a practical problem since gravitation is so weak interaction. When one however tries to quantized general relativity, this kind of sloppiness cannot be

allowed, and the problem reason for the continual failure of the attempts to build a theory of quantum gravity might be tracked down to this kind of conceptual sloppiness.

The way out of the problem is based on assumption that space-times are imbeddable as 4-surfaces to certain 8-dimensional space by replacing the points of 4-D empty Minkowski space with 4-D very small internal space. This space -call it S- is unique from the requirement that the theory has the symmetries of standard model:  $S = CP_2$ , where  $CP_2$  is complex projective space with 4 real dimensions [L1], is the unique choice. Symmetries as isometries of space-time are lifted to those of imbedding space. Symmetry transformation does not move point of space-time along it but moves entire space-time surface. Space-time surface is like rigid body rotated, translated, and Lorentz boosted by symmetries. This means that Noether's theorem predicts the classical conserved charges once general coordinate action principle is written down.

Also now the curvature of space-time codes for gravitation. Now however the number of solutions to field equations is dramatically smaller than in Einstein's theory. An unexpected bonus was that a geometrization classical fields of standard model for  $S = CP_2$ . Later it turned out that also the counterparts for field quanta emerge naturally but this requires profound generalization of the notion of space-time so that topological inhomogenities of space-time surface are identified as particles. This meant a further huge reduction in dynamical field like variables. By general coordinate invariance only four imbedding space coordinates appear as variables analogous to classical fields: in a typical gut their number is hundreds.

 $CP_2$  also codes for the standard model quantum numbers in its geometry in the sense that electromagnetic charge and weak isospin emerge from  $CP_2$  geometry: the corresponding symmetries are not isometries so that electroweak symmetry breaking is coded already at this level. Color quantum numbers which correspond to the isometries of  $CP_2$  and are unbroken symmetry: this also conforms with empirical facts. The color of TGD however differs from that in standard model in several aspects and LHC has began to exhibit these differences via the unexpected behavior of what was believed to be quark gluon plasma. The conservation of baryon and lepton number follows as a prediction. Leptons and quarks correspond to opposite chiralities for fermions at the level of imbedding space.

What remains to be explained is family replication phenomenon for leptons and quarks which means that both quarks and leptons appear as three families which are identical except that they have different masses. Here the identification of particles as 2-D boundary components of 3-D surface inspired the conjecture that fermion families correspond to different topologies for 2-D surfaces characterized by genus telling the number g (genus) of handles attached to sphere to obtain the surface sphere, torus, ..... The identification as boundary component turned out to be too simplistic but can be replaced with partonic 2-surface assignable to light-like 3-surface at which the signature of the induced metric of space-time surface transforms from Minkowskian to Euclidian. This 3-D surfaces replace the lines of Feynman diagrams in TGD Universe in accordance with the replacement of point-like particle with 3-surface.

The problem was that only three lowest genera are observed experimentally. Are the genera g>2 very heavy or don't they exist. One ends up with a possible explanation in terms of conformal symmetries: the genera  $g\leq 2$  allow always two element group as subgroup of conformal symmetries (this is called hyper-ellipticity) whereas higher genera in general do not. Observed 3 particle families would have especially high conformal symmetries. This could explain why higher genera are very massive or not realized as elementary particles in the manner one would expect.

The surprising outcome is that  $M^4 \times CP_2$  codes for the standard model. Much later further arguments in favor of this choice have emerged. The latest one relates to twistorialization. 4-D Minkowski space is unique space-time with Minkowskian signature of metric in the sense that it allows twistor structure. This is a big problem in attempts to introduce twistors to General Relativity Theory (GRT) and very serious obstacle in quantization based on twistor Grassmann approach which has demonstrate its enormous power in the quantization of gauge theories. The obvious idea in TGD framework is whether one could lift also the twistor structure to the level of imbedding space  $M^4 \times CP_2$ .  $M^4$  has twistor structure and so does also  $CP_2$ : which is the only Euclidian 4-manifold allowing twistor space which is also Kähler manifold!

It soon became clear that TGD can be seen as a generalization of hadronic string model - not yet superstring model since this model became fashionable two years after the thesis about TGD. Later it has become clear that string like objects, which look like strings but are actually 3-D are basic stuff of TGD Universe and appear in all scales. Also strictly 2-D string world sheets pop up in the formulation of quantum TGD so that one can say that string model in 4-D space-time is part of TGD.

One can say that TGD generalizes standard model symmetries and provides a proposal for a dynamics which is incredibly simple as compared to the competing theories: only 4 classical field variables and in fermionic sector only quark and lepton like spinor fields. The basic objection against TGD looks rather obvious in the light of afterwisdom. One loses linear superposition of fields which holds in good approximation in ordinary field theories, which are almost linear. The solution of the problem relies on the notion many-sheeted space-time to be discussed below.

#### 3.2.2 Many-sheeted space-time

The replacement of the abstract manifold geometry of general relativity with the geometry of surfaces brings the shape of surface as seen from the perspective of 8-D space-time and this means additional degrees of freedom giving excellent hopes of realizing the dream of Einstein about geometrization of fundamental interactions.

The work with the generic solutions of the field equations assignable to almost any general coordinate invariant variational principle led soon to the realization that the space-time in this framework is much more richer than in general relativity.

1. Space-time decomposes into space-time sheets with finite size: this lead to the identification of physical objects that we perceive around us as space-time sheets. For instance, the outer boundary of the table is where that particular space-time sheet ends. We can directly see the complex topology of many-sheeted space-time! Besides sheets also string like objects and elementary particle like objects appear so that TGD can be regarded also as a generalization of string models obtained by replacing strings with 3-D surfaces.

What does one mean with space-time sheet? Originally it was identified as a piece of slightly deformed  $M^4$  in  $M^4 \times CP_2$  having boundary. It however became gradually clear that boundaries are probably not allowed since boundary conditions cannot be satisfied. Rather, it seems that sheet in this sense must be glued along its boundaries together with its deformed copy to get double covering. Sphere can be seen as simplest example of this kind of covering: northern and southern hemispheres are glued along equator together.

So: what happens to the identification of family replication in terms of genus of boundary of 3-surface and to the interpretation of the boundaries of physical objects as space-time boundaries? Do they correspond to the surfaces at which the gluing occurs? Or do they correspond to 3-D light-like surfaces at which the signature of the induced metric changes. My educated guess is that the latter option is correct but one must keep mind open since TGD is not an experimentally tested theory.

2. Elementary particles are roughly speaking identified as topological inhomogenities glued to these space-time sheets using topological sum contacts. This means roughly drilling a hole to both sheets and connecting with a cylinder. 2-dimensional illustration should give the idea. In this conceptual framework material structures and shapes are not due to some mysterious substance in slightly curved space-time but reduce to space-time topology just as energy-momentum currents reduce to space-time curvature in general relativity.

This view has gradually evolved to much more detailed picture. Without going to details one can say that particles have wormhole contacts as basic building bricks. Wormhole contact is very small Euclidian connecting two Minkowskian space-time sheets with light-like boundaries carrying spinor fields and there particle quantum numbers. Wormhole contact carries magnetic monopole flux through it and there must be second wormhole contact in order to have closed lines of magnetic flux. One might describe particle as a pair of magnetic monopoles with opposite charges. With some natural assumptions the explanation for the family replication phenomenon is not affected and nothing new is predicted. Bosons emerge as fermion anti-fermion pairs with fermion and anti-fermion at the opposite throats of the wormhole contact. In principle family replication phenomenon should have bosonic analog. This picture assigns to particles strings connecting the two wormhole throats at each space-time sheet so that string model mathematics becomes part of TGD.

The notion of classical field differs in TGD framework in many respects from that in Maxwellian theory.

- 1. In TGD framework fields do not obey linear superposition and all classical fields are expressible in terms of four imbedding space coordinates in given region of space-time surface. Superposition for classical fields is replaced with superposition of their effects. Particle can topologically condensed simultaneously to several space-time sheets by generating topological sum contacts. Particle experiences the superposition of the effects of the classical fields at various space-time sheets rather than the superposition of the fields. It is also natural to expect that at macroscopic length scales the physics of classical fields (to be distinguished from that for field quanta) can be explained using only four fields since only four primary field like variables are present. Electromagnetic gauge potential has only four components and classical electromagnetic fields give and excellent description of physics. This relates directly to electroweak symmetry breaking in color confinement which in standard model imply the effective absence of weak and color gauge fields in macroscopic scales. TGD however predicts that copies of hadronic physics and electroweak physics could exist in arbitrary long scales and there are indications that just this makes living matter so different as compared to inanimate matter.
- 2. The notion of induced field means that one induces electroweak gauge potentials defining so called spinor connection to space-time surface. Induction means locally a projection for the imbedding space vectors representing the spinor connection locally. This is essentially dynamics of shadows! The classical fields at the imbedding space level are non-dynamical and fixed and extremely simple: one can say that one has generalization of constant electric field and magnetic fields in  $\mathbb{C}P_2$ . The dynamics of the 3-surface however implies that induced fields can form arbitrarily complex field patterns.

Induced fields are not however equivalent with ordinary free fields. In particular, the attempt to represent constant magnetic or electric field as a space-time time surface has a limited success. Only a finite portion of space-time carrying this field allows realization as 4-surface. I call this topological field quantization. The magnetization of electric and magnetic fluxes is the outcome. Also gravitational field patterns allowing imbedding are very restricted: one implication is that topological with over-critical mass density are not globally imbeddable. This would explain why the mass density in cosmology can be at most critical. This solves one of the mysteries of GRT based cosmology. Quite generally the field patterns are extremely restricted: not only due to imbeddability constraint but also due to the fact that only very restricted set of space-time surfaces can appear solutions of field equations: I speak of preferred extremals. One might speak about archetypes at the level of physics: they are in quite strict sense analogies of Bohr orbits in atomic physics: this is implies by the realization of general coordinate invariance (GCI).

One might of course argue that this kind of simplicity does not conform with what we observed. The way out is many-sheeted space-time. Particles experience superposition of effects from the archetypal field configurations. Basic field patterns are simple but effects are complex!

The important implication is that one can assign to each material system a field identity since electromagnetic and other fields decompose to topological field quanta. Examples are magnetic and electric flux tubes and flux sheets and topological light rays representing light propagating along tube like structure without dispersion and dissipation making em ideal tool for communications [K14]. One can speak about field body or magnetic body of the system.

3. Field body indeed becomes the key notion distinguishing TGD inspired model of quantum biology from competitors but having applications also in particle physics since also leptons and quarks possess field bodies. The is evidence for the Lamb shift anomaly of muonic hydrogen [C1] and the color magnetic body of u quark whose size is somewhat larger than the Bohr radius could explain the anomaly [K10].

#### 3.3 The hierarchy of Planck constants

The motivations for the hierarchy of Planck constants come from both astrophysics and biology [K16, K4]. In astrophysics the observation of Nottale [E1] that planetary orbits in solar system seem to correspond to Bohr orbits with a gigantic gravitational Planck constant motivated the proposal that Planck constant might not be constant after all [K19, K15].



Figure 2: Many-sheeted space-time.

This led to the introduction of the quantization of Planck constant as an independent postulate. It has however turned that quantized Planck constant in effective sense could emerge from the basic structure of TGD alone. Canonical momentum densities and time derivatives of the imbedding space coordinates are the field theory analogs of momenta and velocities in classical mechanics. The extreme non-linearity and vacuum degeneracy of Kähler action imply that the correspondence between canonical momentum densities and time derivatives of the imbedding space coordinates is 1-to-many: for vacuum extremals themselves 1-to-infinite.

A convenient technical manner to treat the situation is to replace imbedding space with its n-fold singular covering. Canonical momentum densities to which conserved quantities are proportional would be same at the sheets corresponding to different values of the time derivatives. At each sheet of the covering Planck constant is effectively  $\hbar = n\hbar_0$ . This splitting to multi-sheeted structure can be seen as a phase transition reducing the densities of various charges by factor 1/n and making it possible to have perturbative phase at each sheet (gauge coupling strengths are proportional to  $1/\hbar$  and scaled down by 1/n). The connection with fractional quantum Hall effect [D1] is almost obvious. At the more detailed level one finds that the spectrum of Planck constants would be given by  $\hbar = n_a n_b \hbar_0$  [K6].

This has many profound implications, which are welcome from the point of view of quantum biology but the implications would be profound also from particle physics perspective and one could say that living matter represents zoome up version of quantum world at elementary particle length scales.

