Schrödinger's cat paradox resolution using GRW collapse model. Von Neumann measurement postulate revisited

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Abstract

In his famous thought experiment,Schrôdinger(1935) imagined a cat that measures the value of an quantum mechanical observable with its life. Since Schrödinger's time, no any interpretations or modifications of quantum mechanics have been proposed which gives clear unambiguous answers to the questions posed by Schrödinger's cat of how long superpositions last and when (or whether) they collapse? In this paper appropriate modification of quantum mechanics is proposed. We claim that canonical interpretation of the wave function $\psi = c_1\psi_1 + c_2\psi_2$ is correct only when the supports the wave functions ψ_1 and ψ_2 essentially overlap. When the wave functions ψ_1 and ψ_2 have separated supports (as in the case of the experiment that we are considering in this paper) we claim that canonical interpretation of the wave function $\psi = c_1\psi_1 + c_2\psi_2$ is no longer valid for a such cat state. Possible solution of the Schrödinger's cat paradox is considered.We pointed out that the collapsed state of the cat always shows definite and predictable outcomes even if cat also consists of a superposition:

$$\text{cat} = c_1\left|\text{live cat}\right\rangle + c_2\left|\text{death cat}\right\rangle.$$

Keywords

Probability representation of quantum states; **Schrödinger's cat**; **GRW collapse model**; **Measurement Von Neumann measurement postulate**.

6mm . Introduction 3mm

As Weinberg recently reminded us cite: Weinberg12[1], the measurement problem remains a fundamental conundrum. During measurement the state vector of the microscopic system collapses in a probabilistic way to one of a number of classical states, in a way that is unexplained, and cannot be described by the time-dependent Schrödinger equation cite: Weinberg12[1].To review the essentials, it is sufficient to consider two-state systems. Suppose a nucleus **n**, whose Hilbert space is spanned by orthonormal states $|s_i(t)\rangle$, $i = 1,2,$where $|s_1(t)\rangle = \big|\text{undecayed nucleus at instant } t\big\rangle$ and $|s_2(t)\rangle = \big|\text{decayed nucleus at instant } t\big\rangle$ is in the superposition state,

$$|\Psi_t\rangle_{\mathbf{n}} = c_1|s_1(t)\rangle + c_2|s_2(t)\rangle, |c_1|^2 + |c_2|^2 = 1. \tag{1.1}$$

A measurement apparatus A, which may be microscopic or macroscopic, is designed to distinguish between states $|s_i(t)\rangle$ by transitioning at each instant t into state $|a_i(t)\rangle$ if it finds **n** is in $|s_i(t)\rangle$, $i = 1,2.$ Assume the detector is reliable, implying the $|a_1(t)\rangle$ and $|a_2(t)\rangle$ are orthonormal at each instant t ,i.e., $\langle a_1(t)||a_2(t)\rangle = 0$ and that the measurement interaction does not disturb states $|s_i\rangle$ -i.e., the measurement is "ideal". When A measures $|\Psi_t\rangle_{\mathbf{n}}$, the Schrödinger equation's unitary time evolution then leads to the "measurement state" $|\Psi_t\rangle_{\mathbf{n}A}$:

$$|\Psi_t\rangle_{\mathbf{n}A} = c_1|a_1(t)\rangle + c_2|a_2(t)\rangle, |c_1|^2 + |c_2|^2 = 1. \tag{1.2}$$

of the composite system **n**A following the measurement.

Standard formalism of continuous quantum measurements cite: BassiLochanSatinSinghUlbricht13, JacobsSteck06, Mensky93, Mensky00 [2, 3, 4, 5] leads to a definite but unpredictable measurement outcome, either $|a_1(t)\rangle$ or $|a_2(t)\rangle$ and that $|\Psi_t\rangle_{\mathbf{n}}$ suddenly "collapses" at instant t' into the corresponding state $|s_i(t')\rangle$. But unfortunately equation (1.2) does not appear to resemble such a collapsed state at instant t'?

The measurement problem is as follows:
(**I**) How do we reconcile canonical collapse models postulate's
(**II**) How do we reconcile the measurement postulate's definite outcomes with the "measurement state" $|\Psi_t\rangle_{\mathbf{n}A}$ at each instant t and
(**III**) how does the outcome become irreversibly recorded in light of the Schrödinger equation's unitary and, hence, reversible evolution?

This paper deals with only the special case of the measurement problem, known as Schrödinger’s cat paradox. For a good and complete explanation of this paradox see Leggett cite: Leggett84[6] and Hobson cite: Hobson13[7].

