The Concept of Mass as Interfering Photons

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Abstract

For most purposes in physics the concept of mass particles and photons are treated as though they are completely separate and distinct entities having little connection accept through collision interactions. This paper explores the concept of a mass particle being viewed as a pair of trapped photons in a mass-less box demonstrating proper relativistic dynamics and Lorentz covariance. The mechanism of trapping of the photons in the particle is not mechanically apparent; however the lack of sufficient spin to properly define a photon would be a containment factor. Although this presentation is primarily focused on electrons, presumably it would be applicable to any particle with primary constituents consisting of particles that travel at light speed along null vectors. This is the perfect way to teach the concept of the equivalence of mass and energy, and why mass velocity cannot exceed the speed of light.

Introduction

The special theory of relative, through the Lorentz transformations, yields the energy velocity relation for photons and particles, one through a shift in frequency, the other through a shift in mass. Considering these particles as different forms of energy, however, bestows a distinction between the forms of energy that is possibly unwarranted. The Lorentz transforms applied to a pair of localized photons can be shown to yield the same results as the transforms applied to a mass particle. If a photon is constrained to a point as the result of annihilation by the emission of a virtual photon by its opposite going companion or is just the three-dimensional interference patterns of two photons that are somehow spatially constrained to their center of mass, the properties match very well the dynamics of real mass particles. One may not subscribe to the details of this, but it does give a useful perspective regarding mass, rest mass, and energy, and offer an intuitive understand of why particles cannot exceed the speed of light.

I Momentum

Consider a thought experiment, in which two photons are placed in a perfectly reflecting mass-less container. Presuming that if the two photons are not aligned in the given frame, there has to be some sub-light speed frame of reference, in which the photons are aligned, and in opposite directions, as well as having equal energy and frequency. This frame is thus the rest frame for the center of mass for the two photons

Using the momentum for the photons to be:

$$\vec{P} = M\vec{c} = \left[\frac{h\nu}{c}\right]\frac{\vec{c}}{c} \ , \tag{1}$$

where we can designate an energy equivalent "mass " for the photon to be $M = h\nu / c^2$. The momentum of the container with respect to a moving frame of reference with velocity v is then:

$$\vec{P} = \left(M_1 + M_2\right)\vec{v} \, . \tag{2}$$

From the perspective of the individual opposite-going photons the momentum is:

$$P = P_1 + P_2 = \frac{h\nu_1 - h\nu_2}{c} = \frac{h\Delta\nu}{c} = \frac{h}{\lambda_B} \, . \tag{3}$$

The wavelength of the difference in the frequency here, or the "beat" frequency, is just the simple deBroglie wavelength.

The total energy, which is the sum of the energy of the photons, and thus sum of the frequencies, yields the simple Compton wavelength:

$$\frac{E_1 + E_2}{c} = \frac{h\nu_1 + h\nu_2}{c} = \frac{h}{\lambda_C} \, . \tag{4}$$

Using the above noted designation for "mass" we can write for the total "mass":

$$M_T = \left(h\nu_1 + h\nu_2\right) / c^2 \, , \tag{5}$$

and

$$P = M_T v = \left(M_1 - M_2\right) c \, . \tag{6}$$

Solving for velocity:

$$\frac{v}{c} = \frac{\left(M_1 - M_2\right)}{\left(M_1 + M_2\right)} \, . \tag{7}$$

This is notably just the velocity for the center of mass for two opposite going photons.

Since for a particle:

$$M_0{}^2 = M^2\left[1-\left(\frac{v}{c}\right)^2\right]. \tag{8}$$

Putting in M_T ,and v/c and solving gives:

$$M_0{}^2 = \left(M_1 + M_2\right)^2 - \left(M_1 - M_2\right)^2 \quad = 4M_1M_2\,. \tag{9}$$

So the square of the rest mass of the particle is four times the product of the "mass" of the individual photons.

II Doppler

Now for the moment, we can shift gears, and look at this same picture from the standpoint of the Doppler shift, on transformation of velocity coordinates for the two photons.

