The Substitute Theory of Massive Photons
(Draft)

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Abstract

This paper is concerned with my second theory on massive photons. The main difference between these two theories is the way kinetic energy and total energy are defined. Because it is difficult to decide which theory is the correct one without solid experimental evidence to discard the incorrect one, we must take both theories into account for the time being.

Keywords: *Black hole, Schwarzschild radius, rest mass, relativistic mass, equivalent mass, event horizon, Proca equations, quantum gravity theories.*

1. Introduction

In this paper I introduced a theory that account for the rest mass of the photon. The idea of massive photons is not new. Proca developed a set of equations for electromagnetism that take into account the rest mass of the photon. These equations, known as the Proca equations, are the Maxwell equations for massive photons.

In his book "Cosmic impressions", the physisist Walter Thirring [1] quotes:

"It is been always said that photons have zero rest mass, but maybe we should say that its mass has a value which is lower than the limits we are able to measure. If the mass [of the photon] were exactly zero, then its Compton wavelength would be infinite, but because the radius of the universe is finite, it is time to begin to work meticulously."

The equations for the energy of the photon are developed from simple modifications carried out to the special theory of relativity.

After reading this paper, the reader will surely observe that in this theory the terms relativistic mass and relativistic energy have no meaning whatsoever. This is because the equivalent mass of a photon depends on its frequency and not on its velocity. In the light of this fact this theory should not be called "The Relativistic Theory of Massive Photons"

Instead, I have called this theory "The Substitute Theory of Massive Photons". This is because of the fact that both the equations of the total equivalent energy and the kinetic equivalent energy of this theory can be derived by substituting ' pc ' in the corresponding Einstein's equations of special relativity by ' hf ' (see Table 1).

2. The Equations for Massive Photons

Because the equivalent mass of a photon depends on its frequency and not on its velocity the Einstein's relativistic mass law is not applicable to massive photons. Therefore, assuming that photons have a non-zero rest mass, they have to be treated differently.

Let us consider the Einstein's total relativistic energy equation

$$E_{rel}^2 = (p_{rel}\, c)^2 + (m_0 c^2)^2 \qquad (2.1\text{-}1)$$

I have used E_{rel} to denote the relativistic energy of a body to avoid confusion with the concept of equivalent energy used in this paper.

According to Einstein the rest mass, m_0 , of a photon is zero, therefore, in special relativity, the total relativistic energy of a photon is

$$E_{rel} = p_{rel}\, c \qquad (2.1\text{-}2)$$

Also according to Einstein, the relativistic kinetic energy is given by

$$K_{rel} = E_{rel} - m_0 c^2 \qquad (2.1\text{-}3)$$

But because $m_0 = 0$ we get

$$K_{rel} = E_{rel} = p_{rel}\, c \qquad (2.1\text{-}4)$$

Therefore we reach the following conclusion for massless photons: the relativistic total energy and the kinetic energy are identical.

However, if photons have non-zero rest mass, their kinetic energy must be different to their equivalent energy. Since the mass of the photon does not depend on its velocity, the concept of relativistic energy does not make any sense for massive photons. Thus, relativistic energy, in this case, must be replaced by the concept of equivalent energy.

I shall start the derivation of the equivalent kinetic energy of a massive photon by re-writing equation (2.1-1) as follows

$$E_{rel}^2 - (m_0 c^2)^2 = (p_{rel}\, c)^2 \qquad (2.1\text{-}5)$$

$$(E_{rel} + m_0 c^2)(E_{rel} - m_0 c^2) = (p_{rel}\, c)^2 \qquad (2.1\text{-}6)$$

$$E_{rel} - m_0 c^2 = \frac{(p_{rel}\, c)^2}{E_{rel} + m_0 c^2} \qquad (2.1\text{-}7)$$

Now I define the equivalent quantities for massive photons as follows

$K =$ Equivalent kinetic energy of the photon

$E =$ Equivalent total energy of the photon

$p =$ Equivalent momentum of the photon

$m_0 =$ photon rest mass

Now we write equation (2.1-7) using the equivalent energies and the equivalent momentum rather than the relativistic counterparts.

$$K = E - m_0 c^2 \tag{2.1-8 a}$$

$$K = \frac{(pc)^2}{E + m_0 c^2} \tag{2.1-8 b}$$

So far there are no changes to Einstein's equations except for the conceptual change.