- 1. Quantum coherence and quantum superposition become possible in arbitrary long length scales. One can speak about zoomed up variants of elementary particles and zoomed up sizes make it possible to satisfy the overlap condition for quantum length parameters used as a criterion for the presence of macroscopic quantum phases. In the case of quantum gravitation the length scale involved are astrophysical. This would conform with Penrose's intuition that quantum gravity is fundamental for the understanding of consciousness and also with the idea that consciousness cannot be localized to brain.
- 2. Photons with given frequency can in principle have arbitrarily high energies by E=hf formula, and this would explain the strange anomalies associated with the interaction of ELF em fields with living matter [J1]. Quite generally the cyclotron frequencies which correspond to energies much below the thermal energy for ordinary value of Planck constant could correspond to energies above thermal threshold.
- 3. The value of Planck constant is a natural characterizer of the evolutionary level and biological evolution would mean a gradual increase of the largest Planck constant in the hierarchy characterizing given quantum system. Evolutionary leaps would have interpretation as phase transitions increasing the maximal value of Planck constant for evolving species. The space-time correlate would be the increase of both the number and the size of the sheets of the covering associated with the system so that its complexity would increase.
- 4. The phase transitions changing Planck constant change also the length of the magnetic flux tubes. The natural conjecture is that biomolecules form a kind of Indra's net connected by the flux tubes and  $\hbar$  changing phase transitions are at the core of the quantum bio-dynamics. The contraction of the magnetic flux tube connecting distant biomolecules would force them near to each other making possible for the bio-catalysis to proceed. This mechanism could be central

for DNA replication and other basic biological processes. Magnetic Indra's net could be also responsible for the coherence of gel phase and the phase transitions affecting flux tube lengths could induce the contractions and expansions of the intracellular gel phase. The reconnection of flux tubes would allow the restructuring of the signal pathways between biomolecules and other subsystems and would be also involved with ADP-ATP transformation inducing a transfer of negentropic entanglement [K8] . The braiding of the magnetic flux tubes could make possible topological quantum computation like processes and analog of computer memory realized in terms of braiding patterns [K5] .

5. p-Adic length scale hypothesis and hierarchy of Planck constants suggest entire hierarchy of zoomed up copies of standard model physics with range of weak interactions and color forces scaling like  $\hbar$ . This is not conflict with the known physics for the simple reason that we know very little about dark matter (partly because we might be making misleading assumptions about its nature). One implication is that it might be someday to study zoomed up variants particle physics at low energies using dark matter.

Dark matter would make possible the large parity breaking effects manifested as chiral selection of bio-molecules [C2]. What is required is that classical  $Z^0$  and W fields responsible for parity breaking effects are present in cellular length scale. If the value of Planck constant is so large that weak scale is some biological length scale, weak fields are effectively massless below this scale and large parity breaking effects become possible.

For the solutions of field equations which are almost vacuum extremals  $Z^0$  field is non-vanishing and proportional to electromagnetic field. The hypothesis that cell membrane corresponds to a space-time sheet near a vacuum extremal (this corresponds to criticality very natural if the cell membrane is to serve as an ideal sensory receptor) leads to a rather successful model for cell membrane as sensory receptor with lipids representing the pixels of sensory qualia chart. The surprising prediction is that bio-photons [I2] and bundles of EEG photons can be identified as different decay products of dark photons with energies of visible photons. Also the peak frequencies of sensitivity for photoreceptors are predicted correctly [K16] .

## 3.4 p-Adic physics and number theoretic universality

p-Adic physics [K11, K21] has become gradually a key piece of TGD inspired biophysics. Basic quantitative predictions relate to p-adic length scale hypothesis and to the notion of number theoretic entropy. Basic ontological ideas are that life resides in the intersection of real and p-adic worlds and that p-adic space-time sheets serve as correlates for cognition and intentionality. Number theoretical universality requires the fusion of real physics and various p-adic physics to single coherent whole. On implication is the generalization of the notion of number obtained by fusing real and p-adic numbers to a larger structure.

#### 3.4.1 p-Adic number fields

p-Adic number fields  $Q_p$  [A5] -one for each prime p- are analogous to reals in the sense that one can speak about p-adic continuum and that also p-adic numbers are obtained as completions of the field of rational numbers. One can say that rational numbers belong to the intersection of real and p-adic numbers. p-Adic number field  $Q_p$  allows also an infinite number of its algebraic extensions. Also transcendental extensions are possible. For reals the only extension is complex numbers.

p-Adic topology defining the notions of nearness and continuity differs dramatically from the real topology. An integer which is infinite as a real number can be completely well defined and finite as a p-adic number. In particular, powers  $p^n$  of prime p have p-adic norm (magnitude) equal to  $p^{-n}$  in  $Q_p$  so that at the limit of very large n real magnitude becomes infinite and p-adic magnitude vanishes.

p-Adic topology is rough since p-adic distance d(x,y) = d(x-y) depends on the lowest pinary digit of x-y only and is analogous to the distance between real points when approximated by taking into account only the lowest digit in the decimal expansion of x-y. A possible interpretation is in terms of a finite measurement resolution and resolution of sensory perception. p-Adic topology looks somewhat strange. For instance, p-adic spherical surface is not infinitely thin but has a finite thickness and p-adic surfaces possess no boundary in the topological sense. Ultrametricity is the technical term characterizing the basic properties of p-adic topology and is coded by the inequality

 $d(x-y) \leq Min\{d(x), d(y)\}$ . p-Adic topology brings in mind the decomposition of perceptive field to objects.

#### 3.4.2 Motivations for p-adic number fields

The physical motivations for p-adic physics came from the observation that p-adic thermodynamics -not for energy but infinitesimal scaling generator of so called super-conformal algebra [A2] acting as symmetries of quantum TGD [K17] - predicts elementary particle mass scales and also masses correctly under very general assumptions [K11] . The calculations are discussed in more detail in the second article of the series. In particular, the ratio of proton mass to Planck mass, the basic mystery number of physics, is predicted correctly. The basic assumption is that the preferred primes characterizing the p-adic number fields involved are near powers of two:  $p \simeq 2^k$ , k positive integer. Those nearest to power of two correspond to Mersenne primes  $M_n = 2^n - 1$ . One can also consider complex primes known as Gaussian primes, in particular Gaussian Mersennes  $M_{G,n} = (1+i)^n - 1$ .

It turns out that Mersennes and Gaussian Mersennes are in a preferred position physically in TGD based world order. What is especially interesting that the length scale range 10 nm-5  $\mu$ m contains as many as four scaled up electron Compton lengths  $L_e(k) = \sqrt{5}L(k)$  assignable to Gaussian Mersennes  $M_k = (1+i)^k - 1$ , k = 151, 157, 163, 167, [K16] . This number theoretical miracle supports the view that p-adic physics is especially important for the understanding of living matter.

The philosophical for p-adic numbers fields come from the question about the possible physical correlates of cognition and intention [K13]. Cognition forms representations of the external world which have finite cognitive resolution and the decomposition of the perceptive field to objects is an essential element of these representations. Therefore p-adic space-time sheets could be seen as candidates of thought bubbles, the mind stuff of Descartes. One can also consider p-adic space-time sheets as correlates of intentions. The quantum jump in which p-adic space-time sheet is replaced with a real one could serve as a quantum correlate of intentional action. This process is forbidden by conservation laws in standard ontology: one cannot even compare real and p-adic variants of the conserved quantities like energy in the general case. In zero energy ontology the net values of conserved quantities for zero energy states vanish so that conservation laws allow these transitions.

Rational numbers belong to the intersection of real and p-adic continua. An obvious generalization of this statement applies to real manifolds and their p-adic variants. When extensions of p-adic numbers are allowed, also some algebraic numbers can belong to the intersection of p-adic and real worlds. The notion of intersection of real and p-adic worlds has actually two meanings.

- 1. The intersection could consist of the rational and possibly some algebraic points in the intersection of real and p-adic partonic 2-surfaces at the ends of CD. This set is in general discrete. The interpretation could be as discrete cognitive representations.
- 2. The intersection could also have a more abstract meaning. For instance, the surfaces defined by rational functions with rational coefficients have a well-defined meaning in both real and p-adic context and could be interpreted as belonging to this intersection. There is strong temptation to assume that intentions are transformed to actions only in this intersection. One could say that life resides in the intersection of real and p-adic worlds in this abstract sense.

Additional support for the idea comes from the observation that Shannon entropy  $S = -\sum p_n log(p_n)$  allows a p-adic generalization if the probabilities are rational numbers by replacing  $log(p_n)$  with  $-log(|p_n|_p)$ , where  $|x|_p$  is p-adic norm. Also algebraic numbers in some extension of p-adic numbers can be allowed. The unexpected property of the number theoretic Shannon entropy is that it can be negative and its unique minimum value as a function of the p-adic prime p it is always negative. Entropy transforms to information!

In the case of number theoretic entanglement entropy there is a natural interpretation for this. Number theoretic entanglement entropy would measure the information carried by the entanglement whereas ordinary entanglement entropy would characterize the uncertainty about the state of either entangled system. For instance, for p maximally entangled states both ordinary entanglement entropy and number theoretic entanglement negentropy are maximal with respect to  $R_p$  norm. Entanglement carries maximal information. The information would be about the relationship between the systems, a rule. Schrödinger cat would be dead enough to know that it is better to not open the bottle completely.

Negentropy Maximization Principle (NMP) [K9] coding the basic rules of quantum measurement theory implies that negentropic entanglement can be stable against the effects of quantum jumps unlike entropic entanglement. Therefore living matter could be distinguished from inanimate matter also by negentropic entanglement possible in the intersection of real and p-adic worlds. In consciousness theory negentropic entanglement could be seen as a correlate for the experience of understanding or any other positively colored experience, say love.

Negentropically entangled states are stable but binding energy and effective loss of relative translational degrees of freedom is not responsible for the stability. Therefore bound states are not in question. The distinction between negentropic and bound state entanglement could be compared to the difference between unhappy and happy marriage. The first one is a social jail but in the latter case both parties are free to leave but do not want to. The special characterics of negentropic entanglement raise the question whether the problematic notion of high energy phosphate bond [I1] central for metabolism could be understood in terms of negentropic entanglement. This would also allow an information theoretic interpretation of metabolism since the transfer of metabolic energy would mean a transfer of negentropy [K8] .

# 3.5 Zero energy ontology

#### 3.5.1 Zero energy state as counterpart of physical event

In standard ontology of quantum physics physical states are assumed to have positive energy. In zero energy ontology physical states decompose to pairs of positive and negative energy states such that all net values of the conserved quantum numbers vanish. The interpretation of these states in ordinary ontology would be as transitions between initial and final states, physical events.

Zero energy ontology conforms with the crossing symmetry of quantum field theories meaning that the final states of the quantum scattering event are effectively negative energy states. As long as one can restrict the consideration to either positive or negative energy part of the state ZEO is consistent with positive energy ontology. This is the case when the observer characterized by a particular CD studies the physics in the time scale of much larger CD containing observer's CD as a sub-CD. When the time scale sub-CD of the studied system is much shorter that the time scale of sub-CD characterizing the observer, the interpretation of states associated with sub-CD is in terms of quantum fluctuations.

ZEO solves the problem which results in any theory assuming symmetries giving rise to to conservation laws. The problem is that the theory itself is not able to characterize the values of conserved quantum numbers of the initial state. In ZEO this problem disappears since in principle any zero energy state is obtained from any other state by a sequence of quantum jumps without breaking of conservation laws. The fact that energy is not conserved in general relativity based cosmologies can be also understood since each CD is characterized by its own conserved quantities. As a matter fact, one must be speak about average values of conserved quantities since one can have a quantum superposition of zero energy states with the quantum numbers of the positive energy part varying over some range.

At the level of principle the implications are quite dramatic. In quantum jump as recreation replacing the quantum Universe with a new one it is possible to create entire sub-universes from vacuum without breaking the fundamental conservation laws. Free will is consistent with the laws of physics. This makes obsolete the basic arguments in favor of materialistic and deterministic world view.

#### 3.5.2 Zero energy states are located inside causal diamond (CD)

By quantum classical correspondence zero energy states must have space-time and imbedding space correlates.

