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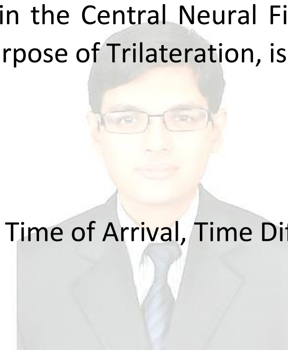
A Novel Trilateration Algorithm for Localization of a Transmitter/Receiver Station in a 2D Plane Using Analytical Geometry

Abstract

Trilateration is the name given to the Algorithm used in Global Positioning System (GPS) technology to localize the position of a Transmitter/Receiver station (also called a Blind Node) in a 2D plane, using the positional knowledge of three non-linearly placed Anchor Nodes. For instance, it may be desired to locate the whereabouts of a mobile phone (Blind Node) lying somewhere within the range of three radio signal transmitting towers (Anchor Nodes). There are various Trilateration Algorithms in the literature that achieve this end using among other methods, linear algebra.

This paper is a direct spin off from prior work by the same author, titled "A Mathematical Treatise on Polychronous Wavefront Computation and its Applications into Modeling Neurosensory Systems". The Geometric Algorithm developed there was originally intended to localize the position of a special class of neurons called Coincidence Detectors in the Central Neural Field. A general outline of how the same methodology can be adapted for the purpose of Trilateration, is presented here.



Keywords

Trilateration, Anchor Node, Blind Node, Time of Arrival, Time Difference of Arrival

List of Abbreviations

TOA – Time of Arrival
TDOA – Time Difference of Arrival

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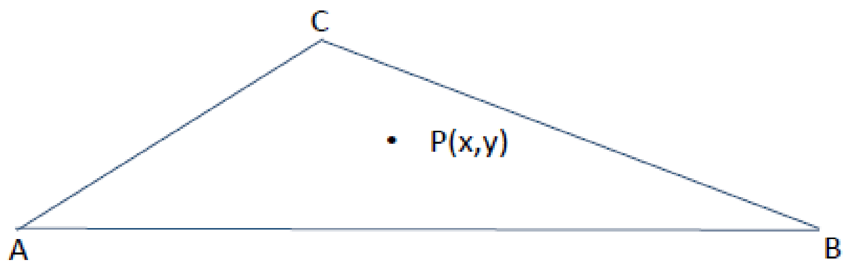
Introduction

The Geometric Elements composing Trilateration

Consider three Anchor Nodes situated at the vertices A, B and C of a scalene triangle with known coordinate positions (see Figure 1). A scalene triangle is chosen so as to maintain generality. The Blind Node P, whose coordinate position (x, y) is to be ascertained, lies somewhere in the same plane as that of the three Anchor Nodes. The algorithm employed inorder to do this, makes use of methods restricted to Analytical Geometry. The quintessence of this algorithm is as follows: *"the Blind Node P lies at the common point of intersection of the branches of three hyperbolas having the sides AB, AC and CB of ΔABC as transverse axes."* The equations of these hyperbolas are presented in the ***Methods*** section along with examples of their graphical simulation using MATLAB coding, in the ***Results*** section.



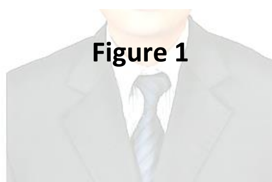


Figure 1

Method and Materials

The detailed derivations of the main equations presented in this section, can be found in the Supplementary Material to the author's Main Paper titled "A Mathematical Treatise on Polychronous Wavefront Computation and its Application into Modeling Neurosensory Systems". Only a summary of the key steps involved in the proposed algorithm are highlighted here.