According to De Broglie a particle's wavelength and its momentum are related by

$$\lambda = \frac{h}{mv} \tag{2.1-9}$$

moreover

$$\lambda = \frac{c}{f} \tag{2.1-10}$$

$$p = \frac{hf}{c} \tag{2.1-11}$$

Then substituting p in equation (2.1-8 b) by the second side of (2.1-11) we get the expression of the kinetic energy for massive photons as a function of f and E.

$$K = \frac{(hf)^2}{E + m_0 c^2} \tag{2.1-12}$$

From equations (2.1-8 a) and (2.1-12) we get the following quadratic equation

$$K^2 + 2 m_0 c^2 K - (hf)^2 = 0 \tag{2.1-13}$$

The solution to this equation is

$$K = \sqrt{(hf)^2 + (m_0 c^2)^2} - m_0 c^2 \tag{2.1-14}$$

This is the expression for the kinetic energy of the photon as a function of its frequency, f.

According to the famous Einstein's equation mass and energy are related as follows

$$m = \frac{E}{c^2} \tag{2.1-15}$$

The total equivalent energy is equal to the kinetic equivalent energy plus the rest energy, (See equation (2.1-8 a)). Then we write

$$E = K + m_0 c^2 \tag{2.1-16}$$

Substituting E in equation (2.1-15) with the second side of equation (2.1-16) yields

$$m = \frac{K + m_0 c^2}{c^2} \tag{2.1-17}$$

Substituting K in equation (2.1-17) with the second side of equation (2.1-14) yields

$$m = \frac{\sqrt{(hf)^2 + (m_0 c^2)^2} - m_0 c^2 + m_0 c^2}{c^2} = \frac{\sqrt{(hf)^2 + (m_0 c^2)^2}}{c^2} \tag{2.1-18}$$

Finally, the equation for the equivalent mass of the photon turns out to be

$$m = \sqrt{\left(\frac{hf}{c^2}\right)^2 + m_0^2} \tag{2.1-19}$$

As expected, this formula reduces to $m = \frac{hf}{c^2}$ when the rest mass of the photon is zero.

A summary of the equations of this theory is given in the second column of table 1. It is interesting to observe that the equations of the second column (total energy and kinetic energy only) can be derived by substituting *pc* in the equations of the first column (Einstein's equations) with *hf.* This substitution is the reason of this theory being entitled “The Substitute Theory of Massive Photons”.

It is worthy to observe (see table 2, columns 1 and 3) that special relativity uses two different names for the energy of a particle according to the value of its rest mass. If the rest mass of a particle is non-zero (e.g. the rest mass of an electron), then special relativity refers to “relativistic energy”. However, if the rest mass of the particle is zero (e.g. the rest mass of photons) special relativity refers to “equivalent energy”.

So that, in special relativity the energy E_{rel} in the equation $E_{rel}^2 = (p_{rel}\, c)^2 + (m_0 c^2)^2$ has two different meanings: relativistic and equivalent (I used E_{rel} for simplicity reasons). This dual concept of special relativity suggests that photons are massive and that they should have a separate set of equations.

Energy type	Einstein's equations for massive subluminar particles (electrons, muons, tauons, hadrons)	Equations for massive luminar particles (photons and gluons)	Einstein's Equations for massless photons
Rest mass	m_0	m_0	0
Mass (m)	Relativistic mass $\frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$	Equivalent mass $m=\sqrt{\left(\frac{hf}{c^2}\right)^2+m_0^2}$	Equivalent mass $m=\frac{hf}{c^2}$
Momentum (p)	Relativistic momentum $mv = m_0\frac{v}{\sqrt{1-\frac{v^2}{c^2}}}$	Equivalent momentum $pc=\left(\sqrt{\left(\frac{hf}{c^2}\right)^2+m_0^2}\right)c$	Equivalent momentum $\frac{hf}{c}$
Rest energy (E_0)	$m_0 c^2$	$m_0 c^2$	0
Total energy (E)	Total relativistic energy $\sqrt{(p_{rel}\,c)^2+(m_0 c^2)^2}$	Total equivalent energy $\sqrt{(hf)^2+(m_0c^2)^2}$	Total equivalent energy hf
Kinetic energy (K)	Kinetic relativistic energy $\sqrt{(p_{rel}\,c)^2+(m_0c^2)^2}-m_0c^2$	Kinetic equivalent energy $\sqrt{(hf)^2+(m_0c^2)^2}-m_0c^2$	Kinetic equivalent energy hf

Table 1: *The equations of the second theory on massive photons. Note that according to Einstein's special relativity the total energy of a photon coincides with its kinetic energy.*

3. Predictions

3.1 Derivation of the Schwarzchild Radius

Now I shall derive the approximate formula for the Schwarzchild radius using the equations shown on Table 1 for massive photons.

