Conjectured Compositeness Tests for Specific Classes of $b^n - b - 1$ and $b^n + b + 1$

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Abstract: Compositeness criteria for specific classes of numbers of the form $b^n + b + 1$ and $b^n - b - 1$ are introduced .
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1 Introduction

In 2008 Ray Melham provided unconditional , probabilistic , lucasian type primality test for generalized Mersenne numbers [1] . In this note I present polynomial time compositeness tests for specific classes of numbers of the form $b^n + b + 1$ and $b^n - b - 1$.

2 The Main Result

Definition 2.1. Let $P_m(x) = 2^{-m} \cdot \left(\left(x - \sqrt{x^2-4}\right)^m + \left(x + \sqrt{x^2-4}\right)^m \right)$, where m and x are nonnegative integers .

Conjecture 2.1. *Let $N = b^n - b - 1$ such that $n > 2$, $b \equiv 0, 6 \pmod 8$.*

$$\text{Let } S_i = P_b(S_{i-1}) \text{ with } S_0 = P_{b/2}(6) \text{ , thus}$$
$$\text{If } N \text{ is prime then } S_{n-1} \equiv P_{(b+2)/2}(6) \pmod N$$

Conjecture 2.2. *Let $N = b^n - b - 1$ such that $n > 2$, $b \equiv 2, 4 \pmod 8$.*

$$\text{Let } S_i = P_b(S_{i-1}) \text{ with } S_0 = P_{b/2}(6) \text{ , thus}$$
$$\text{If } N \text{ is prime then } S_{n-1} \equiv -P_{b/2}(6) \pmod N$$

Conjecture 2.3. *Let $N = b^n + b + 1$ such that $n > 2$, $b \equiv 0, 6 \pmod 8$.*

$$\text{Let } S_i = P_b(S_{i-1}) \text{ with } S_0 = P_{b/2}(6) \text{ , thus}$$
$$\text{If } N \text{ is prime then } S_{n-1} \equiv P_{b/2}(6) \pmod N$$

Conjecture 2.4. *Let* $N = b^n + b + 1$ *such that* $n > 2$, $b \equiv 2, 4 \pmod{8}$.

$$\text{Let } S_i = P_b(S_{i-1}) \text{ with } S_0 = P_{b/2}(6) \text{ , thus}$$
$$\text{If } N \text{ is prime then } S_{n-1} \equiv -P_{(b+2)/2}(6) \pmod{N}$$

References

[1] R. S. Melham , ”Probable prime tests for generalized Mersenne numbers,” , *Bol. Soc. Mat. Mexicana* , 14 (2008) , 7-14 .