Figure

Schrödinger

Schrödinger’s cat adapted to the measurement

of position of an alpha particle cite: Schrodinger35[8]

In his famous thought experiment cite: Schrodinger35[8], Schrôdinger(1935) imagined a cat that measures the value of an quantum mechanical observable with its life. Adapted to the measurement of position of an alpha particle, the experiment is this. A cat, a flask of poison, and a radioactive source are placed in a sealed box. If an internal monitor detects radioactivity (i.e. a single atom decaying), the flask is shattered, releasing the poison that kills the cat. The Copenhagen interpretation of quantum mechanics implies that after a while, the cat is simultaneously alive and dead. Yet, when one looks in the box, one sees the cat either alive or dead, not both alive and dead.

This poses the question of when exactly quantum superposition ends and reality collapses into one possibility or the other?

Since Schrödinger’s time, no any interpretations or extensions of quantum mechanics have been proposed which gives clear unambiguous answers to the questions posed by Schrödinger’s cat of how long superpositions last and when (or whether) they collapse.

The canonical interpretations of the experiment.

Copenhagen interpretation

The most commonly held interpretation of quantum mechanics is the Copenhagen interpretation cite: Wimmel92[9]. In the Copenhagen interpretation, a system stops being a superposition of states and becomes either one or the other when an observation takes place. This thought experiment makes apparent the fact that the nature of measurement, or observation, is not well-defined in this interpretation. The experiment can be interpreted to mean that while the box is closed, the system simultaneously exists in a superposition of the states "decayed nucleus/dead cat" and "undecayed nucleus/living cat", and that only when the box is opened and an observation performed does the wave function collapse into one of the two states.

However, one of the main scientists associated with the Copenhagen interpretation, Niels Bohr, never had in mind the observer-induced collapse of the wave function, so that Schrödinger's cat did not pose any riddle to him. The cat would be either dead or alive long before the box is opened by a conscious observer cite: Faye08[10]. Analysis of an actual experiment found that measurement alone (for example by a Geiger counter) is sufficient to collapse a quantum wave function before there is any conscious observation of the measurement cite: CarpenterAnderson06[11]. The view that the "observation" is taken when a particle from the nucleus hits the detector can be developed into objective collapse theories. The thought experiment requires an "unconscious observation" by the detector in order for magnification to occur.

Objective collapse theories

According to objective collapse theories, superpositions are destroyed spontaneously (irrespective of external observation) when some objective physical threshold (of time, mass, temperature, irreversibility, etc.) is reached. Thus, the cat would be expected to have settled into a definite state long before the box is opened. This could loosely be phrased as "the cat observes itself", or "the environment observes the cat".

Objective collapse theories require a modification of standard quantum mechanics to allow superpositions to be destroyed by the process of time evolution. This process, known as "decoherence", is among the fastest processes currently known to physics cite: Omnes99[12].

Ensemble interpretation

The ensemble interpretation states that superpositions are nothing but subensembles of a larger statistical ensemble. The state vector would not apply to individual cat experiments, but only to the statistics of many similarly prepared cat experiments. Proponents of this interpretation state that this makes the Schrödinger's cat paradox a trivial matter, or a non-issue. This interpretation serves to discard the idea that a single physical system in quantum mechanics has a mathematical description that corresponds to it in any way.

Remark 1.1.Ensemble interpretation in a good agreement with a canonical interpretetion of the wave function (ψ-function) in canonical QM-measurement theory. However under rigorous consideration an dinamics of the Schrödinger's cat this interpretation gives unphysical result, see Proposition 3.2.(ii).

The canonical collapse models.

In order to appreciate how canonical collapse models work, and what they are able to achieve, we briefly review the GRW model. Let us consider a system of n particles which, only for the sake of simplicity, we take to be scalar and spinless; the GRW model is defined by the following postulates: (**1**) The state of the system is represented by a wave function $\psi_t(\mathbf{x}_1,\mathbf{x}_2,\ldots,\mathbf{x}_n)$ belonging to the Hilbert space $\mathcal{L}_2(\mathbb{R}^{3n})$. (**2**) At random times, the wave function experiences a sudden jump of the form:

$$\psi_t(\mathbf{x}_1,\mathbf{x}_2,\ldots,\mathbf{x}_n) \to \psi_t(\mathbf{x}_1,\mathbf{x}_2,\ldots,\mathbf{x}_n;\widetilde{\mathbf{x}}_m) = \frac{\Re_m(\widetilde{\mathbf{x}}_m)\psi_t(\mathbf{x}_1,\mathbf{x}_2,\ldots,\mathbf{x}_n)}{\|\Re_m(\widetilde{\mathbf{x}}_m)\psi_t(\mathbf{x}_1,\mathbf{x}_2,\ldots,\mathbf{x}_n)\|_2}, \tag{1.3}$$

where $\psi_t(\mathbf{x}_1,\mathbf{x}_2,\ldots,\mathbf{x}_n)$ is the state vector of the whole system at time t, immediately prior to the jump process and $\Re_n(\widetilde{\mathbf{x}}_m)$ is a linear operator which is conventionally chosen equal to:

$$\Re_m(\widetilde{\mathbf{x}}_m) = (\pi r_c^2)^{-3/4} \exp\left[-\frac{(\widehat{\mathbf{x}}_m - \widetilde{\mathbf{x}}_m)^2}{2r_c^2} \right], \tag{1.4}$$

where r_c is a new parameter of the model which sets the width of the localization process, and $\widehat{\mathbf{x}}_m$ is the position operator associated to the m-th particle of the system and the random variable $\widetilde{\mathbf{x}}_m$ corresponds to the place where the jump occurs. (**3**) It is assumed that the jumps are distributed in time like a Poissonian process with frequency $\lambda = \lambda_{GRW}$ this is the second new parameter of the model. (4) Between two consecutive jumps, the state vector evolves according to the standard Schrödinger equation.

The 1-particle master equation of the GRW model takes the form

$$\frac{d}{dt}\rho(t) = -\frac{i}{\hbar}\left[\widehat{\mathbf{H}}, \rho(t) \right] - T[\rho(t)]. \tag{1.5}$$

Here $\widehat{\mathbf{H}}$ is the standard quantum Hamiltonian of the particle, and $T[\bullet]$ represents the effect of the spontaneous collapses on the particle's wave function. In the position representation, this operator becomes:

$$\langle \mathbf{x}|T[\rho(t)]|\mathbf{y}\rangle = \lambda\left\{ 1 - \exp\left[-\frac{(\mathbf{x}-\mathbf{y})^2}{4r_c^2} \right] \right\}\langle \mathbf{x}|\rho(t)|\mathbf{y}\rangle. \tag{1.6}$$

Another modern approach to stochastic reduction is to describe it using a stochastic nonlinear Schrödinger equation, an elegant simplied example of which is the following one particle case known as Quantum Mechanics with Universal Position Localization [QMUPL]:

$$d|\psi_t(x)\rangle = \left[-\frac{i}{\hbar}\widehat{\mathbf{H}} - k(\widehat{q} - \langle q_t\rangle)^2 dt \right]|\psi_t(x)\rangle dt + \sqrt{2k}\,(\widehat{q} - \langle q_t\rangle) dW_t|\psi_t(x)\rangle. \tag{1.7}$$

Here $\widehat{q}$ is the position operator, $\langle q_t\rangle = \langle \psi_t|\widehat{q}|\psi_t\rangle$ it is its expectation value, and k is a constant, characteristic of the model, which sets the strength of the collapse mechanics, and it is chosen proportional to the mass m of the particle according to the formula: $k = (m/m_0)\lambda_0$, where m_0 is the nucleon's mass and λ_0 measures the collapse strength. It is easy to see that Eqn.(1.5) contains both non-linear and stochastic terms, which are necessary to induce the collapse of the wave function. For an examle let us consider a free particle ($\widehat{\mathbf{H}} = p^2/2m$), and a Gaussian state:

$$\psi_t(x) = \exp\left\{ -a_t(x - \bar{x}_t)^2 + i\bar{k}_t x \right\}. \tag{1.8}$$

It is easy to see that $\psi_t(x)$ given by Eq.(1.6) is solution of Eq.(1.5), where

$$\frac{da_t}{dt} = k - \frac{2i\hbar}{m}a_t^2, \frac{d\bar{x}_t}{dt} = \frac{\hbar}{m}\bar{k}_t + \frac{\sqrt{k}}{2\,\mathrm{Re}(a_t)}\dot{W}_t, \frac{d\bar{k}_t}{dt} = -\sqrt{k}\,\frac{\mathrm{Im}(a_t)}{\mathrm{Re}(a_t)}\dot{W}_t. \tag{1.9}$$

The CSL model is defined by the following stochastic differential equation in the Fock space:

$$d|\psi_t(\mathbf{x})\rangle = \left[-\frac{i}{\hbar}\widehat{\mathbf{H}} - k\left(\widehat{M}(\mathbf{x}) - \langle M_t(\mathbf{x})\rangle\right)^2 dt\right]|\psi_t(\mathbf{x})\rangle dt + \tag{1.10}$$
$$+\sqrt{2k}\left(\widehat{M}(\mathbf{x}) - \langle M_t(\mathbf{x})\rangle\right)dW_t(\mathbf{x})|\psi_t(\mathbf{x})\rangle.$$

6mm . **Generalized Gamov theory of the alpha decay via tunneling using GRW collapse model** 3mm

By 1928, George Gamow had solved the theory of the alpha decay via tunneling cite: Gamov28[13]. The alpha particle is trapped in a potential well by the nucleus. Classically, it is forbidden to escape, but according to the (then) newly discovered principles of quantum mechanics, it has a tiny (but non-zero) probability of "tunneling" through the barrier and appearing on the other side to escape the nucleus. Gamow solved a model potential for the nucleus and derived, from first principles, a relationship between the half-life of the decay, and the energy of the emission.