Let the Anchor Nodes be situated at the vertices A, B and C of a Scalene Triangle with known co-ordinates $(-a, 0)$, $(b, 0)$ and $(0, c)$, respectively. Here, a, b and c are chosen to be non-negative numbers. Also let the Blind Node P be situated in the same plane as A, B and C, at an unknown position (x, y). Finally, denote the distances of P from the vertices $A(-a, 0)$, $B(b, 0)$ and $C(0, c)$ as r_1, r_2 and r_3, respectively (see Figure 2). If v be the speed of signal propagation, then the time it takes for the signals emitted from A, B and C to reach P are $\frac{r_1}{v}, \frac{r_2}{v}$ and $\frac{r_3}{v}$ respectively. These quantities are referred to as Time of Arrivals (TOAs), in the literature. However, for the particular algorithm that is formulated in this paper, it is the Differences in the Time of Arrivals (TDOAs) of the signals at P that is exploited (denoted by Δt_1, Δt_2 and Δt_3). By considering the Anchor Nodes in a pair-wise fashion, the expressions for TDOAs can be written as follows:

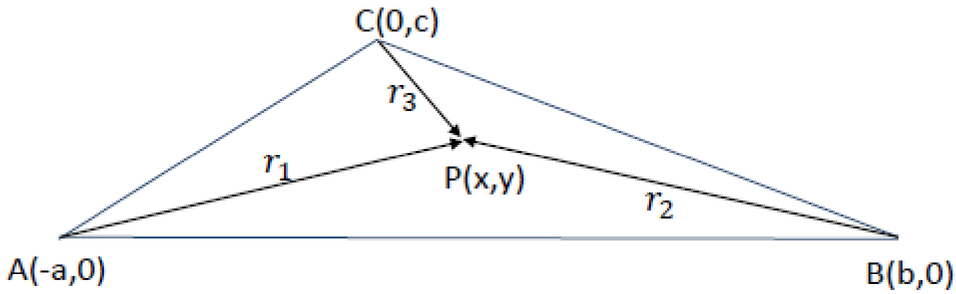


Figure 2

The TDOA between signals transmitted from Anchor Nodes A and B at the Blind Node P, is given by:

$$\Delta t_1 = |t_{BP} - t_{AP}| = \left|\frac{r_2}{v} - \frac{r_1}{v}\right| = \left|\frac{r_2 - r_1}{v}\right|$$

The TDOA between signals transmitted from Anchor Nodes C and B at the Blind Node P, is given by:

$$\Delta t_2 = |t_{BP} - t_{CP}| = \left|\frac{r_2}{v} - \frac{r_3}{v}\right| = \left|\frac{r_2 - r_3}{v}\right|$$

The TDOA between signals transmitted from Anchor Nodes A and C at the Blind Node P, is given by:

$$\Delta t_3 = |t_{CP} - t_{AP}| = \left|\frac{r_3}{v} - \frac{r_1}{v}\right| = \left|\frac{r_3 - r_1}{v}\right|$$

Recall that the Blind Node P is located at the common point of intersection of three hyperbolas with sides AB, AC and CB as transverse axes.

These hyperbolas are characterized by the constants $|r_2 - r_1|$, $|r_2 - r_3|$ and $|r_3 - r_1|$ and their equations are given by:

(i) Equation of Hyperbola with side AB as transverse axis ($|r_2 - r_1| = constant$),

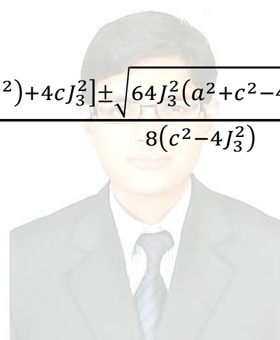
$$y = \pm\sqrt{\left(\left(\frac{a+b}{2}\right)^2 - {J^2}_1\right)}\sqrt{\left(\frac{\left(x-\frac{b-a}{2}\right)^2}{{J^2}_1} - 1\right)}$$

(ii) Equation of Hyperbola with side CB as transverse axis ($|r_2 - r_3| = constant$),

$$y = \frac{-4\left[-2bcx+c\left(b^2-c^2\right)+4cJ_2^2\right]\pm\sqrt{64J_2^2\left(b^2+c^2-4J_2^2\right)(4x^2-4bx+b^2+c^2-4J_2^2)}}{8\left(c^2-4J_2^2\right)}$$

