Conjectured Primality Criteria for Specific Classes of $k \cdot b^n - 1$

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Abstract: Conjectured polynomial time primality tests for specific classes of numbers of the form $k \cdot b^n - 1$ are introduced .
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1 Introduction

In 1969 Hans Riesel provided polynomial time primality test for numbers of the form $3 \cdot 2^n - 1$ with $n > 2$, see Theorem 5 in [1] . In this note I present polynomial time primality tests for specific classes of numbers of the form $k \cdot b^n - 1$ that may be considered as generalization of the Riesel primality test for $3 \cdot 2^n - 1$.

2 The Main Result

Definition 2.1. Let $P_m(x) = 2^{-m} \cdot \left(\left(x - \sqrt{x^2-4}\right)^m + \left(x + \sqrt{x^2-4}\right)^m \right)$, where m and x are nonnegative integers .

Conjecture 2.1. *Let $N = k \cdot b^n - 1$ such that $n > 2$, $k < b^n$ and*

$$\begin{cases} k \equiv 3 \pmod{30} \; with \; b \equiv 2 \pmod{10} \; \textit{and} \; n \equiv 0,3 \pmod 4 \\ k \equiv 3 \pmod{30} \; with \; b \equiv 4 \pmod{10} \; \textit{and} \; n \equiv 0,2 \pmod 4 \\ k \equiv 3 \pmod{30} \; with \; b \equiv 6 \pmod{10} \; \textit{and} \; n \equiv 0,1,2,3 \pmod 4 \\ k \equiv 3 \pmod{30} \; with \; b \equiv 8 \pmod{10} \; \textit{and} \; n \equiv 0,1 \pmod 4 \end{cases}$$

Let $S_i = P_b(S_{i-1})$ with $S_0 = P_{bk/2}(P_{b/2}(5778))$, then
N is prime iff $S_{n-2} \equiv 0 \pmod N$

Conjecture 2.2. *Let $N = k \cdot b^n - 1$ such that $n > 2$, $k < b^n$ and*

$$\begin{cases} k \equiv 9 \pmod{30} \; with \; b \equiv 2 \pmod{10} \; and \; n \equiv 0,1 \pmod{4} \\ k \equiv 9 \pmod{30} \; with \; b \equiv 4 \pmod{10} \; and \; n \equiv 0,2 \pmod{4} \\ k \equiv 9 \pmod{30} \; with \; b \equiv 6 \pmod{10} \; and \; n \equiv 0,1,2,3 \pmod{4} \\ k \equiv 9 \pmod{30} \; with \; b \equiv 8 \pmod{10} \; and \; n \equiv 0,3 \pmod{4} \end{cases}$$

Let $S_i = P_b(S_{i-1})$ *with* $S_0 = P_{bk/2}(P_{b/2}(5778))$ *, then*
N *is prime iff* $S_{n-2} \equiv 0 \pmod{N}$

References

[1] Riesel, Hans (1969) , "Lucasian Criteria for the Primality of $k \cdot 2^n - 1$" , *Mathematics of Computation* (AmericanMathematical Society), 23 (108): 869-875 .