In order to do that let us consider the gravitational potential energy of the photon at a distance r from the center of a star of mass M

$$U = -\frac{GMm}{r} \qquad (3.1\text{- }1)$$

The kinetic energy, K, of a photon is

$$K = \sqrt{(hf)^2 + (m_0 c^2)^2} - m_0 c^2 \qquad (3.1\text{- }2)$$

According to Einstein

$$E = m c^2 \qquad (3.1\text{- }3)$$

where

$m = m_\gamma =$ equivalent mass of the proton

Substituting hf in equation (3.1- 2) with the second side of equation (3.1- 3) yields the kinetic energy as a function of the equivalent mass of the photon

$$K = \sqrt{(m^2 c^4) + (m_0^2 c^4)} - m_0 c^2 \qquad (3.1\text{- }4)$$

Now we invoke the principle of conservation of energy which can be written as

$$K + U = k \qquad (3.1\text{- }5)$$

where k is a constant.

Substituting K with equation (3.1- 4) , U with equation (3.1- 1) and the constant k with 0 and the non-emission condition, we get

$$\sqrt{(m^2 c^4) + (m_0^2 c^4)} - m_0 c^2 \leq \frac{GM\, m}{r} \qquad (3.1\text{- }6)$$

$$c^2\left(\sqrt{m^2 + m_0^2} - m_0\right) \leq \frac{GM\, m}{r} \qquad (3.1\text{- }7)$$

$$r \leq \frac{GM}{c^2} \frac{m}{\sqrt{m^2 + m_0^2} - m_0} \qquad (3.1\text{- }8)$$

Now we assume that when the gravitational force exerted by the star on the photon is sufficiently large, the photon will not be able to escape from the star. Under this conditions the distance from the center of the star to the photon will be not bigger than R (the photon cannot escape to empty space). Let us assume that the distance, r , equals the radius of the star, $R,$ that under these extreme conditions we shall call it R_S instead

Now we define the cutoff radius, R_{cutoff} , as

$$R_{cutoff} \equiv \frac{GM}{c^2} \frac{m}{\sqrt{m^2+m_0^2} - m_0} \qquad (3.1\text{- } 9\text{ a})$$

thus equation (3.1- 8) can be rewritten as

$$r \leq R_{cutoff} \qquad (3.1\text{- }10)$$

Equation (3.1- 9 a) and (3.1- 10) tells us that the cutoff radius depends on both the equivalent mass of the photon and on its rest mass.

Under these conditions I shall assume that

Assumption

due to the intense gravitational field inside the black hole, the equivalent mass of the photon, $m = m_\gamma$, will be the same as its rest mass, $m_0 = m_{\gamma_0}$. Thus the cutoff radius will be the

If this assumption is true then and only then equation (3.1- 10) will transform into

$$R_{cutoff} = \frac{GM}{c^2} \frac{m_0}{\sqrt{m_0^2+m_0^2} - m_0} \qquad (3.1\text{- }10)$$

$$R_{cutoff} = \frac{1}{\sqrt{2}-1} \frac{GM}{c^2} \qquad (3.1\text{- }11)$$

$$R_{cutoff} = \left(\sqrt{2}+1\right) \frac{GM}{c^2} \qquad (3.1\text{- }12)$$

Under these conditions we can call this particular cutoff radius the Schwarzschild radius $R_{cutoff} = R_S$ and therefore we can write

$$R_S = R_{S\,massive} \simeq 2.4142\, \frac{GM}{c^2} \qquad (3.1\text{- }13)$$

(Result from this theory of massive photons: 20.7 % higher than GR)

Using the equations of this new theory (assuming that photons are massive particles), and assuming that gravity reduces the equivalent mass of the photons the value of their rest masses, we have proved that the equation for the Schwarzschild radius is closer to the equation given by general relativity (GR) than the corresponding result obtained with Einstein's special relativity (photons with zero rest mass).

