Conjectured Polynomial Time Compositeness Tests for Numbers of the Form $k \cdot 2^n \pm 1$

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August 11 , 2014

Abstract: Conjectured polynomial time compositeness tests for numbers of the form $k \cdot 2^n - 1$ and $k \cdot 2^n + 1$ are introduced .
Keywords: Compositeness test , Polynomial time , Prime numbers .
AMS Classification: 11A51 .

1 Introduction

Let p be an odd prime . Define the sequence $\{S_n\}_{n \geq 0}$ by

$$S_0 = 6 ,$$
$$S_{k+1} = S_k^2 - 2 , k \geq 0$$

The compositeness test for $(2^p + 1)/3$ states :

Theorem 1.1. *If N_p is prime then $S_{p-1} \equiv -34 \pmod{N_p}$*

See Theorem 2 in [1] .

2 The Main Result

Definition 2.1. Let $P_m(x) = 2^{-m} \cdot \left(\left(x - \sqrt{x^2-4}\right)^m + \left(x + \sqrt{x^2-4}\right)^m\right)$, where m and x are positive integers .

Conjecture 2.1. *Let $N = k \cdot 2^n - 1$ such that $n > 2$ and $k > 0$.*

Let $S_i = P_2(S_{i-1})$ with $S_0 = P_k(6)$, thus
If N is prime then $S_{n-1} \equiv 6 \pmod{N}$

Conjecture 2.2. *Let $N = k \cdot 2^n + 1$ such that $n > 2$ and $k > 0$.*

Let $S_i = P_2(S_{i-1})$ with $S_0 = P_k(6)$, thus
If N is prime then $S_{n-1} \equiv 2 \pmod{N}$

References

[1] Pedro Berrizbeitia ,Florian Luca ,Ray Melham , ”On a Compositeness Test for $(2^p + 1)/3$”, *Journal of Integer Sequences*,Vol. 13 (2010), Article 10.1.7 .