The axiomatic definition of infinitesimals

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Abstract

In this paper, we redefine the infinitesimals by using axiomatic method.

1 Introduction and results

There are several definitions of the infinitesimals:

- 1. Archimedes' definition in the method of exhaustion.
- 2. Leibniz's definition in differential calculus.
- 3. Cauchy's definition in the theory of limit.
- 4. Robinson's definition in non-standard analysis.

We try to propose the redefinition of infinitesimals in the following axiom:

Axiom 1. Let e be the infinitesimal. Then:

- 1. 0 < e < 1.
- 2. $|e-0| \in \mathbf{R} \setminus \mathbf{Z}$.
- $3. 1/e < \infty.$

We hope the definition can help us to understand the behavior of infinitesimals.