(iii) Equation of Hyperbola with side AC as transverse axis ($|r_3 - r_1| = constant$),

$$y = \frac{-4\left[2acx+c\left(a^2-c^2\right)+4cJ_3^2\right]\pm\sqrt{64J_3^2\left(a^2+c^2-4J_3^2\right)(4x^2+4ax+a^2+c^2-4J_3^2)}}{8\left(c^2-4J_3^2\right)}$$



where $J_1 = \frac{v\Delta t_1}{2}$, $J_2 = \frac{v\Delta t_2}{2}$ and $J_3 = \frac{v\Delta t_3}{2}$

Results

For the purpose of graphical simulation, the following numerical values are adopted:

$$a = 10\, meters, b = 20\, meters, c = 30\, meters, v = 300\, meters/second$$

Please refer to the ***Appendix*** section for the ***MATLAB coding*** used in the examples below for arbitrary signal TDOAs at the Blind Node P.

Example 1:

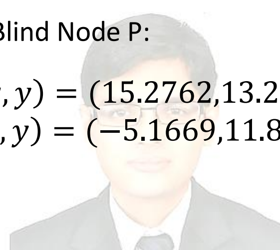
For the following choice of TDOA values:

$$\Delta t_1 = 0.05\, seconds$$

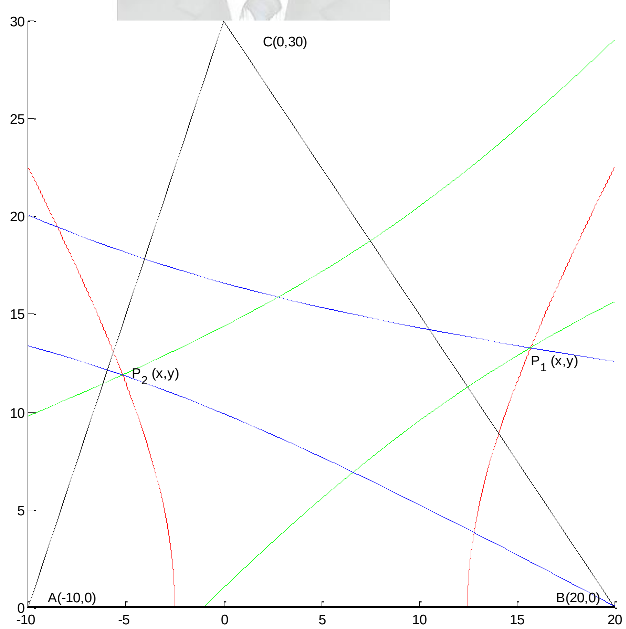
$$\Delta t_2 = 0.03\, seconds$$

$$\Delta t_3 = 0.02\, seconds$$

There are two possible positions of the Blind Node P:



$$P_1(x,y) = (15.2762,13.2817)$$
$$P_2(x,y) = (-5.1669,11.8889)$$



Example 2:

For the following choice of TDOA values:

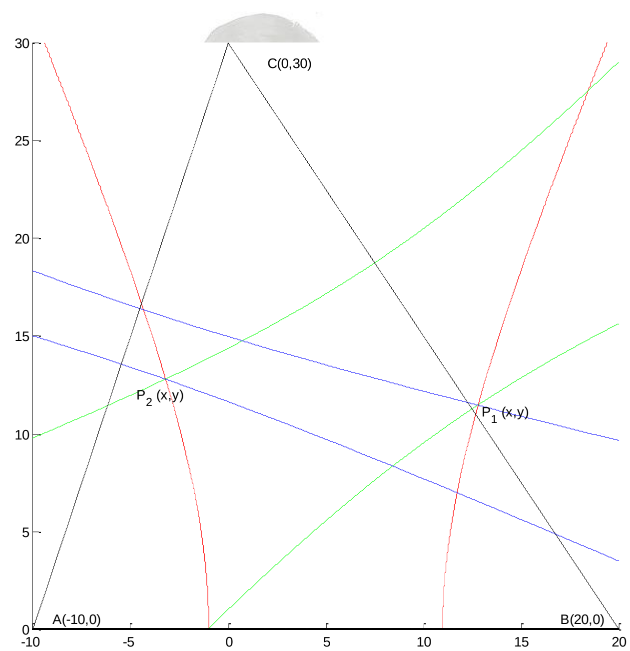
$$\Delta t_1 = 0.04\ seconds$$

$$\Delta t_2 = 0.03\ seconds$$

$$\Delta t_3 = 0.01\ seconds$$

There are two possible positions of the Blind Node P:

$$P_1(x,y) = (12.8128,11.4656)$$
$$P_2(x,y) = (-3.198,12.7998)$$



Example 3:

For the following choice of TDOA values:

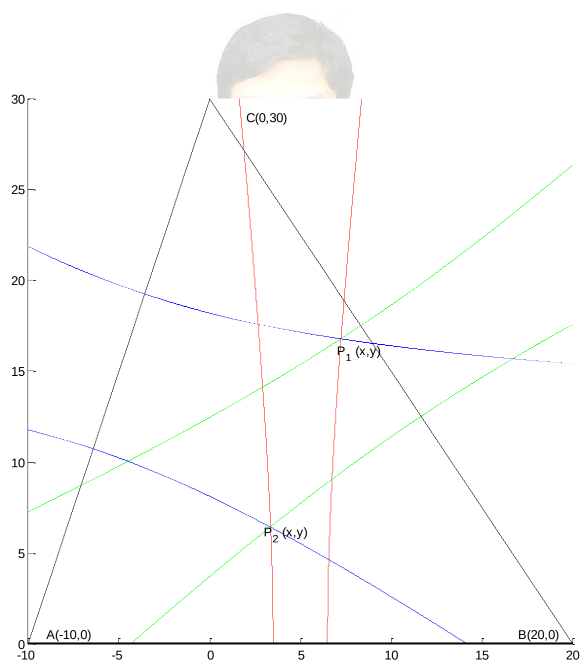
$$\Delta t_1 = 0.01 \; seconds$$

$$\Delta t_2 = 0.02 \; seconds$$

$$\Delta t_3 = 0.03 \; seconds$$

There are two possible positions of the Blind Node P:

$$P_1(x, y) = (7.2575, 16.7867)$$
$$P_2(x, y) = (3.3674, 6.413)$$



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Example 4:

For the following choice of TDOA values:

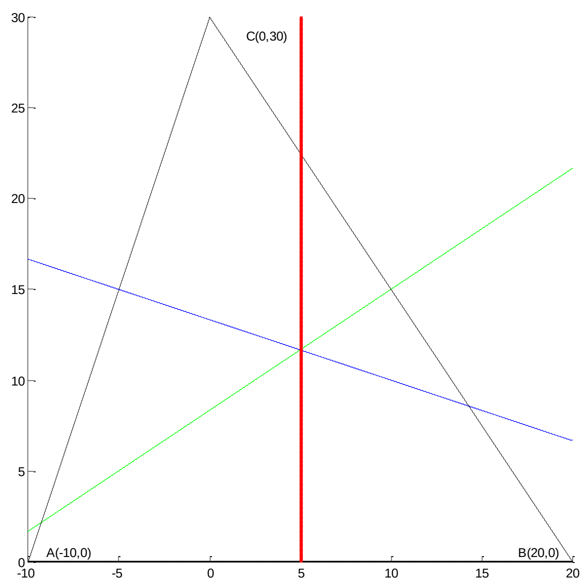
$$\Delta t_1 = 0 seconds$$

$$\Delta t_2 = 0 \ seconds$$

$$\Delta t_3 = 0 \ seconds$$

There is only a single positions for the Blind Node P:

$$P(x, y) = (5, 11.6667)$$



Discussion

The novelty of the proposed algorithm consists in the use of analytical geometry to arrive at the equations of the three hyperbolas, with each hyperbola having one side of the scalene triangle for a transverse axis. The common point of intersections of their branches defines the position of the Blind Node. The accuracy of the solution can be increased by simply increasing the point resolution along the X-axis.