The α-particle has total energy E and is incident on the barrier from the right to left.

Figure

The

The particle has total energy E and
is incident on the barrier $V(x)$ from right to left.
Adapted from cite: Gamov28[13]

The Schrödinger equation in each of regions $\mathbf{I} = \{x|x < 0\}, \mathbf{II} = \{x|0 \leq x \leq l\}$ and $\mathbf{III} = \{x|x > l\}$ takes the following form

$$\frac{\partial^2\Psi(x)}{\partial x^2} + \frac{2m}{\hbar^2}[E - U(x)]\Psi(x) = 0, \tag{2.1}$$

where

$$U(x) = \begin{cases} 0 \text{ for } x < 0 \\ U_0 \text{ for } 0 \leq x \leq l \\ 0 \text{ for } x > l \end{cases} \tag{2.2}$$

The solutions reads [8]:

$$\begin{aligned} \Psi_{\mathbf{III}}(x) &= C_+ \exp(ikx) + C_- \exp(-ikx), \\ \Psi_{\mathbf{II}}(x) &= B_+ \exp(k'x) + B_- \exp(-k'x), \\ \Psi_{\mathbf{I}}(x) = A\cos(kx) &= \frac{A}{2}[\exp(ikx) + \exp(-ikx)], \end{aligned} \tag{2.3}$$

where

$$\begin{aligned} k &= \frac{2\pi}{\hbar}\sqrt{2mE}\,, \\ k' &= \frac{2\pi}{\hbar}\sqrt{2m(U_0 - E)}\,. \end{aligned} \tag{2.4}$$

At the boundary $x = 0$ we have the following boundary conditions:

$$\Psi_{\mathbf{I}}(0)|_{x=0} = \Psi_{\mathbf{II}}(0)|_{x=0}, \frac{\partial \Psi_{\mathbf{I}}(x)}{\partial x}\bigg|_{x=0} = \frac{\partial \Psi_{\mathbf{II}}(x)}{\partial x}\bigg|_{x=0}. \tag{2.5}$$

At the boundary $x = l$ we have the following boundary conditions

$$\Psi_{\mathbf{II}}(l)|_{x=l} = \Psi_{\mathbf{III}}(l)|_{x=l}, \frac{\partial \Psi_{\mathbf{II}}(x)}{\partial x}\bigg|_{x=l} = \frac{\partial \Psi_{\mathbf{III}}(x)}{\partial x}\bigg|_{x=l}. \tag{2.6}$$

From the boundary conditions (2.5)-(2.6) one obtains cite: Gamov28[13]:

$$\begin{gathered} B_+ = \frac{A}{2}\left(1 + i\frac{k}{k'}\right), B_- = \frac{A}{2}\left(1 - i\frac{k}{k'}\right), \\ C_+ = A[ch(k'l) + iDsh(k'l)], C_- = i(ASsh(k'l)\exp(ikl)), \\ D = \frac{1}{2}\left(\frac{k}{k'} - \frac{k'}{k}\right), S = \frac{1}{2}\left(\frac{k}{k'} + \frac{k'}{k}\right). \end{gathered} \tag{2.7}$$

From (2.7) one obtain the conservation law

$$|A|^2 = |C_+|^2 - |C_-|^2.$$

Let us introduce now a function $E_{\mathbf{II}}(x,l) = \theta_2(x,l)E_2(x,l)$ where

$$E_2(x,l) = \begin{cases} (\pi r_c^2)^{-1/4}\exp\left(-\frac{x^2}{2r_c^2}\right) \text{ for } -\infty < x < \frac{l}{2} \\ (\pi r_c^2)^{-1/4}\exp\left(-\frac{(x-l)^2}{2r_c^2}\right) \text{ for } \frac{l}{2} \leq x < \infty \end{cases} \tag{2.8}$$

$$\theta_2(x,l) = \begin{cases} 1 \text{ for } x \in [0,l] \\ 0 \text{ for } x \notin [0,l] \end{cases}$$

Assumption 2.1. We assume now that:

(i) at instant $t = 0$ the wave function $\Psi_{\mathbf{I}}(x)$ experiences a sudden jump of the form

$$\Psi_{\mathbf{I}}(x) \to \Psi_{\mathbf{I}}^{\#}(x) = \frac{\Re_{\mathbf{I}}(\hat{x})\Psi_{\mathbf{I}}(x)}{\|\Re_{\mathbf{I}}(\hat{x})\Psi_{\mathbf{I}}(x)\|_2}, \tag{2.9}$$

where $\Re_{\mathbf{I}}(\hat{x})$ is a linear operator which is chosen equal to:

$$\Re_{\mathbf{I}}(\hat{x}) = (\pi r_c^2)^{-1/4}\theta_1(\hat{x},l)\exp\left[-\frac{\hat{x}^2}{2r_c^2}\right]; \tag{2.10}$$

where

$$\theta_1(x,l) = \begin{cases} 1 \text{ for } x \in [-l,0], \\ 0 \text{ for } x \notin [-l,0]. \end{cases}$$