Mathematically the result from GR is

$$R_{S-GR} = \frac{2\,GM}{c^2} \qquad (3.1\text{- }14)$$

(result from GR)

The result from special relativity (SR) is

$$R_{S-SR} = \frac{GM}{c^2} \qquad (3.1\text{- }15)$$

(Result form special relativity (massless photons): 50 % lower than GR)

Table 2 includes the last three equations (3.1- 13), (3.1- 14) and (3.1- 15) for comparison purposes

Special relativity (massless photons)	**General relativity**	**This theory (massive photons)**
$R_{S-SR} = \frac{GM}{c^2}$	$R_{S-GR} = 2\frac{GM}{c^2}$	$R_{S\,massive} \simeq 2.4142\frac{GM}{c^2}$
$R_{S-SR} = 50\,\%\,R_{S-GR}$	–	$R_{S\,massive} = 120.7\,\%\,R_{S-GR}$

Table 2: *The three results at a glance. First row: the first column shows the result from special relativity (SR). The second column shows the result from general relativity (GR). The third column shows the result from this theory (massive). Second row: this row shows the above values for the Schwarzschild radius as a percentage of the value predicted by GR.*

Taking into account that, regardless of the model we use (massless or massive photons), the theory of special relativity cannot produce the correct equation for the Schwarzschild radius of a black hole, we conclude that the present formulation improves the results by a 29.3 %. This improvement is due to the fact that this formulation assumes that photons are massive particles.

Equation (3.1- 10) can be expressed in terms of the frequency of the photons through equation (2.1-19) as follows

$$R_{cutoff} = \frac{GM}{c^2}\frac{\sqrt{\left(\frac{hf}{c^2}\right)^2 + m_0^2}}{\sqrt{\left(\frac{hf}{c^2}\right)^2 + 2m_0^2} - m_0^2} \qquad (3.1\text{- }16)$$

This equation (3.1- 16) tells us something new and fundamental:

The cutoff radius of a black hole is a function of the frequency of the photons.

As expected, equation (3.1- 16) reduces to $R_{cutoff} = R_S = \frac{GM}{c^2}$ when we take the limit of the photon rest mass tending to zero. This is the result we obtain with SR.

3.2 The Photoelectric Effect

The photoelectric effect was explained by Albert Einstein with the following equation

$$\frac{1}{2} m_e v_{max}^2 = hf - W \qquad (3.2\text{-}1)$$

where

m_e = electron rest mass
v_{max} = maximum emission velocity of photoelectrons for a monochromatic frequency f
h = Planck's constant
f = frequency of monochromatic light
W = work function. Work done to eject one of the electrons from the surface of the material.

The first side of equation (3.2-1) is the maximum kinetic energy of the emitted electrons. Einstein assumed that these electrons were the least firmly bound to an atom of the material. This energy is also known as the maximum emission energy.

Taking into account equation (2.1-14) for the equivalent kinetic energy of the photon the new equation for the photoelectric effect is

$$\frac{1}{2} m_e v_{max}^2 = \sqrt{(hf)^2 + (m_0 c^2)^2} - m_0 c^2 - W \qquad (3.2\text{-}2)$$

As expected, equation (3.2-2) reduces to equation (3.2-1) when the photon rest mass is zero.

3. Conclusions

In a previous article [2] I introduced my first theory on massive photons. In this paper I have presented my second theory on this subject. The reader has the right to ask: why are there two theories? After all, Nature will either behave one way or the other. The reason is that I don't know which theory is the correct one. To decide which is the right formulation we need to test them thoroughly, both theoretically and experimentally. On the theoretical side, one way of testing these two competing formulations, is to incorporate them into the standard model and see what the outcome is. Another way is to incorporate them into quantum gravity theories and evaluate, among other things, the cosmological outcomes.

Despite their differences, both theories predict the same behaviour for black holes: The cutoff radius of a black hole is a function of the frequency of the photons generated inside the black hole. The exact mechanism that light suffers inside the black hole is unclear, however, we have now two new fresh formulations that eventually will help us to unveil the mistery.

REFERENCES
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[2] R. A. Frino, *“The Theory of Massive Photons (Theory 1)”*, viXra: 1409.xxxx, (2014)