The TDOAs are determined from the knowledge of the TOAs. The latter are found by means of time tracking devices implanted in the Anchor Nodes. As can be seen in each graphical example, there are two possible locations for the Blind Node to be located for a given triplet set of TDOAs (the exception to this rule is when all the TDOAs are zero simultaneously, in which case the Blind Node will be located at a single point, namely the Circumcenter of the scalene triangle – see Example 4). In order to identify at which one of the two possible general point locations, the required Blind Node actually lies, it is necessary to first determine the ordering in the magnitudes of TOAs. And since the TOA is proportional to distance r of the Blind Node from the Anchor Node, (because signal speed v is constant), the ordering of Anchor Node-Blind Node Distances (r_1, r_2, r_3) will help ascertain the required Blind Node position.

Acknowledgements

Gloria in excelsis Deo

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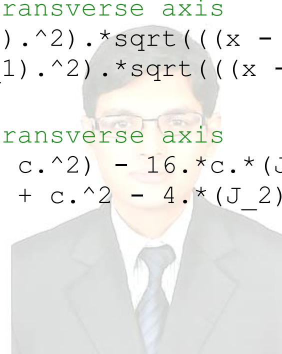
Appendix

MATLAB Coding

```
% parameter values for anchor node coordinates and signal speed
a = 10 ;
b = 20 ;
c = 30 ;
v = 300 ;

% TDOA of Signals at Blind Node
del_t_1 = 0.05 ; % input example
del_t_2 = 0.03 ; % input example
del_t_3 = 0.02 ; % input example

J_1 = (v.*del_t_1)./2 ;
J_2 = (v.*del_t_2)./2 ;
J_3 = (v.*del_t_3)./2 ;

x = linspace(-10, 20, 10000) ;

% Hyperbolas with side AB as transverse axis
y_1 = sqrt(((a+b)./2)^2 - (J_1).^2).*sqrt(((x - (b-a)./2)./(J_1)).^2 - 1) ;
y_2 = -sqrt(((a+b)./2)^2 - (J_1).^2).*sqrt(((x - (b-a)./2)./(J_1)).^2 - 1) ;

% Hyperbolas with side AC as transverse axis
A = 8.*b.*c.*x - 4.*c.*(b.^2 - c.^2) - 16.*c.*(J_2).^2 ;
B = sqrt(64.*((J_2).^2).*(b.^2 + c.^2 - 4.*(J_2).^2).*(4.*x.^2 - 4.*b.*x +
b.^2 + c.^2 - 4.*(J_2).^2)) ;
C = 8.*(c.^2 - 4.*(J_2).^2) ;
y_3 = (A+B)./C ;
y_4 = (A-B)./C ;

% Hyperbolas with side CB as axis
E = -8.*a.*c.*x - 4.*c.*(a.^2 - c.^2) - 16.*c.*(J_3).^2 ;
F = sqrt(64.*((J_3).^2).*(a.^2 + c.^2 - 4.*(J_3).^2).*(4.*x.^2 + 4.*a.*x +
a.^2 + c.^2 - 4.*(J_3).^2)) ;
G = 8.*(c.^2 - 4.*(J_3).^2) ;
y_5 = (E+F)./G ;
y_6 = (E-F)./G ;

hold on
plot(x,y_1,'r',x,y_2,'r')
plot(x,y_3,'g',x,y_4,'g')
plot(x,y_5,'b',x,y_6,'b')
plot(x,0,'k',x,3.*x+30,'k',x,-(1.5).*x+30,'k')
axis square
axis([-10 20 0 30])
text(-9,0.5,'A(-10,0)')
text(2,29,'C(0,30)')
text(17,0.5,'B(20,0)')
text(15.75,12.5,'P_1 (x,y)')
text(-4.7,11.88,'P_2 (x,y)')
hold off
```



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