1. Positive and negative energy parts reside at future and past light-like boundaries of causal diamond (CD) defined as intersection of future and past directed light-cones and visualizable as double cone. The analog of CD in cosmology is big bang followed by big crunch. CDs for a fractal hierarchy containing CDs within CDs. Disjoint CDs are possible and CDs can also intersect.

The interpretation of CD in TGD inspired theory of consciousness is as an imbedding space correlate for the spot-light of consciousness: the contents of conscious experience is about the region defined by CD. At the level of space-time sheets the experience come from space-time sheets restricted to the interior of CD. Whether the sheets can continue outside CD is still unclear.

2. p-Adic length scale hypothesis [K12] motivates the hypothesis that the temporal distances between the tips of the intersecting light-cones come as octaves  $T=2^nT_0$  of a fundamental time scale  $T_0$  defined by  $CP_2$  size R as  $T_0=R/c$ . One prediction is that in the case of electron this time scale is .1 seconds defining the fundamental biorhythm. Also in the case u and d quarks the time scales correspond to biologically important time scales given by 10 ms for u quark and by and 2.5 ms for d quark [K1]. This means a direct coupling between microscopic and macroscopic scales.

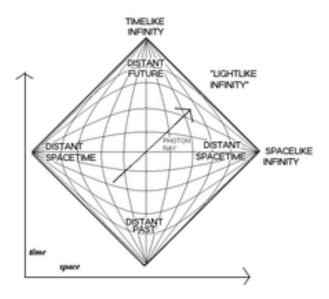


Figure 3: The 2-D variant of CD is equivalent with Penrose diagram in empty Minkowski space.

#### 3.5.3 Quantum theory as square root of thermodynamics

Quantum theory in ZEO can be regarded as a "complex square root" of thermodynamics obtained as a product of positive diagonal square root of density matrix and unitary S-matrix. M-matrix defines time-like entanglement coefficients between positive and negative energy parts of the zero energy state and replaces S-matrix as the fundamental observable. Various M-matrices define the rows of the unitary U matrix characterizing the unitary process part of quantum jump.

The fact that M-matrices are products of Hermitian square roots (operator analog for real variable) of Hermitian density matrix multiplied by a unitary S-matrix S with they commute implies that possible U-matrices for an algebra generalizing Kac-Moody algebra defining Kac-Moody type symmetries of the S-matrix. This might mean final step in the reduction of theories to their symmetries since the states predicted by the theory would generate its symmetries!

#### 3.5.4 State function reduction, arrow of time in ZEO, and Akaschic records

From the point of view of consciousness theory the importance of ZEO is that conservation laws in principle pose no restrictions for the new realities created in quantum jumps: free will is maximal. In standard quantum measurement theory this time-like entanglement would be reduced in quantum measurement and regenerated in the next quantum jump if one accepts Negentropy Maximization Principle (NMP) [K9] as the fundamental variational principle.

CD as two light-like boundaries corresponding to the positive and negative energy parts of zero energy states which correspond to initial and final states of physical event. State function reduction can occur to both of these boundaries.

- 1. If state function reductions occur alternately- one at time- then it is very difficult to understand why we experience same arrow of time continually: why not continual flip-flop at the level of perceptions. Some people claim to have actually experienced a temporary change of the arrow of time: I belong to them and I can tell that the experience is frightening. Why we experience the arrow of time as constant?
- 2. One possible way to solve this problem perhaps the simplest one is that state function reduction to the same boundary of CD can occur many times repeatedly. This solution is so absolutely trivial that I could perhaps use this triviality to defend myself for not realizing it immediately! I made this totally trivial observation only after I had realized that also in this process the wave function in the moduli space of CDs could change in these reductions. Zeno effect in ordinary measurement theory relies on the possibility of repeated state function reductions. In the ordinary quantum measurement theory repeated state function reductions don't affect the state in this kind of sequence but in ZEO the wave function in the moduli space labelling different CDs with the same boundary could change in each quantum jump. It would be natural that this sequence of quantum jumps give rise to the experience about flow of time?
- 3. This option would allow the size scale of CD associated with human consciousness be rather short, say .1 seconds. It would also allow to understand why we do not observe continual change of arrow of time. Maybe living systems are working hardly to keep the personal arrow of time changed and that it would be extremely difficult to live against the collective arrow of time.

NMP implies that negentropic entanglement generated in state function reductions tends to increase. This tendency is mirror image of entropy growth for ensembles and would provide a natural explanation for evolution as something real rather than just thermodynamical fluctuation as standard thermodynamics suggests. Quantum Universe is building kind of Akashic records. The history would be recorded in a huge library and these books could might be read by interaction free quantum measurements giving conscious information about negentropically entangled states and without changing them: as a matter fact, this is an idealization. Conscious information would require also now state function reduction but it would occur for another system. Elitzur-Vaidman bomb tester(http://en.wikipedia.org/wiki/ElitzurVaidman\_bomb-testing\_problem) is a down-to-earth representation for what is involved.

# 4 Different visions about TGD as mathematical theory

There are two basic vision about Quantum TGD: physics as infinite-dimensional geometry and physics as generalized number theory.

# 4.1 Quantum TGD as spinor geometry of World of Classical Worlds

A turning point in the attempts to formulate a mathematical theory was reached after seven years from the birth of TGD. The great insight was "Do not quantize". The basic ingredients to the new approach have served as the basic philosophy for the attempt to construct Quantum TGD since then and have been the following ones:

1. Quantum theory for extended particles is free(!), classical(!) field theory for a generalized Schrödinger amplitude in the WCW CH consisting of all possible 3-surfaces in H. "All possible" means that surfaces with arbitrary many disjoint components and with arbitrary internal topology and also singular surfaces topologically intermediate between two different manifold topologies are included. Particle reactions are identified as topology changes [A7, A8, A9]. For instance, the decay of a 3-surface to two 3-surfaces corresponds to the decay  $A \rightarrow B + C$ . Classically this corresponds to a path of WCW leading from 1-particle sector to 2-particle sector. At quantum level this corresponds to the dispersion of the generalized Schrödinger amplitude localized to 1-particle sector to two-particle sector. All coupling constants should result as predictions of the theory since no nonlinearities are introduced.

- 2. During years this naive and very rough vision has of course developed a lot and is not anymore quite equivalent with the original insight. In particular, the space-time correlates of Feynman graphs have emerged from theory as Euclidian space-time regions and the strong form of General Coordinate Invariance has led to a rather detailed and in many respects un-expected visions. This picture forces to give up the idea about smooth space-time surfaces and replace space-time surface with a generalization of Feynman diagram in which vertices represent the failure of manifold property. I have also introduced the word "world of classical worlds" (WCW) instead of rather formal "WCW". I hope that "WCW" does not induce despair in the reader having tendency to think about the technicalities involved!
- 3. WCW is endowed with metric and spinor structure so that one can define various metric related differential operators, say Dirac operator, appearing in the field equations of the theory. The most ambitious dream is that zero energy states correspond to a complete solution basis for the Dirac operator of WCW so that this classical free field theory would dictate M-matrices which form orthonormal rows of what I call U-matrix. Given M-matrix in turn would decompose to a product of a hermitian density matrix and unitary S-matrix.
  - M-matrix would define time-like entanglement coefficients between positive and negative energy parts of zero energy states (all net quantum numbers vanish for them) and can be regarded as a hermitian square root of density matrix multiplied by a unitary S-matrix. Quantum theory would be in well-defined sense a square root of thermodynamics. The orthogonality and hermiticity of the complex square roots of density matrices commuting with S-matrix means that they span infinite-dimensional Lie algebra acting as symmetries of the S-matrix. Therefore quantum TGD would reduce to group theory in well-defined sense: its own symmetries would define the symmetries of the theory. In fact the Lie algebra of Hermitian M-matrices extends to Kac-Moody type algebra obtained by multiplying hermitian square roots of density matrices with powers of the S-matrix. Also the analog of Yangian algebra involving only non-negative powers of S-matrix is possible.
- 4. By quantum classical correspondence the construction of WCW spinor structure reduces to the second quantization of the induced spinor fields at space-time surface. The basic action is so called modified Dirac action in which gamma matrices are replaced with the modified gamma matrices defined as contractions of the canonical momentum currents with the imbedding space gamma matrices. In this manner one achieves super-conformal symmetry and conservation of fermionic currents among other things and consistent Dirac equation. This modified gamma matrices define as anticommutators effective metric, which might provide geometrization for some basic observables of condensed matter physics. The conjecture is that Dirac determinant for the modified Dirac action gives the exponent of Kähler action for a preferred extremal as vacuum functional so that one might talk about bosonic emergence in accordance with the prediction that the gauge bosons and graviton are expressible in terms of bound states of fermion and antifermion.

The evolution of these basic ideas has been rather slow but has gradually led to a rather beautiful vision. One of the key problems has been the definition of Kähler function. Kähler function is Kähler action for a preferred extremal assignable to a given 3-surface but what this preferred extremal is? The obvious first guess was as absolute minimum of Kähler action but could not be proven to be right or wrong. One big step in the progress was boosted by the idea that TGD should reduce to almost topological QFT in which braids wold replace 3-surfaces in finite measurement resolution, which could be inherent property of the theory itself and imply discretization at partonic 2-surfaces with discrete points carrying fermion number.

1. TGD as almost topological QFT vision suggests that Kähler action for preferred extremals reduces to Chern-Simons term assigned with space-like 3-surfaces at the ends of space-time (recall the notion of causal diamond (CD)) and with the light-like 3-surfaces at which the signature of the induced metric changes from Minkowskian to Euclidian. Minkowskian and Euclidian regions would give at wormhole throats the same contribution apart from coefficients and in Minkowskian regions the  $\sqrt{g_4}$  factor would be imaginary so that one would obtain sum of real term identifiable as Kähler function and imaginary term identifiable as the ordinary action giving rise to interference effects and stationary phase approximation central in both classical

and quantum field theory. Imaginary contribution - the presence of which I realized only after 33 years of TGD - could also have topological interpretation as a Morse function. On physical side the emergence of Euclidian space-time regions is something completely new and leads to a dramatic modification of the ideas about black hole interior.

- 2. The manner to achieve the reduction to Chern-Simons terms is simple. The vanishing of Coulombic contribution to Kähler action is required and is true for all known extremals if one makes a general ansatz about the form of classical conserved currents. The so called weak form of electric-magnetic duality defines a boundary condition reducing the resulting 3-D terms to Chern-Simons terms. In this manner almost topological QFT results. But only "almost" since the Lagrange multiplier term forcing electric-magnetic duality implies that Chern-Simons action for preferred extremals depends on metric.
- 3. A further quite recent hypothesis inspired by effective 2-dimensionality is that Chern-Simons terms reduce to a sum of two 2-dimensional terms. An imaginary term proportional to the total area of Minkowskian string world sheets and a real term proportional to the total area of partonic 2-surfaces or equivalently strings world sheets in Euclidian space-time regions. Also the equality of the total areas of strings world sheets and partonic 2-surfaces is highly suggestive and would realize a duality between these two kinds of objects. String world sheets indeed emerge naturally for the proposed ansatz defining preferred extremals. Therefore Kähler action would have very stringy character apart from effects due to the failure of the strict determinism meaning that radiative corrections break the effective 2-dimensionality.

The definition of spinor structure - in practice definition of so called gamma matrices of WCW-and WCW Kähler metric define by their anti-commutators has been also a very slow process. The progress in the physical understanding of the theory and the wisdom that has emerged about preferred extremals of Kähler action and about general solution of the field equations for modified Dirac operator during last decade have led to a considerable progress in this respect quite recently.

- 1. Preferred extremals of Kähler action [K2] seem to have slicing to string world sheets and partonic 2-surfaces such that points of partonic 2-surface slice parametrize different world sheets. I have christened this slicing as Hamilton-Jacobi structure. This slicing brings strongly in mind string models.
- 2. The modes of the modified Dirac action fixed uniquely by Kähler action by the requirement of super-conformal symmetry and internal consistency must be localized to 2-dimensional string world sheets with one exception: the modes of right handed neutrino which do not mix with left handed neutrino, which are delocalized into entire space-time sheet. The localization follows from the condition that modes have well-defined em charge in presence of classical W boson fields. This implies that string model in 4-D space-time becomes part of TGD.