The relativistic Doppler shift of the photons from one velocity frame to another is:

$$\nu_1' = \nu_1\left[\frac{1-\frac{v}{c}}{1+\frac{v}{c}}\right] \qquad \nu_2' = \nu_2\left[\frac{1+\frac{v}{c}}{1-\frac{v}{c}}\right] \quad , \tag{10}$$

or using the above noted conventions for energy equivalent mass:

$$M_1' = M_1\left[\frac{1-\frac{v}{c}}{1+\frac{v}{c}}\right] \qquad M_2' = M_2\left[\frac{1+\frac{v}{c}}{1-\frac{v}{c}}\right] \quad , \tag{11}$$

Multiplying the two relations gives:

$$M_1'M_2' = M_1M_2 = \text{constant}\,, \tag{12}$$

and simple math gets:

$$M_1M_2 = \frac{\left(M_1 + M_2\right)^2 - \left(M_1 - M_2\right)^2}{4}\,, \tag{13}$$

and:

$$\left[1-\left(\frac{v}{c}\right)^2\right] = \frac{4M_1M_2}{\left(M_1+M_2\right)^2} = \frac{M_0^2}{M^2}, \tag{14}$$

which is the same as the above relation, found for conformance to relativistic kinematics, the model thus transforms properly.

Four Momentum

Defining the photon mass as in Eq.(5), moving along null vectors in the opposite direction the null four-momentum of two opposite going photons previously defined for the particle in the geometric algebra matrix form is:

$$\overrightarrow{P_1} = \gamma_1 m_1 c_x + \gamma_2 m_1 c_x + \gamma_4 m_1 c_z + \gamma_4 \left(m_1 c\right) \tag{15}$$

$$\overrightarrow{P_2} = -\gamma_1 m_2 c_x - \gamma_2 m_2 c_x - \gamma_4 m_2 c_z + \gamma_4 \left(m_2 c\right) \tag{16}$$

Presuming these two photons are co-located, the square of the sum of the null vectors is necessarily constant and is:

$$\left(m_1+m_2\right)^2 - \left(m_1-m_2\right)^2 = 4m_1m_2 = m_R{}^2 \tag{17}$$

The magnitude of each of these null four-momentum is zero for covariance, and the sum of two such moments must be constant. Thus $m_R{}^2$ must be invariant fixed quantity associated with the pair of opposite going photons. If this is defined as a rest mass then it is easy to identify:

$$\left(m_1+m_2\right)^2 = m_T, \tag{18}$$

as the total mass. Factoring the total mass from Eq.(17), gives:

$$\left(m_1 + m_2\right)^2 \left[1 - \frac{\left(m_1 - m_2\right)^2}{\left(m_1 + m_2\right)^2}\right] = m_R{}^2 \tag{19}$$

Noting that:

$$\frac{\left(m_1 - m_2\right)}{\left(m_1 + m_2\right)} \tag{20}$$

Is the ratio of the velocity of each photon to the velocity of the center of mass.

$$\left(m_1 - m_2\right) v_c = \left(m_1 + m_2\right) c \tag{21}$$

This makes Eq.(19), the relativistic energy equation for a mass particle.

$$m^2 \left[1 - \frac{v^2}{c^2}\right] = m_R{}^2 \tag{22}$$

It can thus be asserted that two light speed photons, or other confined zero rest mass particles, have the property of a mass particle with mass energy equivalent to the energy of the individual particles.

Electron-Positron Annihilation

The above relations give the proper Mechanical result for the electron-positron annihilation. That is, as two particles merge the velocity of each increase to c and become two opposite going photons. In a simplistic description as the pair merge, the left going photons in the two particles, and the right going photons in the two particles, constructively and destructively interfere giving two opposite going free photons. Spin considerations for pair annihilation is treated in Appendix I.

Conclusion

The concurring points of similarity of the opposite photons in a massless box model and the particles are then:

1) The deBroglie wavelength.
2) The Compton wavelength
3) The zero velocity or rest mass
4) The total energy
5) Velocity transforms
6) Annihilation process

Using a reflecting container is somewhat artificial, but as in the case of the transformation of momentum between velocity frames, the gross mechanics do not depend on the internal structure. All of the real internal constraints such as spin, energy, etc, which may be important to the actual mechanics of holding a particle together are not necessary to understand the concept.