Remark 2.1. Note that: $\operatorname{supp}(\Psi_{\mathbf{I}}^{\#}(x)) \subseteq [-l,0]$

(ii) at instant $t = 0$ the wave function $\Psi_{\mathbf{II}}(x)$ experiences a sudden jump of the form

$$\Psi_{\mathbf{II}}(x) \to \Psi_{\mathbf{II}}^{\#}(x) = \frac{\Re_{\mathbf{II}}(\hat{x})\Psi_{\mathbf{II}}(x)}{\|\Re_{\mathbf{II}}(\hat{x})\Psi_{\mathbf{II}}(x)\|_2}, \tag{2.11}$$

where $\Re_{\mathbf{II}}(\hat{x})$ is a linear operator which is chosen equal to:

$$\Re_{\mathbf{II}}(\hat{x}) = E_{\mathbf{II}}(\hat{x},l); \tag{2.12}$$

Remark 2.2. Note that: $\operatorname{supp}(\Psi_{\mathbf{II}}^{\#}(x)) \subseteq [0,l]$.

(iii) at instant $t = 0$ the wave function $\Psi_{\mathbf{III}}(x)$ experiences a sudden jump of the form

$$\Psi_{\mathbf{III}}(x) \to \Psi_{\mathbf{III}}^{\#}(x) = \frac{\Re_{\mathbf{III}}(\hat{x})\Psi_{\mathbf{III}}(x)}{\|\Re_{\mathbf{III}}(\hat{x})\Psi_{\mathbf{III}}(x)\|_2}, \tag{2.13}$$

where $\Re_{\mathbf{III}}(\hat{x})$ is a linear operator which is chosen equal to:

$$\Re_{\mathbf{III}}(\hat{x}) = (\pi r_c^2)^{-1/4}\exp\left[-\frac{(\hat{x}-l)^2}{2r_c^2}\right]. \tag{2.14}$$

Remark 2.3. Note that. We have choose operators (2.10),(2.12) and (2.14) such that the boundary conditions (2.5),(2.6) are satisfied.

Definition 2.1. Let $\Psi(x)$ be an solution of the Schrödinger equation (2.1). The stationary

Schrödinger equation (2.1) is a weakly well preserved in region $\Gamma \subseteq \mathbb{R}$ by collapsed wave function $\Psi^{\#}(x)$ if there exist an wave function $\Psi(x)$ such that the estimate

$$\int_{\Gamma}\left\{\frac{\partial^2\Psi^{\#}(x)}{\partial x^2}+\frac{2m}{\hbar^2}[E-U(x)]\Psi^{\#}(x)\right\}dx = O(\hbar^{2+\alpha}), \tag{2.15}$$

where $\alpha \geq 1,$ is satisfied.

Proposition 2.1.The Schrödinger equation in each of regions $\mathbf{I},\mathbf{II},\mathbf{III}$ is a weakly well

preserved by collapsed wave function $\Psi^{\#}_{\mathbf{I}}(x),\Psi^{\#}_{\mathbf{II}}(x)$ and $\Psi^{\#}_{\mathbf{III}}(x)$ correspondingly.

Proof. See Appendix B.

Definition 2.2.Let us consider the time-dependent Schrödinger equation:

$$i\hbar\frac{\partial\Psi(\mathbf{x},t)}{\partial t} = \widehat{\mathbf{H}}\Psi(\mathbf{x},t), \tag{2.16}$$
$$t \in [0,T], \mathbf{x} \in \mathbb{R}^{3n}.$$

The time-dependent Schrödinger equation (2.16) is a weakly well preserved by corresponding to $\Psi(\mathbf{x},t)$ collapsed wave function $\Psi^{\#}(\mathbf{x},t)$

$$\Psi^{\#}(\mathbf{x}_1,\mathbf{x}_2,\ldots,\mathbf{x}_n,t) =$$

$$\Psi(\mathbf{x}_1,\mathbf{x}_2,\ldots,\mathbf{x}_n,t;\widetilde{\mathbf{x}}_{m_1},\ldots,\widetilde{\mathbf{x}}_{m_k}) =$$

$$= \frac{\Re_{m_1,\ldots,m_k}(\widetilde{\mathbf{x}}_{m_1},\ldots,\widetilde{\mathbf{x}}_{m_k})\Psi(\mathbf{x}_1,\mathbf{x}_2,\ldots,\mathbf{x}_n,t)}{\|\Re_{m_1,\ldots,m_k}(\widetilde{\mathbf{x}}_{m_1},\ldots,\widetilde{\mathbf{x}}_{m_k})\Psi(\mathbf{x}_1,\mathbf{x}_2,\ldots,\mathbf{x}_n,t)\|_2},$$
$$\Re_{m_1,\ldots,m_k}(\widetilde{\mathbf{x}}_{m_1},\ldots,\widetilde{\mathbf{x}}_{m_k}) = \prod_{i=1}^{k}\Re_{m_i}(\widetilde{\mathbf{x}}_{m_i})$$