This input leads to a modification of the earlier construction allowing to overcome its features vulnerable to critics. The earlier proposal forced strong form of holography in sense which looked too strong. The data about WCW geometry was localized at partonic 2-surfaces rather than 3-surfaces. The new formulations uses data also from interior of 3-surfaces and this is due to replacement of point-like particle with string: point of partonic 2-surface -wormhole throat- is replaced with a string connecting it to another wormhole throat. The earlier approach used only single mode of induced spinor field: right-handed neutrino. Now all modes of induced spinor field are used and one obtains very concrete connection between elementary particle quantum numbers and WCW geometry.

#### 4.2 TGD as a generalized number theory

Quantum T(opological)D(ynamics) as a classical spinor geometry for infinite-dimensional WCW, padic numbers and quantum TGD, and TGD inspired theory of consciousness, have been for last ten years the basic three strongly interacting threads in the tapestry of quantum TGD. The fourth thread deserves the name 'TGD as a generalized number theory'. It involves three separate threads: the fusion of real and various p-adic physics to a single coherent whole by requiring number theoretic universality discussed already, the formulation of quantum TGD in terms of hyper-counterparts of classical number fields identified as sub-spaces of complexified classical number fields with Minkowskian signature of the metric defined by the complexified inner product, and the notion of infinite prime.

#### 4.2.1 p-Adic TGD and fusion of real and p-adic physics to single coherent whole

The p-adic thread emerged for roughly ten years ago as a dim hunch that p-adic numbers might be important for TGD. Experimentation with p-adic numbers led to the notion of canonical identification mapping reals to p-adics and vice versa. The breakthrough came with the successful p-adic mass calculations using p-adic thermodynamics for Super-Virasoro representations with the super-Kac-Moody algebra associated with a Lie-group containing standard model gauge group. Although the details of the calculations have varied from year to year, it was clear that p-adic physics reduces not only the ratio of proton and Planck mass, the great mystery number of physics, but all elementary particle mass scales, to number theory if one assumes that primes near prime powers of two are in a physically favored position. Why this is the case, became one of the key puzzles and led to a number of arguments with a common gist: evolution is present already at the elementary particle level and the primes allowed by the p-adic length scale hypothesis are the fittest ones.

It became very soon clear that p-adic topology is not something emerging in Planck length scale as often believed, but that there is an infinite hierarchy of p-adic physics characterized by p-adic length scales varying to even cosmological length scales. The idea about the connection of p-adics with cognition motivated already the first attempts to understand the role of the p-adics and inspired 'Universe as Computer' vision but time was not ripe to develop this idea to anything concrete (p-adic numbers are however in a central role in TGD inspired theory of consciousness). It became however obvious that the p-adic length scale hierarchy somehow corresponds to a hierarchy of intelligences and that p-adic prime serves as a kind of intelligence quotient. Ironically, the almost obvious idea about p-adic regions as cognitive regions of space-time providing cognitive representations for real regions had to wait for almost a decade for the access into my consciousness.

There were many interpretational and technical questions crying for a definite answer.

- 1. What is the relationship of p-adic non-determinism to the classical non-determinism of the basic field equations of TGD? Are the p-adic space-time region genuinely p-adic or does p-adic topology only serve as an effective topology? If p-adic physics is direct image of real physics, how the mapping relating them is constructed so that it respects various symmetries? Is the basic physics p-adic or real (also real TGD seems to be free of divergences) or both? If it is both, how should one glue the physics in different number field together to get *The Physics*? Should one perform p-adicization also at the level of the WCW of 3-surfaces? Certainly the p-adicization at the level of super-conformal representation is necessary for the p-adic mass calculations.
- 2. Perhaps the most basic and most irritating technical problem was how to precisely define p-adic definite integral which is a crucial element of any variational principle based formulation of the field equations. Here the frustration was not due to the lack of solution but due to the too large number of solutions to the problem, a clear symptom for the sad fact that clever inventions rather than real discoveries might be in question. Quite recently I however learned that the problem of making sense about p-adic integration has been for decades central problem in the frontier of mathematics and a lot of profound work has been done along same intuitive lines as I have proceeded in TGD framework. The basic idea is certainly the notion of algebraic continuation from the world of rationals belonging to the intersection of real world and various p-adic worlds.

Despite these frustrating uncertainties, the number of the applications of the poorly defined p-adic physics grew steadily and the applications turned out to be relatively stable so that it was clear that the solution to these problems must exist. It became only gradually clear that the solution of the problems might require going down to a deeper level than that represented by reals and p-adics.

The key challenge is to fuse various p-adic physics and real physics to single larger structures. This has inspired a proposal for a generalization of the notion of number field by fusing real numbers and various p-adic number fields and their extensions along rationals and possible common algebraic numbers. This leads to a generalization of the notions of imbedding space and space-time concept and one can speak about real and p-adic space-time sheets. The quantum dynamics should be such that it allows quantum transitions transforming space-time sheets belonging to different number fields to each other. The space-time sheets in the intersection of real and p-adic worlds are of special interest and the hypothesis is that living matter resides in this intersection. This leads to surprisingly detailed predictions and far reaching conjectures. For instance, the number theoretic generalization of entropy concept allows negentropic entanglement central for the applications to living matter.

The basic principle is number theoretic universality stating roughly that the physics in various number fields can be obtained as completion of rational number based physics to various number fields. Rational number based physics would in turn describe physics in finite measurement resolution and cognitive resolution. The notion of finite measurement resolution has become one of the basic principles of quantum TGD and leads to the notions of braids as representatives of 3-surfaces and inclusions of hyper-finite factors as a representation for finite measurement resolution.

The proposal for a concrete realization of this program at space-time level is in terms of the notion of p-adic manifold [K25] generalising the notion of real manifold. Chart maps of p-adic manifold are however not p-adic but real and mediated by a variant of canonical correspondence between real and p-adic numbers. This modification of the notion of chart map allows to circumvent the grave difficulties caused by p-adic topology. Also p-adic manifolds can serve as charts for real manifolds and now the interpretation is as cognitive representation. The coordinate maps are characterized by finite measurement/cognitive resolution and are not completely unique. The basic principle reducing part of the non-uniqueness is the condition that preferred extremals are mapped to preferred extremals: actually this requires finite measurement resolution.

#### 4.2.2 The role of classical number fields

The vision about the physical role of the classical number fields relies on the notion of number theoretic compactification stating that space-time surfaces can be regarded as surfaces of either  $M^8$  or  $M^4 \times CP_2$ . As surfaces of  $M^8$  identifiable as space of hyper-octonions they are hyper-quaternionic or co-hyper-quaternionic- and thus maximally associative or co-associative. This means that their tangent space is either hyper-quaternionic plane of  $M^8$  or an orthogonal complement of such a plane. These surface can be mapped in natural manner to surfaces in  $M^4 \times CP_2$  [K21] provided one can assign to each point of tangent space a hyper-complex plane  $M^2(x) \subset M^4$ . One can also speak about  $M^8 - H$  duality.

This vision has very strong predictive power. It predicts that the extremals of Kähler action correspond to either hyper-quaternionic or co-hyper-quaternionic surfaces such that one can assign to tangent space at each point of space-time surface a hyper-complex plane  $M^2(x) \subset M^4$ . As a consequence, the  $M^4$  projection of space-time surface at each point contains  $M^2(x)$  and its orthogonal complement. These distributions are integrable implying that space-time surface allows dual slicings defined by string world sheets  $Y^2$  and partonic 2-surfaces  $X^2$ . The existence of this kind of slicing was earlier deduced from the study of extremals of Kähler action and christened as Hamilton-Jacobi structure. The physical interpretation of  $M^2(x)$  is as the space of non-physical polarizations and the plane of local 4-momentum.

One can fairly say, that number theoretical compactification is responsible for most of the understanding of quantum TGD that has emerged during last years. This includes the realization of Equivalence Principle at space-time level, dual formulations of TGD as Minkowskian and Euclidian string model type theories, the precise identification of preferred extremals of Kähler action as extremals for which second variation vanishes (at least for deformations representing dynamical symmetries) and thus providing space-time correlate for quantum criticality, the notion of number theoretic braid implied by the basic dynamics of Kähler action and crucial for precise construction of quantum TGD as almost-topological QFT, the construction of WCW metric and spinor structure in terms of second quantized induced spinor fields with modified Dirac action defined by Kähler action realizing automatically the notion of finite measurement resolution and a connection with inclusions of hyper-finite factors of type II<sub>1</sub> about which Clifford algebra of WCW represents an example.

The two most important number theoretic conjectures relate to the preferred extremals of Kähler action. The general idea is that classical dynamics for the preferred extremals of Kähler action should reduce to number theory: space-time surfaces should be either associative or co-associative in some sense

1. The first meaning for associativity (co-associativity) would be that tangent (normal) spaces of space-time surfaces are quaternionic in some sense and thus associative. This can be formulated in terms of octonionic representation of the imbedding space gamma matrices possible in dimension D=8 and states that induced gamma matrices generate quaternionic sub-algebra at each space-time point. It seems that induced rather than modified gamma matrices must be in question.

- 2. Second meaning for associative (co-associativity) would be following. In the case of complex numbers the vanishing of the real part of real-analytic function defines a 1-D curve. In octnionic case one can decompose octonion to sum of quaternion and quaternion multiplied by an octonionic imaginary unit. Quaternionicity could mean that space-time surfaces correspond to the vanishing of the imaginary part of the octonion real-analytic function. Co-quaternionicity would be defined in an obvious manner. Octonionic real analytic functions form a function field closed also with respect to the composition of functions. Space-time surfaces would form the analog of function field with the composition of functions with all operations realized as algebraic operations for space-time surfaces. Co-associativity could be perhaps seen as an additional feature making the algebra in question also co-algebra.
- 3. The third conjecture is that these conjectures are equivalent.

#### 4.2.3 Infinite primes

The discovery of the hierarchy of infinite primes and their correspondence with a hierarchy defined by a repeatedly second quantized arithmetic quantum field theory gave a further boost for the speculations about TGD as a generalized number theory. The work with Riemann hypothesis led to further ideas.

After the realization that infinite primes can be mapped to polynomials representable as surfaces geometrically, it was clear how TGD might be formulated as a generalized number theory with infinite primes forming the bridge between classical and quantum such that real numbers, p-adic numbers, and various generalizations of p-adics emerge dynamically from algebraic physics as various completions of the algebraic extensions of rational (hyper-)quaternions and (hyper-)octonions. Complete algebraic, topological and dimensional democracy would characterize the theory.

What is especially satisfying is that p-adic and real regions of the space-time surface could emerge automatically as solutions of the field equations. In the space-time regions where the solutions of field equations give rise to in-admissible complex values of the imbedding space coordinates, p-adic solution can exist for some values of the p-adic prime. The characteristic non-determinism of the p-adic differential equations suggests strongly that p-odic regions correspond to 'mind stuff', the regions of space-time where cognitive representations reside. This interpretation implies that p-adic physics is physics of cognition. Since Nature is probably an extremely brilliant simulator of Nature, the natural idea is to study the p-adic physics of the cognitive representations to derive information about the real physics. This view encouraged by TGD inspired theory of consciousness clarifies difficult interpretational issues and provides a clear interpretation for the predictions of p-adic physics.

# 5 Guiding Principles

# 5.1 Physics is unique from the mathematical existence of WCW

- 1. The conjecture inspired by the geometry of loop spaces [A6] is that H is fixed from the mere requirement that the infinite-dimensional Kähler geometry exists. WCW must reduce to a union of symmetric spaces having infinite-dimensional isometry groups and labeled by zero modes having interpretation as classical dynamical variables.
  - This requires infinite-dimensional symmetry groups. At space-time level super-conformal symmetries are possible only if the basic dynamical objects can be identified as light-like or space-like 3-surfaces. At imbedding space level there are extended super-conformal symmetries assignable to the light-cone of H if the Minkowski space factor is four-dimensional.
- 2. The great vision has been that the second quantization of the induced spinor fields can be understood geometrically in terms of the WCW spinor structure in the sense that the anti-commutation relations for WCW gamma matrices require anti-commutation relations for the oscillator operators for free second quantized induced spinor fields defined at space-time surface. This means geometrization of Fermi statistics usually regarded as one of the purely quantal features of quantum theory.

#### 5.2 Number theoretical Universality

The original view about physics as the geometry of WCW is not enough to meet the challenge of unifying real and p-adic physics to a single coherent whole. This inspired "physics as a generalized number theory" approach [K20].

#### 5.2.1 Fusion of real and padic physics to single coherent whole

Fusion of real and p-adic physics to single coherent whole is the first part in the program aiming to realize number theoretical universality.