The dynamics of the center of mass of the two photons is the same whether the photons are confined or not, accept in one case there is a localized mass particle. It is also easy to understand from this model why mass particles do not exceed the speed of light.

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Appendix I

Electron Photon Spin Relations
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The purpose of this paper is to show that the superposition of the spin representation, of four properly arranged photons, is equivalent to the spin representation of an electron positron pair, colocated at the same position. This is in furtherance of the notion that a mass particle such an electrons can be viewed as a pair of photons trapped in a mass-less box, held together by a lack of sufficient spin angular momentum to escape.

Photon spin represented in the four dimensional, rank three 4x4 matrix as.

$$\sigma_{p1} = \frac{i}{\sqrt{2}}\begin{bmatrix} & -i & & \\ -i & & -i & \\ & -i & & \\ & & & \end{bmatrix} \sigma_{p2} = \frac{i}{\sqrt{2}}\begin{bmatrix} & -1 & & \\ 1 & & -1 & \\ & 1 & & \\ & & & \end{bmatrix} \quad \sigma_{p3} = i\begin{bmatrix} -i & & & \\ & & & \\ & & +i & \\ & & & \end{bmatrix} \tag{1}$$

A second Photon could be represented similarly

$$\sigma'_{p1} = \frac{i}{\sqrt{2}}\begin{bmatrix} & & & \\ & & i & \\ & i & & -i \\ & & -i & \end{bmatrix} \sigma'_{p2} = \frac{i}{\sqrt{2}}\begin{bmatrix} & & & \\ & & 1 & \\ & -1 & & -1 \\ & & 1 & \end{bmatrix} \quad \sigma'_{p3} = i\begin{bmatrix} & & & \\ & i & & \\ & & & \\ & & & -i \end{bmatrix} \tag{2}$$

The spin magnitudes of these photons are:

$$\sigma^2 = \sigma_x{}^2 + \sigma_y{}^2 + \sigma_z{}^2$$

$$= \frac{1}{2}\begin{bmatrix} 1 & & 1 & \\ & 2 & & \\ 1 & & 1 & \\ & & & \end{bmatrix} \begin{bmatrix} 1 & & -1 & \\ & 2 & & \\ -1 & & +1 & \\ & & & \end{bmatrix} \begin{bmatrix} 2 & & & \\ & & & \\ & & 2 & \\ & & & \end{bmatrix} = \begin{bmatrix} 2 & & & \\ & 2 & & \\ & & 2 & \\ & & & \end{bmatrix} \tag{4}$$

and

$$= \frac{1}{2}\begin{bmatrix} & & & \\ & 1 & & -1 \\ & & 2 & \\ & -1 & & 1 \end{bmatrix} \begin{bmatrix} & & & \\ & 1 & & 1 \\ & & 2 & \\ & 1 & & 1 \end{bmatrix} \begin{bmatrix} & & & \\ & 2 & & \\ & & & \\ & & & 2 \end{bmatrix} = \begin{bmatrix} & & & \\ & 2 & & \\ & & 2 & \\ & & & 2 \end{bmatrix} \tag{5}$$

Where the spins are aligned along the Z axis.

We can also represent the photons as being circularly polarized as:

$$\sigma_{p1} = \frac{i}{\sqrt{2}}\begin{bmatrix} & -i & & \\ -i & & -i & \\ & -i & & \\ & & & \end{bmatrix} \sigma_{p2} = \frac{i}{\sqrt{2}}\begin{bmatrix} & -1 & & \\ 1 & & -1 & \\ & 1 & & \\ & & & \end{bmatrix} \quad \sigma_{p3} = \frac{i}{\sqrt{2}}\begin{bmatrix} 1-i & & & \\ & & & \\ & & -i+i & \\ & & & \end{bmatrix} \tag{6}$$

$$\sigma'_{p1}=\frac{i}{\sqrt{2}}\begin{bmatrix} & i & \\ i & & -i \\ & -i & \end{bmatrix}\sigma'_{p2}=\frac{i}{\sqrt{2}}\begin{bmatrix} & 1 & \\ -1 & & -1 \\ & 1 & \end{bmatrix}\quad \sigma'_{p3}=i\begin{bmatrix} & i-1 & \\ & & \\ & & -i+1 \end{bmatrix} \tag{7}$$