in region $\Gamma \subseteq \mathbb{R}^{3d}$ if there exist an

wave function $\Psi(\mathbf{x},t)$ such that the estimate

$$\int_{\Gamma}\left\{i\hbar\frac{\partial\Psi^{\#}(\mathbf{x},t)}{\partial t}-\widehat{\mathbf{H}}\Psi^{\#}(\mathbf{x},t)\right\}d^{3d}x = O(\hbar^{\alpha}), \tag{2.17}$$
$$t \in [0,T], \mathbf{x} \in \mathbb{R}^{3d},$$

where $\alpha \geq 1,$ is satisfied.

Definition 2.3. Let $\Psi^{\#}(\mathbf{x},t)$=$\Psi^{\#}(\mathbf{x}_1,\mathbf{x}_2,\ldots,\mathbf{x}_d,t)$ be a function

$\Psi(\mathbf{x}_1,\mathbf{x}_2,\ldots,\mathbf{x}_d,t;\widetilde{\mathbf{x}}_1,\ldots,\widetilde{\mathbf{x}}_d)$. Let us consider the Probability Current Law

$$\frac{\partial}{\partial t}P(\Gamma,t)+\int\limits_{\partial\Gamma}\mathbf{J}(\mathbf{x}_1,\mathbf{x}_2,\ldots,\mathbf{x}_d,t)\cdot\mathbf{n}d^{2d}x=O(\hbar^{\alpha}),$$
$$\mathbf{J}(\mathbf{x}_1,\mathbf{x}_2,\ldots,\mathbf{x}_d,t)=\Psi(\mathbf{x},t)\nabla\overline{\Psi(\mathbf{x},t)}-\overline{\Psi(\mathbf{x},t)}\nabla\Psi(\mathbf{x},t),\tag{2.18}$$
$$t\in[0,T],\mathbf{x}\in\mathbb{R}^{3d},$$

corresponding to Schrödinger equation (2.16). Probability Current Law (2.18) is a weakly well preserved by corresponding to $\Psi(\mathbf{x},t)$ collapsed wave function $\Psi^{\#}(\mathbf{x},t)$ in region $\Gamma\subseteq\mathbb{R}^{3d}$ if there exist an wave function $\Psi(\mathbf{x},t)$ such that the estimate

$$\frac{\partial}{\partial t}P(\Gamma,t)+\int\limits_{\partial\Gamma}\mathbf{J}^{\#}(\mathbf{x}_1,\mathbf{x}_2,\ldots,\mathbf{x}_d,t)\cdot\mathbf{n}d^{2d}x=O(\hbar^{\alpha}),$$
$$\mathbf{J}^{\#}(\mathbf{x}_1,\mathbf{x}_2,\ldots,\mathbf{x}_d,t)=\Psi^{\#}(\mathbf{x},t)\nabla\overline{\Psi^{\#}(\mathbf{x},t)}-\overline{\Psi^{\#}(\mathbf{x},t)}\nabla\Psi^{\#}(\mathbf{x},t)\tag{2.19}$$
$$=O(\hbar^{\alpha}),$$
$$t\in[0,T],\mathbf{x}\in\mathbb{R}^{3d},$$

where $\alpha\geq 1$, is satisfied.

Proposition 2.2. Assume that there exist an wave function $\Psi(\mathbf{x},t)$ such that the estimate (2.17) is satisfied. Then Probability Current Law (2.18) is a weakly well preserved by corresponding to $\Psi(\mathbf{x},t)$ collapsed wave function $\Psi^{\#}(\mathbf{x},t)$ in region $\Gamma\subseteq\mathbb{R}^{3d}$, i.e. the estimate (2.19) is satisfied on the wave function $\Psi^{\#}(\mathbf{x},t)$.

6mm . Schrödinger's cat paradox resolution 3mm

In this section we shall consider the problem of the collapse of the cat state vector on the basis of two different hypotheses:

(**A**) The canonical postulate of QM is correct in all cases.

(**B**) The canonical interpretation of the wave function $\psi=c_1\psi_1+c_2\psi_2$ is correct only when the supports the wave functions ψ_1 and ψ_2 essentially overlap. When the wave functions ψ_1 and ψ_2 have separated supports (as in the case of the experiment that we are considering in section II) we claim that canonical interpretation of the wave function $\psi=c_1\psi_1+c_2\psi_2$ is no longer valid for a such cat state, for details see section 4.