- 1. The first element is a generalization of the notion of number obtained by "gluing" reals and various p-adic number fields and their algebraic extensions along common rationals and algebraics to form a larger structure.
- 2. At the level of imbedding space this gluing corresponds to a gluing of real and p-adic variants of the imbedding space together along rational and common algebraic points (the number of which depends on algebraic extension of p-adic numbers used) to what could be seen as a book like structure. General Coordinate Invariance restricted to rationals or their extension requires preferred coordinates for  $CD \times CP_2$  and this kind coordinates can be fixed by isometries of H. The coordinates are however not completely unique since non-rational isometries produce new equally good choices. Whether this can be seen as an objection against the approach is not clear.
- 3. The analogous gluing of real and various p-adic physics to a larger structure forces to ask what are the common points of WCWs associated with real and various p-adic worlds. What it is to be a partonic 2-surface belonging to the intersection of real and p-adic variants of WCW? The natural answer is that partonic 2-surfaces which have a mathematical representation making sense both for real numbers and p-adic numbers or their algebraic extensions can be regarded as "common points" or identifiable points of p-adicity and reality. This of course applies also to partonic 2-surfaces corresponding to two different p-adic number fields. This mathematical property means a representability in terms of ratios of polynomials with rational (or possibly even algebraic) coefficients in the preferred imbedding space coordinates.
- 4. The intersections of WCWs and partonic 2-surfaces in different number fields are involved. An attractive idea is that only the information about common points of surfaces belonging to different number fields code for physics so that number-theoretically universal part of physics is number theoretical physics relying only on rationals and their algebraic extensions. For instance, the transition amplitudes between p-adic and real variants of partonic 2-surface can involve only the data at these points. This suggests the existence of what might be called number theoretical QFT. At space-time level this extension of introduce a discretization at space-time level in terms of rational and algebraic points common to real space-time sheets and their p-adic variants. The number of these points is in general finite for a given CD and the proposed interpretation is in terms of cognitive representations. The discrete intersections would define the initial and final points of number theoretical braids central for the formulation of the theory in finite measurement resolution.
- 5. Much later came the realization that living matter or what makes living matter living could be interpreted as something in this intersection of real and p-adic worlds so that number theoretic QFT might apply to crucial aspects of living matter.

# 5.2.2 Classical number fields and associativity and commutativity as fundamental law of physics

The dimensions of classical number fields appear as dimensions of basic objects in quantum TGD. Imbedding space has dimension 8, space-time has dimension 4, light-like 3-surfaces are orbits of 2-D partonic surfaces. If conformal QFT applies to 2-surfaces (this is questionable), one-dimensional structures would be the basic objects. The lowest level would correspond to discrete sets of points identifiable as intersections of real and p-adic space-time sheets. This suggests that besides p-adic

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number fields also classical number fields (reals, complex numbers, quaternions, octonions [A4]) are involved [K21] and the notion of geometry generalizes considerably. In the recent view about quantum TGD the dimensional hierarchy defined by classical number field indeed plays a key role.  $H = M^4 \times CP_2$  has a number theoretic interpretation and standard model symmetries can be understood number theoretically as symmetries of hyper-quaternionic planes of hyper-octonionic space.

The associativity condition A(BC) = (AB)C suggests itself as a fundamental physical law of both classical and quantum physics. Commutativity can be considered as an additional condition. In conformal field theories associativity condition indeed fixes the n-point functions of the theory. At the level of classical TGD space-time surfaces could be identified as maximal associative (hyper-quaternionic) submanifolds of the imbedding space whose points contain a preferred hyper-complex plane  $M^2$  in their tangent space and the hierarchy finite fields-rationals-reals-complex numbers-quaternions-octonions could have direct quantum physical counterpart [K21]. This leads to the notion of number theoretic compactification analogous to the dualities of M-theory: one can interpret space-time surfaces either as hyper-quaternionic 4-surfaces of  $M^8$  or as 4-surfaces in  $M^4 \times CP_2$ . As a matter fact, commutativity in number theoretic sense is a further natural condition and leads to the notion of number theoretic braid naturally as also to direct connection with super string models.

At the level of modified Dirac action the identification of space-time surface as a hyper-quaternionic submanifold of H means that the modified gamma matrices of the space-time surface defined in terms of canonical momentum currents of Kähler action using octonionic representation for the gamma matrices of H span a hyper-quaternionic sub-space of hyper-octonions at each point of space-time surface (hyper-octonions are the subspace of complexified octonions for which imaginary units are octonionic imaginary units multiplied by commutating imaginary unit). Hyper-octonionic representation leads to a proposal for how to extend twistor program to TGD framework [K7, K22].

### 5.3 Symmetries

#### 5.3.1 Magic properties of light cone boundary and isometries of WCW

The special conformal, metric and symplectic properties of the light cone of four-dimensional Minkowski space:  $\delta M_+^4$ , the boundary of four-dimensional light cone is metrically 2-dimensional(!) sphere allowing infinite-dimensional group of conformal transformations and isometries(!) as well as Kähler structure. Kähler structure is not unique: possible Kähler structures of light cone boundary are parameterized by Lobatchevski space SO(3,1)/SO(3). The requirement that the isotropy group SO(3) of  $S^2$  corresponds to the isotropy group of the unique classical 3-momentum assigned to  $X^4(Y^3)$  defined as a preferred extremum of Kähler action, fixes the choice of the complex structure uniquely. Therefore group theoretical approach and the approach based on Kähler action complement each other.

- 1. The allowance of an infinite-dimensional group of isometries isomorphic to the group of conformal transformations of 2-sphere is completely unique feature of the 4-dimensional light cone boundary. Even more, in case of δM<sub>+</sub><sup>4</sup> × CP<sub>2</sub> the isometry group of δM<sub>+</sub><sup>4</sup> becomes localized with respect to CP<sub>2</sub>! Furthermore, the Kähler structure of δM<sub>+</sub><sup>4</sup> defines also symplectic structure. Hence any function of δM<sub>+</sub><sup>4</sup> × CP<sub>2</sub> would serve as a Hamiltonian transformation acting in both CP<sub>2</sub> and δM<sub>+</sub><sup>4</sup> degrees of freedom. These transformations obviously differ from ordinary local gauge transformations. This group leaves the symplectic form of δM<sub>+</sub><sup>4</sup> × CP<sub>2</sub>, defined as the sum of light cone and CP<sub>2</sub> symplectic forms, invariant. The group of symplectic transformations of δM<sub>+</sub><sup>4</sup> × CP<sub>2</sub> is a good candidate for the isometry group of the WCW.
- 2. The approximate symplectic invariance of Kähler action is broken only by gravitational effects and is exact for vacuum extremals. If Kähler function were exactly invariant under the symplectic transformations of  $CP_2$ ,  $CP_2$  symplectic transformations would correspond to zero modes having zero norm in the Kähler metric of WCW. This does not make sense since symplectic transformations of  $\delta M^4 \times CP_2$  actually parameterize the quantum fluctuation degrees of freedom.
- 3. The groups G and H, and thus WCW itself, should inherit the complex structure of the light cone boundary. The diffeomorphisms of  $M^4$  act as dynamical symmetries of vacuum extremals. The radial Virasoro localized with respect to  $S^2 \times CP_2$  could in turn act in zero modes perhaps inducing conformal transformations: note that these transformations lead out from the symmetric space associated with given values of zero modes.

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# 5.3.2 Symplectic transformations of $\delta M_{+}^{4} \times CP_{2}$ as isometries of WCW

The symplectic transformations of  $\delta M_+^4 \times CP_2$  are excellent candidates for inducing symplectic transformations of the WCW acting as isometries. There are however deep differences with respect to the Kac Moody algebras.

- 1. The conformal algebra of the WCW is gigantic when compared with the Virasoro + Kac Moody algebras of string models as is clear from the fact that the Lie-algebra generator of a symplectic transformation of  $\delta M_+^4 \times CP_2$  corresponding to a Hamiltonian which is product of functions defined in  $\delta M_+^4$  and  $CP_2$  is sum of generator of  $\delta M_+^4$ -local symplectic transformation of  $CP_2$  and  $CP_2$ -local symplectic transformations of  $\delta M_+^4$ . This means also that the notion of local gauge transformation generalizes.
- 2. The physical interpretation is also quite different: the relevant quantum numbers label the unitary representations of Lorentz group and color group, and the four-momentum labelling the states of Kac Moody representations is not present. Physical states carrying no energy and momentum at quantum level are predicted. The appearance of a new kind of angular momentum not assignable to elementary particles might shed some light to the longstanding problem of baryonic spin (quarks are not responsible for the entire spin of proton). The possibility of a new kind of color might have implications even in macroscopic length scales.
- 3. The central extension induced from the natural central extension associated with  $\delta M_+^4 \times CP_2$  Poisson brackets is anti-symmetric with respect to the generators of the symplectic algebra rather than symmetric as in the case of Kac Moody algebras associated with loop spaces. At first this seems to mean a dramatic difference. For instance, in the case of  $CP_2$  symplectic transformations localized with respect to  $\delta M_+^4$  the central extension would vanish for Cartan algebra, which means a profound physical difference. For  $\delta M_+^4 \times CP_2$  symplectic algebra a generalization of the Kac Moody type structure however emerges naturally.

The point is that  $\delta M_+^4$ -local  $CP_2$  symplectic transformations are accompanied by  $CP_2$  local  $\delta M_+^4$  symplectic transformations. Therefore the Poisson bracket of two  $\delta M_+^4$  local  $CP_2$  Hamiltonians involves a term analogous to a central extension term symmetric with respect to  $CP_2$  Hamiltonians, and resulting from the  $\delta M_+^4$  bracket of functions multiplying the Hamiltonians. This additional term could give the entire bracket of the WCW Hamiltonians at the maximum of the Kähler function where one expects that  $CP_2$  Hamiltonians vanish and have a form essentially identical with Kac Moody central extension because it is indeed symmetric with respect to indices of the symplectic group.

#### 5.3.3 Attempts to identify WCW Hamiltonians

The construction of WCW geometry reduces to that for complexified WCW gamma matrices expressible in terms of fermionic oscillator operators for second quantized induced spinor fields. The contractions of the gamma matrices with isometry generators are the natural operators. The hypothesis is that the isometry generators at the level of WCW correspond to the symplectic algebra at the boundary of CD that is at  $\delta M^4 \pm \times CP_2$  defining WCW Hamiltonians acting as isometries. Therefore WCW gamma matrices have interpretation as super charges of infinite-D conformal supersymmetry. The matrix elements of Kähler metric in the basis defined by isometry generators are obtained as anticommutators of the gamma matrices and reduce to Poisson brackets of corresponding Hamiltonians. A direct connection with physics results since Hamiltonians correspond to irreducible representations of the rotation group SO(3) and color group SU(3).

I have made several attempts to identify explicit representations of WCW Hamiltonians acting as isometries [?] The first two candidates referred to as magnetic and electric Hamiltonians, emerged in a relatively early stage: this approach is discussed in this chapter. Magnetic option is the simplest one but plagued by what looks too strong form of effective 2-dimensionality (inspired by strong form of holography) and lacking connection with the dynamics of Kähler action and modified Dirac action.

The most recent approach identifies super Hamiltonians as Noether super charges and is motivated by the QFT analogy. This proposal discussed in detail in [K26] feeds in the wisdom gained about preferred extremals of Kähler action and solutions of the modified Dirac action: in particular, about their localization to string worlds sheets (right handed neutrino is an exception).

The basic formulas however generalize as such: the only modification is that the super-Hamiltonian of  $\delta M_{\pm}^4 \times CP_2$  at given point of partonic 2-surface is replaced with the Noether super charge associated with the Hamiltonian obtained by integrating the 1-D super current over string emanating from partonic 2-surface. Covariantly constant right handed neutrino spinor is replaced with any mode of the modified Dirac operator localized at string world sheet in the case of Kac-Moodysub-algebra of super-symplectic algebra. In the case of right- handed neutrino one obtains entire super-symplectic algebra and the direct sum of these algebras is used to construct physical states. This step is analogous to the replacement of point like particle with string.

# 5.4 Quantum classical correspondence

Quantum classical correspondence (QCC) has been the basic guiding principle in the construction of TGD. Below are some basic examples about its application.