If these Photons are colocated and aligned the superposition of the functions is:

$$\sigma_{p1}=\frac{i}{\sqrt{2}}\begin{bmatrix} & -i & & \\ -i & & & \\ & & & -i \\ & & -i & \end{bmatrix}\quad \sigma_{p2}=\frac{i}{\sqrt{2}}\begin{bmatrix} & -1 & & \\ 1 & & & \\ & & & -1 \\ & & 1 & \end{bmatrix}\quad \sigma_{p3}=\frac{i}{\sqrt{2}}\begin{bmatrix} -1-i & & & \\ & 1+i & & \\ & & 1+i & \\ & & & -1-i \end{bmatrix}$$

Though we don’t want to suggest this represents the spin of a single particle for it is still the superposition of two photons, note that the value of this is:

$$=\frac{1}{2}\begin{bmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{bmatrix}+\frac{1}{2}\begin{bmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{bmatrix}+\frac{1}{2}\begin{bmatrix} 2 & & & \\ & 2 & & \\ & & 2 & \\ & & & 2 \end{bmatrix}=\begin{bmatrix} 2 & & & \\ & 2 & & \\ & & 2 & \\ & & & 2 \end{bmatrix} \tag{9}$$

applying a time and space inversion of this, by applying the psudoscalar

$$\tau=\begin{bmatrix} 1 & & & \\ & 1 & & \\ & & -1 & \\ & & & -1 \end{bmatrix}$$

Gives

$$\sigma_{p1}=\frac{i}{\sqrt{2}}\begin{bmatrix} & -i & & \\ -i & & & \\ & & & i \\ & & i & \end{bmatrix}\quad \sigma_{p2}=\frac{i}{\sqrt{2}}\begin{bmatrix} & -1 & & \\ 1 & & & \\ & & & 1 \\ & & -1 & \end{bmatrix}\quad \sigma_{p3}=\frac{i}{\sqrt{2}}\begin{bmatrix} -1-i & & & \\ & 1+i & & \\ & & -1-i & \\ & & & 1+i \end{bmatrix}$$

(11)

Making some permutations of the above representations, a second representation of can be:

$$\sigma_{p1} = \frac{i}{\sqrt{2}} \begin{bmatrix} & -i & & \\ -i & & & \\ & & & -i \\ & & -i & \end{bmatrix} \quad \sigma_{p2} = \frac{i}{\sqrt{2}} \begin{bmatrix} & -1 & & \\ 1 & & & \\ & & & -1 \\ & & 1 & \end{bmatrix} \quad \sigma_{p3} = \frac{i}{\sqrt{2}} \begin{bmatrix} -1-i & & & \\ & 1+i & & \\ & & -1-i & \\ & & & 1+i \end{bmatrix} \tag{12}$$

If these four photons are colocated then the sum of these representations can be resegregated into:

$$\sigma_{p1} = \frac{i}{\sqrt{2}} \begin{bmatrix} & -i & & \\ -i & & & \\ & & & i \\ & & i & \end{bmatrix} \quad \sigma_{p2} = \frac{i}{\sqrt{2}} \begin{bmatrix} & -1 & & \\ 1 & & & \\ & & & 1 \\ & & -1 & \end{bmatrix} \quad \sigma_{p3} = \frac{i}{\sqrt{2}} \begin{bmatrix} -i & & & \\ & i & & \\ & & i & \\ & & & i \end{bmatrix} \tag{13}$$

and

$$\sigma_{p1} = \frac{i}{\sqrt{2}} \begin{bmatrix} & -i & & \\ -i & & & \\ & & & -i \\ & & -i & \end{bmatrix} \quad \sigma_{p2} = \frac{i}{\sqrt{2}} \begin{bmatrix} & -1 & & \\ 1 & & & \\ & & & -1 \\ & & 1 & \end{bmatrix} \quad \sigma_{p3} = \frac{i}{\sqrt{2}} \begin{bmatrix} -i & & & \\ & i & & \\ & & -i & \\ & & & i \end{bmatrix} \tag{14}$$

Which is just the spin representation of two half spin particles, i.e. an electron and positron. QED the premise has been shown.