5mm . Consideration of the Schrödinger's cat paradox using canonical von

Neumann postulate 2mm
Let $|s_1(t)\rangle$ and $|s_2(t)\rangle$ be the states

$$|s_1(t)\rangle = \left|\text{undecayed nucleus at instant } t\right\rangle,$$
$$|s_2(t)\rangle = \left|\text{decayed nucleus at instant } t\right\rangle. \tag{3.1}$$

We assume now that

$$|s_1(0)\rangle = \int_{-\infty}^{+\infty} \Psi^{\#}_{\mathbf{II}}(x)|x\rangle dx \tag{3.2}$$

and

$$|s_2(0)\rangle = \int_{-\infty}^{+\infty} \Psi^{\#}_{\mathbf{I}}(x)|x\rangle dx. \tag{3.3}$$

Remark 3.1. Note that: (i) $|s_2(0)\rangle = \left|\text{decayed nucleus at instant } 0\right\rangle =$

$=\left|\text{free } \alpha\text{-particle at instant } 0\right\rangle$. (ii) Feynman propagator of a free α-particle are cite: FeynmanHibbs05[14]:

$$K_2(x,t,x_0) = \left(\frac{m}{2\pi i\hbar t}\right)^{1/2} \exp\left\{\frac{i}{\hbar}\left[\frac{m(x-x_0)^2}{2t}\right]\right\}. \tag{3.4}$$

Therefore from Eq.(3.3),Eq.(2.9) and Eq.(3.4) we obtain

$$|s_2(t)\rangle = \int_{-\infty}^{+\infty} \Psi_{\mathbf{I}}^{\#}(x,t)|x\rangle dx,$$

$$\Psi_{\mathbf{I}}^{\#}(x,t) = \int_{-\infty}^{0} \Psi_{\mathbf{I}}^{\#}(x_0) K_2(x,t,x_0) dx_0 =$$

$$(\pi r_c^2)^{-1/4} \times \left(\frac{m}{2\pi i\hbar t}\right)^{1/2} \times \int_{-\infty}^{0} \theta_1(x_0,l) \exp\left(-\frac{x_0^2}{2r_c^2}\right) \exp\left(-i\frac{2\pi}{\hbar}\sqrt{2mE}\,x_0\right) \times$$

$$\times \exp\left\{\frac{i}{\hbar}\left[\frac{m(x-x_0)^2}{2t}\right]\right\} dx_0 =$$

$$(3.5)$$

$$(\pi r_c^2)^{-1/4} \times \left(\frac{m}{2\pi i\hbar t}\right)^{1/2} \times \int_{-l}^{0} \theta_1(x_0,l) \exp\left(-\frac{x_0^2}{2r_c^2}\right) \times$$

$$\times \exp\left\{\frac{i}{\hbar}\left[\frac{m(x-x_0)^2}{2t} - \pi\sqrt{4mE}\,x_0\right]\right\} dx_0 =$$

$$(\pi r_c^2)^{-1/4} \times \left(\frac{m}{2\pi i\hbar t}\right)^{1/2} \times \int_{-l}^{0} \theta_1(x_0,l) \exp\left(-\frac{x_0^2}{2r_c^2}\right) \times \exp\left\{\frac{i}{\hbar}[S(t,x,x_0)]\right\} dx_0,$$

where

$$S(t,x,x_0) = \frac{m(x-x_0)^2}{2t} - \pi\sqrt{8mE}\,x_0. \tag{3.6}$$

We assume now that

$$\hbar \ll 2r_c^2 \ll l^2 \ll 1. \tag{3.7}$$

Oscillatory integral in RHS of Eq.(3.5) is calculated now directly using stationary phase approximation. The phase term $S(x,x_0)$ given by Eq.(3.6) is stationary when

$$\frac{\partial S(t,x,x_0)}{\partial x_0} = -\frac{m(x-x_0)}{t} - \pi\sqrt{8mE} = 0. \tag{3.8}$$

Therefore

$$-\frac{m(x-x_0)}{t} - \pi\sqrt{8mE} = 0,$$
$$-(x-x_0) = \pi t\sqrt{8E/m}, \tag{3.9}$$

and thus stationary point $x_0(t,x)$ is

$$x_0(t,x) = \pi t\sqrt{8E/m} + x. \tag{3.10}$$

From Eq.(3.5) and Eq.(3.10) using stationary phase approximation we obtain

$$\begin{gathered}|s_2(t)\rangle = \int_{-\infty}^{+\infty} \Psi_{\mathbf{I}}^{\#}(x,t)|x\rangle dx,\\ \Psi_{\mathbf{I}}^{\#}(x,t) =\\ (\pi r_c^2)^{-1/4} \times \theta_1(x_0(t,x),l)\exp\left[-\frac{x_0^2(t,x)}{2r_c^2}\right] \times \exp\left\{\frac{i}{\hbar}[S(t,x,x_0(t,x))]\right\} + O(\hbar),\end{gathered} \tag{3.11}$$

where

$$S(x,x_0(t,x)) = \frac{m(x-x_0(t,x))^2}{2t} - \pi\sqrt{8mE}\,x_0(t,x). \tag{3.12}$$