- 1. QCC led to the idea that Kähler function for poitn  $X^3$  of WCW must have interpretation as classical action for a preferred extremal  $X^4(X^3)$  assignable to Kähler action assumed to be unique: this assumption can of course be criticized because the dynamics is not strictly deterministic. This criticism led to ZEO. The interpretation of preferred extremal is as analog of Bohr orbit so that Bohr orbitology usually believed to be an outcome of stationary phase approximations would be an exact part of quantum TGD.
- 2. QCC suggests a correlation between 4-D geometry of space-time sheet and quantum numbers. This could result if the classical charges in Cartan algebra are identical with the quantal ones. This would give very powerful constraint on the allowed space-time sheets in the superposition of space-time sheets defining WCW spinor field. An even strong condition would be that classical correlation functions are equal to quantal ones.
  - The equality of quantal and classical Cartan charges could be realized by adding constraint terms realized using Lagrange multipliers at the space-like ends of space-time surface at the boundaries of CD. This procedure would be very much like the thermodynamical procedure used to fix the average energy or particle number of the the system using Lagrange multipliers identified as temperature or chemical potential. Since quantum TGD can be regarded as square root of thermodynamics in zero energy ontology (ZEO), the procedure looks logically sound.
  - One aspect of quantum criticality is the condition that the eigenvalues of quantal Noether charges in Cartan algebra associated with the Kähler Dirac action have correspond to the Noether charges for Kähler action in the sense that for given eigenvalue the space-time surfaces have same Kähler Noether charge.
- 3. A stronger form of QCC requires that classical correlation functions for general coordinate invariance observables as functions of two points of imbedding space are equal to the quantal ones at least in the length cale resolution considered. This would give a very powerful maybe too powerful constraint on the zero energy states.
- 4. Measurement interaction term can be added also to the modified Dirac action as a term associated with 3-D space-like 3-surfaces at the boundaries of CDs and at light-like 3-surfaces at which the induced metric changes its signature from Minkowskian to Euclidian. This term implies that Chern-Simons Dirac operator is effectively equivalent with the Dirac operator of  $M^4$  at the lines defining boundaries of string world sheets so that a connection with ordinary Dirac equation would result. This would simplify enormously perturbation theory since fermion propagator would be simply the ordinary Dirac propagator for massless particle as indeed assumed in the twistor Grassmann approach [K18].

#### 5.5 Quantum criticality

The notion of quantum criticality of TGD Universe was originally inspired by the question about how to make TGD unique if Kähler function  $K(X^3)$  in WCW is defined by the Kähler action for a preferred extremal  $X^4(X^3)$  assignable to a given 3-surface. Vacuum functional defined by the exponent of Kähler function is analogous to thermodynamical weight and the obvious idea with Kähler coupling strength taking the role of temperature. The obvious idea was that the value of Kähler coupling strength

is analogous to critical temperature so that TGD would be more or less uniquely defined. One cannot exclude the possibility that  $\alpha_K$  has several values, and the doomsday scenario is that there is infinite number of critical values converging towards  $\alpha_K = 0$ , which corresponds to vanishing temperature).

To understand the delicacies it is convenient to consider various variations of Kähler action first.

- 1. The variation can leave 3-surface invariant but modify space-time surface in such a manner that Kähler action remains invariant. In this case infinitesimal deformation reduces to a diffeomorphism at space-like 3-surface  $X^3$  and perhaps also at light-like 3-surfaces representing partonic orbits. The correspondence between  $X^3$  and  $X^4(X^3)$  would not be unique. Actually this is suggested by that the non-deterministic dynamics characteristic for critical systems. Also the failure of the strict classical determinism implying spin glass type vacuum degeneracy forces to consider this possibility. This criticality would correspond to criticality of Kähler action at  $X^3$  but not that of Kähler function. Note that the original working hypothesis was that  $X^4(X^3)$  is unique.
- 2. The variation could act on zero modes which do not affect Kähler metric, which corresponds to (1,1) part of Hessian in complex coordinates for WCW. Only the zero modes characterizing 3-surface appearing as parameters in the metric of WCW would be affected, and the result would be a generalization of modification of conformal scaling factor. Kähler function would change but only due to the change in zero modes. These transformations do not correspond to critical transformations since Kähler function changes.
- 3. The variation could act on 3-surface both in zero modes and dynamical degrees of freedom represented by complex coordinates. It would affect also the space-time surface. Criticality for Kähler function would mean that Kähler metric has zero modes at  $X^3$  meaning that (1,1) part of Hessian is degenerate. This would mean that in the vicinity of  $X^3$  the Hessian has non-definite signature: same could be true also for the (1,1) part. Physically this is unacceptable since the inner product in Hilbert space should be positive definite.

Consider now critical deformations (the first option). Critical deformations are expected to relate closely to the coset space decomposition of WCW to a union of coset spaces G/H labelled by zero modes.

- 1. Critical deformations leave 3-surface  $X^3$  invariant as do also the transformations of H associated with  $X^3$ . If H affects  $X^4(X^3)$  and corresponds to critical deformations then critical they would allow to extend WCW to a bundle for which 3-surfaces  $X^3$  would be base points and preferred extremals  $X^4(X^3)$  would define the fiber. Gauge invariance with respect to H would generalize the assumption that  $X^4(X^3)$  is unique.
- 2. Critical deformations could correspond to H or sub-group of H (which depends on  $X^3$ ). For other 3-surfaces than  $X^3$  the action of H is non-trivial: to see this consider the simple finite-dimensional case  $CP_2 = SU(3)/U(2)$ . The groups  $H(X^3)$  are symplectic conjugates of each other for given values of zero modes which are symplectic invariants.
- 3. A possible identification of Lie-algebra of H is as a sub-algebra of Virasoro algebra associated with the symplectic transformations of  $\delta M^4 \times CP_2$  and acting as diffeomorphisms for the light-like radial coordinate of  $\delta M^4_+$ . The sub-algebras of Virasoro algebra have conformal weights coming as integer multiplies  $=km,\ k\in Z,$  of given conformal weight m and form inclusion hierarchies suggesting a direct connection with finite measurement resolution realized in terms of inclusions of hyperfinite factors of type  $\Pi_1$ .
  - For m>1 one would have breaking of maximal conformal symmetry. The action of these Virasoro algebra on symplectic algebra would make the corresponding sub-algebras gauge degrees of freedom so that the number of symplectic generators generating non-gauge transformations would be finite. This result is not surprising since also for 2-D critical systems criticality corresponds to conformal invariance acting as local scalings.

The vanishing of the second variation for some deformations means that the system is critical, in the recent case quantum critical [K3, K7]. Basic example of criticality is the bifurcation diagram for cusp catastrophe [A1]. Quantum criticality realized as the vanishing of the second variation gives hopes

about a more or less unique identification of preferred extremals. I have considered also alternative identifications such as absolute minimization of Kähler action. which is just the opposite of criticality.



Figure 4: Matter makes space-time curved and leads to the loss of Poincare invariance so that momentum and energy are not well-defined notions in GRT.

One must be very cautious here: there are two criticalities: one for the extremals of Kähler action with respect to the deformations of four-surface and second for the Kähler function itself with respect to the deformations of 3-surface: these criticalities are not equivalent since in the latter case variation respects preferred extremal property unlike in the first case.

- 1. The criticality for preferred extremals (G/H) option) would make 4-D criticality a property of all physical systems. Conformal symmetry breaking would however break criticality below some scale.
- 2. The criticality for Kähler function would be 3-D and might hold only for very special systems. In fact, the criticality means that some eigenvalues for the Hessian of Kähler function vanish and for nearby 3-surfaces some eigenvalues are negative. On the other hand the Kähler metric defined by (1,1) part of Hessian in complex coordinates must be positive definite. Thus criticality might therefore imply problems.
  - This allows and suggests non-criticality of Kähler function coming from Kähler action for Euclidian space-time regions: this is mathematically the simplest situation since in this case there are no troubles with Gaussian approximation to the functional integral. The Morse function coming from Kähler action in Minkowskian as imaginary contribution analogous to that appearing in path integral could however be critical and allow non-definite signature in principle. In fact this is expected by the defining properties of Morse function. Kähler function would make WCW integral mathematically existing and Morse function would imply the typical quantal interference effects.
- 3. The almost 2-dimensionality implied by strong form of holography suggests that the interior degrees of freedom of 3-surface can be regarded as almost gauge degrees of freedom and that this relates directly to generalised conformal symmetries associated with symplectic isometries of WCW. These degrees of freedom are not critical in the sense inspired by G/H decomposition. The only plausible interpretation seems to be that these degrees of freedom correspond to deformations in zero modes.

This is a brief summary about quantum criticality in bosonic degrees of freedom. One must formulate quantum criticality for the modified Dirac action [K7]. The new element is that critical deformations with vanishing second variation of Kähler action define vanishing first variation of Kähler Dirac action so that second order Noether charges correspond to first order Noether charges in fermionic sector. It seems that the formulation in terms of hierarchy of broken conformal symmetries is the most promising one mathematically and also correspond to physical intuition.

#### 5.6 The notion of finite measurement resolution

Finite measurement resolution has become one of the basic principles of quantum TGD. Finite measurement resolution has two realizations: the quantal realization in terms of inclusions of von Neumann algebras and the classical realization in terms of discretization having a nice description in number theoretic approach. The notion of p-adic manifold relying on the canonical correspondence between real and p-adic physics forces finite cognitive and measurement resolution automatically and implies that p-adic preferred extremals are cognitive representations for real preferred extremals in finite cognitive representations [K25].

Von Neumann introduced three types of algebras as candidates for the mathematics of quantum theory. These algebras are known as von Neumann algebras and the three factors (kind of basic building bricks) are known as factors of type I,II, and III. The factors of type I are simplest and apply in wave mechanics where classical system has finite number of degrees of freedom. Factors of type III apply to quantum field theory where the number of degrees of freedom is infinite. Von Neumann himself regarded factors of type III somehow pathological.

Factors of type II contains as sub-class hyper-finite factors of type  $II_1$  (HFFs). The naive definition of trace of unit matrix as infinite dimension of the Hilbert space involved is replaced with a definition in which unit matrix has finite trace equal to 1 in suitable normalization. One cannot anymore select single ray of Hilbert space but one must always consider infinite-dimensional sub-space. The interpretation is in terms of finite measurement resolution: the sub-Hilbert space representing non-detectable degrees of freedom is always infinite-dimensional and the inclusion to larger Hilbert space is accompanied by inclusion of corresponding von Neumann algebras.

HFFs are between factors of type I and III in the sense that approximation of the system as a finite-dimensional system can be made arbitrary good: this motivates the term hyper-finite.

The realization that HFFs [K23] re tailor made for quantum TGD has led to a considerable progress in the understanding of the mathematical structure of the theory and these algebras provide a justification for several ideas introduced earlier on basis of physical intuition.

HFF has a canonical realization as an infinite-dimensional Clifford algebra and the obvious guess is that it corresponds to the algebra spanned by the gamma matrices of WCW. Also the local Clifford algebra of the imbedding space  $H = M^4 \times CP_2$  in octonionic representation of gamma matrices of H is important and the entire quantum TGD emerges from the associativity or co-associativity conditions for the sub-algebras of this algebra which are local algebras localized to maximal associative or co-associate sub-manifolds of the imbedding space identifiable as space-time surfaces.

The notion of inclusion for hyper-finite factors provides an elegant description for the notion of measurement resolution absent from the standard quantum measurement theory.

- 1. The included sub-factor creates in zero energy ontology states not distinguishable from the original one and the formally the coset space of factors defining quantum spinor space defines the space of physical states modulo finite measurement resolution.
- 2. The quantum measurement theory for hyperfinite factors differs from that for factors of type I since it is not possible to localize the state into single ray of state space. Rather, the ray is replaced with the sub-space obtained by the action of the included algebra defining the measurement resolution. The role of complex numbers in standard quantum measurement theory is taken by the non-commutative included algebra so that a non-commutative quantum theory is the outcome.
- 3. The inclusions of HFFs are closely related to quantum groups studied in recent modern physics but interpreted in terms of Planck length scale exotics formulated in terms of non-commutative space-time. The formulation in terms of finite measurement resolution brings this mathematics to physics in all scales.
  - For instance, the finite measurement resolution means that the components of spinor do not commute anymore and it is not possible to reduce the state to a precise eigenstate of spin. It is however perform a reduction to an eigenstate of an observable which corresponds to the probability for either spin state.
- 4. The realization for quantum measurement theory modulo finite measurement resolution is in terms of M-matrices defined in terms of Connes tensor product which essentially means that the included hyper-finite factor N takes the role of complex numbers.

Discretization defines a classical space-time correlate for the finite measurement resolution.

- 1. The dynamics of TGD itself might realize finite measurement resolution automatically in the sense that the quantum states at partonic 2-surfaces are always defined in terms of fermions localized at discrete points defined the ends of braids defined as the ends of string world sheets.
- 2. The condition that these selected points are common to reals and some algebraic extension of p-adic numbers for some p allows only algebraic points. General coordinate invariance requires the special coordinates and natural coordinate systems are possible thanks to the symmetries of WCW. A restriction of general coordinate invariance to discrete subgroup might well occur and have interpretation in terms of the constraints from the presence of cognition. One might say that the world in which mathematician uses Cartesian coordinates is different from the world in mathematician uses spherical coordinates.
- 3. The realization at the level of WCW would be number theoretical. In given resolution all parameters characterizing the mathematical representation of partonic 2-surfaces would belong to some algebraic extension of rational numbers. Same would hold for their 4-D tangent space data. This would imply that WCW would be effectively discrete space so that finite measurement resolution would be realized.

# 5.7 Weak form of electric magnetic duality

The notion of electric-magnetic duality [B1] was proposed first by Olive and Montonen and is central in  $\mathcal{N}=4$  supersymmetric gauge theories. It states that magnetic monopoles and ordinary particles are two different phases of theory and that the description in terms of monopoles can be applied at the limit when the running gauge coupling constant becomes very large and perturbation theory fails to converge.

The notion of electric-magnetic self-duality is more natural in TGD since for  $CP_2$  geometry Kähler form is self-dual and Kähler magnetic monopoles are also Kähler electric monopoles and Kähler coupling strength is by quantum criticality renormalization group invariant rather than running coupling constant

In TGD framework one must adopt a weaker form of the self-duality applying at partonic 2-surfaces [K7]. The principle is statement about boundary values of the induced Kähler form analogous to Maxwell field at the light-like 3-surfaces, at which the situation is singular since the induced metric for four-surface has a vanishing determinant because the signature of the the induced metric changes from Minkowskian to Euclidian. What the principle says is that Kähler electric field in the normal space is the dual of Kähler magnetic field in the 4-D tangent space of the light-like 3-surface. One can consider even weaker formulation assuming this only at partonic 2-surfaces at the intersection of light-like 3-surfaces and space-like 3-surfaces at the boundaries of CD.

Every new idea must be taken with a grain of salt but the good sign is that this concept leads to precise predictions.

- 1. Elementary particles do not generate monopole fields in macroscopic length scales: at least when one considers visible matter. The first question is whether elementary particles could have vanishing magnetic charges: this turns out to be impossible. The next question is how the screening of the magnetic charges could take place and leads to an identification of the physical particles as string like objects identified as pairs magnetic charged wormhole throats connected by magnetic flux tubes. The string picture was later found to emerge naturally from Kähler Dirac action.
- 2. Second implication is a new view about electro-weak massivation reducing it to weak confinement in TGD framework. The second end of the string contains particle having electroweak isospin neutralizing that of elementary fermion and the size scale of the string is electro-weak scale would be in question. Hence the screening of electro-weak force takes place via weak confinement realized in terms of magnetic confinement.
- 3. This picture generalizes to the case of color confinement. Also quarks correspond to pairs of magnetic monopoles but the charges need not vanish now. Rather, valence quarks would be connected by flux tubes of length of order hadron size such that magnetic charges sum up to

zero. For instance, for baryonic valence quarks these charges could be (2, -1, -1) and could be proportional to color hyper charge.

- 4. The highly non-trivial prediction making more precise the earlier stringy vision is that elementary particles are string like objects in electro-weak scale: this should become manifest at LHC energies. Stringy character is manifested in two manners: as string like objects defined by Kähler magnetic flux tubes and 2-D string world sheets.
- 5. The weak form electric-magnetic duality together with Beltrami flow property of Kähler leads to the reduction of Kähler action to Chern-Simons action so that TGD reduces to almost topological QFT and that Kähler function is explicitly calculable. This has enormous impact concerning practical calculability of the theory.
- 6. One ends up also to a general solution ansatz for field equations from the condition that the theory reduces to almost topological QFT. The solution ansatz is inspired by the idea that all isometry currents are proportional to Kähler current which is integrable in the sense that the flow parameter associated with its flow lines defines a global coordinate. The proposed solution ansatz would describe a hydrodynamical flow with the property that isometry charges are conserved along the flow lines (Beltrami flow). A general ansatz satisfying the integrability conditions is found.
  - The solution ansatz applies also to the extremals of Chern-Simons action and and to the conserved currents associated with the modified Dirac equation defined as contractions of the modified gamma matrices between the solutions of the modified Dirac equation. The strongest form of the solution ansatz states that various classical and quantum currents flow along flow lines of the Beltrami flow defined by Kähler current (Kähler magnetic field associated with Chern-Simons action). Intuitively this picture is attractive. A more general ansatz would allow several Beltrami flows meaning multi-hydrodynamics. The integrability conditions boil down to two scalar functions: the first one satisfies massless d'Alembert equation in the induced metric and the the gradients of the scalar functions are orthogonal. The interpretation in terms of momentum and polarization directions is natural.
- 7. The general solution ansatz works for induced Kähler Dirac equation and Chern-Simons Dirac equation and reduces them to ordinary differential equations along flow lines. The induced spinor fields are simply constant along flow lines of indued spinor field for Dirac equation in suitable gauge. Also the generalized eigen modes of the modified Chern-Simons Dirac operator can be deduced explicitly if the throats and the ends of space-time surface at the boundaries of CD are extremals of Chern-Simons action. Chern-Simons Dirac equation reduces to ordinary differential equations along flow lines and one can deduce the general form of the spectrum and the explicit representation of the Dirac determinant in terms of geometric quantities characterizing the 3-surface (eigenvalues are inversely proportional to the lengths of strands of the flow lines in the effective metric defined by the modified gamma matrices).

# 5.8 TGD as almost topological QFT

Topological QFTs (TQFTs) represent examples of the very few quantum field theories which exist in mathematically rigorous manner. TQFTs are of course physically non-realistic since the notion of distance is lacking and one cannot assign to the particles observables like mass. This raises the hope that TGD could be as near as possible to TQFT.

The vision about TGD as almost topological QFT is very attractive. Almost topological QFT property would naturally correspond to the reduction of Kähler action for preferred extremals to Chern-Simons form integrated over boundary of space-time and over the light-like 3-surfaces means. This is achieved if weak form of em duality vanishes and  $j \cdot A$  term in the decomposition of Kähler action to 4-D integral and 3-D boundary term vanishes. Almost topological QFT would suggests conformal field theory at partonic 2-surface or at their light-like orbits. Strong form of holography states that also conformal field theory associated with space-like 3-surfaces at the ends of CDs describes the physics. These facts suggest that almost 2-dimensional QFT coded by data given at partonic 2-surfaces and their 4-D tangent space is enough to code for physics.

Topological QFT property would mean description in terms of braids. Braids would correspond to the orbits of fermions at partonic 2-surfaces identifiable as ends of string world sheets at which the modes of induced spinor field are localized with one exception: right-handed neutrino. This follows from well-definedness of electromagnetic charge in presence of induce W boson fields. The first guess is that induced W boson field must vanish at string world sheet. "Almost" could mean the replacement of the ends of strings defining braids with strings and duality for the descriptions based on string world sheets resp. partonic 2-surfaces analogous to AdS/CFT duality.

#### 5.9 Twistor revolution and TGD

During last decade so called twistor revolution has revived theoretical physics and has also had strong impact on TGD.

#### 5.9.1 Twistor revolution

There are classical papers by several authors such as Witten and Nima Arkani-Hamed who is one of the leading theoreticians driving the twistor revolution [B4, B5, B2, B3].

- 1. The notion of twistor is due to Penrose and is very convenient notion in theories describing massless particles and therefore possessing conformal invariance extending Poincare symmetries by the inclusion of scalings and so called special conformal transformation which is analogous to reflections in spherical mirror.
- 2. Twistor kinematics means that one can express massless four-momentum and helicity in terms of two massless spinors combining to a twistor living in 4-D complex space which reduces to  $CP_3$  because of projective invariance of the description. The beauty of the twistor kinematics is that non-linear action of special conformal transformations linearizes in  $CP_3$ . What one does is to replace light-like geodesic in Minkowski space with points in the twistor space  $CP_3$  whereas the complex lines of  $CP_3$  correspond to points of Minkowski space. To be honest, complexified Minkowski space is in question and this is one of the technical difficulties involved.
- 3. Gauge theories without fermions and scalar fields are such theories and the applications of twistorial methods to  $\mathcal{N}=4$  super-symmetric Yang-Mills theory has produced amazingly strong results demonstrating that Feynman diagrams sum up to stunningly single twistorial expressions. The key idea is that four-dimensional integrals over loop momenta are interpreted as residue integrals in the complexified space of four-momenta so that they reduce to residues from poles. The surprising discovery is that using Yangian invariants one can express the planar loop amplitudes for given number of external states with given helicities and momenta in terms of on mass shell amplitudes for smaller number of particles and with smaller number of loops by using recursion formulas.
- 4. Twistor revolution has led to a discovery of what is known as dual twistors. The massless momenta associated with incoming states of twistor diagram and expressible in terms of ordinary twistors, can be also expressed as differences of so called region momenta propagating in the edges of polygon characterizing twistor diagram. The massless momenta correspond to intersections of complex lines in what is called momentum twistor space so that one diagram has interpretation also as a diagram in momentum twistor space. The theory possesses conformal invariance also in the momentum twistor space and the two conformal symmetries combine to form a large infinite-dimensional symmetry known as Yangian symmetry [A3] associated with the conformal group of Minkowski space.
- 5. The work of Nima Arkani-Hamed and others [B3] has revealed that the integrands for the twistor amplitudes for planar diagrams can be expressed as residue integrals over Grassmannians G(n,k) where n is the number of massless external particles (gluons or gluinos) and k is the number of negative (say) helicities. The integrands appearing in these integrals are Yangian invariants and there are recipes for their construction. The generalization of BCFW formula gives a recursion formula allowing to deduce the l-loop construction to the scattering of n particles with k negative helicities. The vision of Arkani-Hamed is that this approach allows to get rid of space-time altogether.

#### 5.9.2 The latest view about twistors in TGD

The TGD view about twistors has evolved gradually anbd I have written three chapters about twistors and TGD [K22, K24, K18] giving a view about development of ideas. Some of the conjectures have turned out to be wrong and the original idea about QFT type twistorial description of scattering amplitudes has been replaced by its stringy variant so that also problems caused by non-planar diagrams disappear. Below I summarize only the latest view about the situation [K18].

First the deep result making TGD Universe unique from twistorial point of view. Both  $M^4$  and  $CP_2$  are highly unique in that they allow twistor structure and in TGD one can overcome the fundamental "googly" problem of the standard twistor program preventing twistorialization in general space-time metric by lifting twistorialization to the level of the imbedding space containg  $M^4$  as a Cartesian factor. Also  $CP_2$  allows twistor space identifiable as flag manifold  $SU(3)/U(1) \times U(1)$  as the self-duality of Weyl tensor indeed suggests. This provides an additional "must" in favor of sub-manifold gravity in  $M^4 \times CP_2$ . Both octonionic interpretation of  $M^8$  and triality possible in dimension 8 play a crucial role in the proposed twistorialization of  $H = M^4 \times CP_2$ . It also turns out that  $M^4 \times CP_2$  allows a natural twistorialization respecting Cartesian product: this is far from obvious since it means that one considers space-like geodesics of H with light-like  $M^4$  projection as basic objects. p-Adic mass calculations however require tachyonic ground states and in generalized Feynman diagrams fermions propagate as massless particles in  $M^4$  sense. Furthmore, light-like H-geodesics lead to non-compact candidates for the twistor space of H. Hence the twistor space would be 12-dimensional manifold  $CP_3 \times SU(3)/U(1) \times U(1)$ .

Generalisation of 2-D conformal invariance extending to infinite-D variant of Yangian symmetry; light-like 3-surfaces as basic objects of TGD Universe and as generalised light-like geodesics; light-likeness condition for momentum generalized to the infinite-dimensional context via super-conformal algebras. These are the facts inspiring the question whether also the "world of classical worlds" (WCW) could allow twistorialization. It turns out that center of mass degrees of freedom (imbedding space) allow natural twistorialization: twistor space for  $M^4 \times CP_2$  serves as moduli space for choice of quantization axes in Super Virasoro conditions. Contrary to the original optimistic expectations it turns out that although the analog of incidence relations holds true for Kac-Moody algebra, twistorialization in vibrational degrees of freedom does not look like a good idea since incidence relations force an effective reduction of vibrational degrees of freedom to four.

The Grassmannian formalism for scattering amplitudes is expected to generalize for generalized Feynman diagrams: the basic modification is due to the possible presence of  $CP_2$  twistorialization and the fact that 4-fermion vertex -rather than 3-boson vertex- and its super counterparts define now the fundamental vertices. Both QFT type BFCW and stringy BFCW can be considered.

- 1. For QFT type BFCW BFF and BBB vertices would be an outcome of bosonic emergence (bosons idealized as wormhole contacts) and 4-fermion vertex is proportional to factor with dimensions of inverse mass squared and naturally identifiable as proportional to the factor  $1/p^2$  assignable to each boson line. This predicts a correct form for the bosonic propagators for which mass squared is in general non-vanishing unlike for fermion lines. The usual BFCW construction would emerge naturally in this picture. There is however a problem: the emergent bosonic propagator diverges or vanishes depending on whether one assumes SUSY at the level of single wormhole throat or not. By the special properties of  $\mathcal{N}=4$  SUSY generated by right handed neutrino the SUSY cannot be applied to single wormhole throat but only to a pair of wormhole throats.
- 2. This as also the fact that physical particles are necessarily pairs of wormhole contacts connected by fermionic strings forces stringy variant of BFCW avoiding the problems caused by non-planar diagrams. Now boson line BFCW cuts are replaced with stringy cuts and loops with stringy loops. By generalizing the earlier QFT twistor Grassmannian rules one ends up with their stringy variants in which super Virasoro generators G,  $G^{\dagger}$  and L bringing in CP<sub>i</sub>sub¿2<sub>i</sub>/sub¿ scale appear in propagator lines: most importantly, the fact that G and  $G^{\dagger}$  carry fermion number in TGD framework ceases to be a problem since a string world sheet carrying fermion number has 1/G and  $1/G^{\dagger}$  at its ends. Twistorialization applies because all fermion lines are light-like.
- 3. A more detailed analysis of the properties of right-handed neutrino demonstrates that modified gamma matrices in the modified Dirac action mix right and left handed neutrinos but that

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this happens markedly only in very short length scales comparable to  $CP_2$  scale. This makes neutrino massive and also strongly suggests that SUSY generated by right-handed neutrino emerges as a symmetry at very short length scales so that spartners would be very massive and effectively absent at low energies. Accepting  $CP_2$  scale as cutoff in order to avoid divergent gauge boson propagators QFT type BFCW makes sense. The outcome is consistent with conservative expectations about how QFT emerges from string model type description.

## REFERENCES

# Mathematics

- [A1] Catastrophe Theory. http://en.wikipedia.org/wiki/Catastrophe\_theory#Cusp\_catastrophe.
- [A2] Super Virasoro algebra. http://en.wikipedia.org/wiki/Super\_Virasoro\_algebra.
- [A3] Yangian symmetry. http://en.wikipedia.org/wiki/Yangian.
- [A4] John C. Baez. The Octonions. Bull. Amer. Math. Soc. http://math.ucr.edu/home/baez/ Octonions/octonions.html, 39(2002), 2001.
- [A5] L. Brekke and P. G. O. Freund. p-Adic Numbers in Physics. Phys. Rep., 233(1), 1993.
- [A6] D. S. Freed. The Geometry of Loop Groups, 1985.
- [A7] J. Milnor. Topology form Differential Point of View. The University Press of Virginia, Virginia, 1965
- [A8] R. Thom. Commentarii Math. Helvet., 28, 1954.
- [A9] Wallace. Differential Topology. W. A. Benjamin, New York, 1968.

# Theoretical Physics

- [B1] Montonen Olive Duality. http://en.wikipedia.org/wiki/Montonen-Olive\_duality.
- [B2] N. Arkani-Hamed et al. A duality for the S-matrix. http://arxiv.org/abs/0907.5418, 2009.
- [B3] Nima Arkani-Hamed et al. The All-Loop Integrand For Scattering Amplitudes in Planar N=4 SYM. http://arxiv.org/find/hep-th/1/au:+Bourjaily\_J/0/1/0/all/0/1, 2010.
- [B4] P. Svrcek F. Cachazo and E. Witten. MHV Vertices and Tree Amplitudes In Gauge Theory. http://arxiv.org/abs/hep-th/0403047, 2004.
- [B5] B. Feng R. Britto, F. Cachazo and E. Witten. Direct Proof of Tree-Level Recursion Relation in Yang-Mills Theory. *Phys. Rev.*. http://arxiv.org/abs/hep-th/0501052, 94:181602, 2005.

# Particle and Nuclear Physics

- [C1] Lamb shift. http://en.wikipedia.org/wiki/Lamb\_shift.
- [C2] J. D. Bjorken. Acta Phys. Polonica B, 28:2773, 1997.

# Condensed Matter Physics

[D1] Fractional quantum Hall Effect. http://en.wikipedia.org/wiki/Fractional\_quantum\_Hall\_effect.

# Cosmology and Astro-Physics

[E1] D. Da Roacha and L. Nottale. Gravitational Structure Formation in Scale Relativity. http://arxiv.org/abs/astro-ph/0310036, 2003.

# **Biology**

- [I1] High energy phosphate. http://en.wikipedia.org/wiki/High-energy\_phosphate.
- [I2] F. A. Popp et al. Emission of Visible and Ultraviolet Radiation by Active Biological Systems. Collective Phenomena, 3, 1981.

# Neuroscience and Consciousness

[J1] C. F. Blackman. Effect of Electrical and Magnetic Fields on the Nervous System, pages 331–355. Plenum, New York, 1994.

## Books related to TGD

- [K1] M. Pitkänen. About the New Physics Behind Qualia. In Quantum Hardware of Living Matter. Onlinebook. http://tgdtheory.fi/public\_html/bioware/bioware.html#newphys, 2006.
- [K2] M. Pitkänen. Basic Extremals of Kähler Action. In *Physics in Many-Sheeted Space-Time*. Onlinebook. http://tgdtheory.fi/public\_html/tgdclass/tgdclass.html#class, 2006.
- [K3] M. Pitkänen. Construction of Configuration Space Kähler Geometry from Symmetry Principles. In Quantum Physics as Infinite-Dimensional Geometry. Onlinebook. http://tgdtheory.fi/public\_html/tgdgeom/tgdgeom.html#compl1, 2006.
- [K4] M. Pitkänen. Dark Matter Hierarchy and Hierarchy of EEGs. In *TGD and EEG*. Onlinebook. http://tgdtheory.fi/public\_html/tgdeeg/tgdeeg.html#eegdark, 2006.
- [K5] M. Pitkänen. DNA as Topological Quantum Computer. In *Genes and Memes*. Onlinebook. http://tgdtheory.fi/public\_html/genememe/genememe.html#dnatqc, 2006.
- [K6] M. Pitkänen. Does TGD Predict the Spectrum of Planck Constants? In Hyper-finite Factors and Dark Matter Hierarchy. Onlinebook. http://tgdtheory.fi/public\_html/neuplanck/neuplanck.html#Planck, 2006.
- [K7] M. Pitkänen. Does the Modified Dirac Equation Define the Fundamental Action Principle? In Quantum Physics as Infinite-Dimensional Geometry. Onlinebook. http://tgdtheory.fi/public\_html/tgdgeom/tgdgeom.html#Dirac, 2006.
- [K8] M. Pitkänen. Evolution in Many-Sheeted Space-Time. In *Genes and Memes*. Onlinebook. http://tgdtheory.fi/public\_html/genememe/genememe.html#prebio, 2006.
- [K9] M. Pitkänen. Negentropy Maximization Principle. In *TGD Inspired Theory of Consciousness*. Onlinebook. http://tgdtheory.fi/public\_html/tgdconsc/tgdconsc.html#nmpc, 2006.
- [K10] M. Pitkänen. New Particle Physics Predicted by TGD: Part I. In p-Adic Length Scale Hypothesis and Dark Matter Hierarchy. Onlinebook. http://tgdtheory.fi/public\_html/paddark/ paddark.html#mass4, 2006.
- [K11] M. Pitkänen. p-Adic length Scale Hypothesis and Dark Matter Hierarchy. Onlinebook. http://tgdtheory.fi/public\_html/paddark/paddark.html, 2006.
- [K12] M. Pitkänen. p-Adic Numbers and Generalization of Number Concept. In TGD as a Generalized Number Theory. Onlinebook. http://tgdtheory.fi/public\_html/tgdnumber/tgdnumber.html#padmat, 2006.

- [K13] M. Pitkänen. p-Adic Physics as Physics of Cognition and Intention. In *TGD Inspired The-ory of Consciousness*. Onlinebook. http://tgdtheory.fi/public\_html/tgdconsc/tgdconsc. html#cognic, 2006.
- [K14] M. Pitkänen. Quantum Antenna Hypothesis. In *Quantum Hardware of Living Matter*. Online-book. http://tgdtheory.fi/public\_html/bioware/bioware.html#tubuc, 2006.
- [K15] M. Pitkänen. Quantum Astrophysics. In *Physics in Many-Sheeted Space-Time*. Onlinebook. http://tgdtheory.fi/public\_html/tgdclass/tgdclass.html#qastro, 2006.
- [K16] M. Pitkänen. Quantum Model for Nerve Pulse. In *TGD and EEG*. Onlinebook. http://tgdtheory.fi/public\_html//tgdeeg/tgdeeg/tgdeeg.html#pulse, 2006.
- [K17] M. Pitkänen. Quantum Physics as Infinite-Dimensional Geometry. Onlinebook.http://tgdtheory.fi/public\_html/tgdgeom/tgdgeom.html, 2006.
- [K18] M. Pitkänen. Some Fresh Ideas about Twistorialization of TGD. In Towards M-Matrix. Onlinebook. http://tgdtheory.fi/public\_html/tgdquant/tgdquantum.html#twistornotices, 2006.
- [K19] M. Pitkänen. TGD and Astrophysics. In *Physics in Many-Sheeted Space-Time*. Onlinebook. http://tgdtheory.fi/public\_html/tgdclass/tgdclass.html#astro, 2006.
- [K20] M. Pitkänen. TGD as a Generalized Number Theory. Onlinebook. http://tgdtheory.fi/public\_html/tgdnumber/tgdnumber.html, 2006.
- [K21] M. Pitkänen. TGD as a Generalized Number Theory: Quaternions, Octonions, and their Hyper Counterparts. In TGD as a Generalized Number Theory. Onlinebook. http://tgdtheory.fi/public\_html/tgdnumber/tgdnumber.html#visionb, 2006.
- [K22] M. Pitkänen. Twistors, N=4 Super-Conformal Symmetry, and Quantum TGD. In Towards M-Matrix. Onlinebook. http://tgdtheory.fi/public\_html/tgdquant/tgdquantum.html#twistor, 2006.
- [K23] M. Pitkänen. Was von Neumann Right After All. In *Hyper-finite Factors and Dark Matter Hierarchy*. Onlinebook. http://tgdtheory.fi/public\_html/neuplanck/neuplanck.html#vNeumann, 2006.
- [K24] M. Pitkänen. Yangian Symmetry, Twistors, and TGD. In *Towards M-Matrix*. Onlinebook. http://tgdtheory.fi/public\_html/tgdquant/tgdquantum.html#Yangian, 2006.
- [K25] M. Pitkänen. What p-Adic Icosahedron Could Mean? And What about p-Adic Manifold? In TGD as a Generalized Number Theory. Onlinebook. http://tgdtheory.fi/public\_html/tgdnumber/tgdnumber.html#picosahedron, 2013.
- [K26] M. Pitkänen. Recent View about Kähler Geometry and Spin Structure of "World of Classical Worlds". In *Quantum Physics as Infinite-Dimensional Geometry*. Onlinebook. http://tgdtheory.fi/public\_html/tgdgeom/tgdgeom.html#wcwnew, 2014.

## Articles about TGD

[L1] M. Pitkänen. The Geometry of  $CP_2$  and its Relationship to Standard Model. https://www.createspace.com/3569411, July 2010.