From Eq.(3.10) and Eq.(3.11) we obtain

$$\overline{\Psi_{\mathbf{I}}^{\#}(x,t)}\Psi_{\mathbf{I}}^{\#}(x,t) \simeq (\pi r_c^2)^{-1/2} \times \theta_1\left(x + \pi t\sqrt{8E/m}\,,l\right)\exp\left[-\frac{\left(x + \pi t\sqrt{8E/m}\right)^2}{r_c^2}\right]. \tag{3.13}$$

Remark 3.2. From the inequality (3.7) and Eq.(3.13) follows that α-particle at each instant $t \geq 0$ moves quasiclassically from right to left by the law

$$x(t) = -\pi t\sqrt{8E/m}\,, \tag{3.14}$$

i.e., the result is obtained by estimating the position $x_{\mathbf{est}}(t)$ at each instant $t \geq 0$ with final error r_c gives $|x_{\mathbf{est}}(t) - x(t)| \leq r_c, i = 1,\ldots,d$ with a probability 1,i.e.,$\mathbf{P}\{|x_{\mathbf{est}}(t) - x(t)| \leq r_c\} = 1$.

Remark 3.3. We assume now that a distance between radioactive source and internal monitor which detects a single atom decaying (see Pic.1) is equal to L.

Proposition 3.1. We assume now that:

(i) α-decay arises at instant $t = 0$ with a probability 1, and therefore a nucleus $\mathbf{n}$ at instant $t = 0$ is in the state

$$|\text{decayed nucleus at instant } 0\rangle = |\text{free } \alpha\text{-particle at instant } 0\rangle.$$

(ii) Schrödinger's cat at instant $t = 0$ is in the state

$$|\text{Schrödinger's cat}\rangle = |\text{a live cat}\rangle.$$

Then after α-decay at instant $t = 0$ the collapse

$$|\text{Schrödinger's cat}\rangle \to |\text{death cat}\rangle$$

arises at instant

$$T = \frac{L}{\pi\sqrt{8E/m}} \tag{3.15}$$

with a probability $\mathbf{P}_T\left(\left|\text{death cat}\right\rangle\right)$ to observe a state $\left|\text{death cat}\right\rangle$ at instant T is

$$\mathbf{P}_T\left(\left|\text{death cat}\right\rangle\right) = 1.$$

Proof. Note that. In this case Schrödinger's cat in fact performs the single measurement of α-particle position with accuracy of $\delta x = l$ at instant $t = T$ (given by Eq.(3.15)) by internal monitor (see Pic.1.1). The probability of getting a result L with accuracy of $\delta x = l$ given by

$$\int_{|L-x|\leq l/2} |\langle x|s_2(T)\rangle|^2 dx = 1. \tag{3.16}$$

Therefore (see Remark 3.2)at instant T the α-particle kills Schrödinger's cat with a probability

$$\mathbf{P}_T\left(\left|\text{death cat}\right\rangle\right) = 1.$$

Remark 3.4.Note that. When Schrödinger's cat has performed this measurement the immediate post measurement state of α-particle (by classical von Neumann measurement postulate P.3.5) will end up in the state

$$\left|\breve{\Psi}_T\right\rangle = \frac{\int_{|L-x|\leq l/2} |x\rangle\langle x|s_2(T)\rangle dx}{\sqrt{\int_{|L-x|\leq l/2} |\langle x|s_2(T)\rangle|^2 dx}} = \int_{|L-x|\leq l/2} |x\rangle\langle x|s_2(T)\rangle dx \tag{3.17}$$

From Eq.(3.17) one obtains

$$\langle x'\left|\breve{\Psi}_T\right\rangle = \int_{|L-x|\leq l/2} \langle x'|x\rangle\langle x|s_2(T)\rangle dx = \int_{|L-x|\leq l/2} \delta(x'-x)\langle x|s_2(T)\rangle dx = \Psi^{\#}_{\mathbf{I}}(x',t). \tag{3.18}$$

Therefore immediate post measurement state $|\Psi_T\rangle$ again kills Schrödinger's cat with a probability $\mathbf{P}_T\left(\left|\text{death cat}\right\rangle\right) = 1$.

Suppose now that a nucleus $\mathbf{n}$, whose Hilbert space is spanned by orthonormal states $|s_i(t)\rangle$, $i = 1,2$, where

$$|s_1(t)\rangle = \left|\text{undecayed nucleus at instant } t\right\rangle$$

and

$$|s_2(t)\rangle = \left|\text{decayed nucleus at instant } t\right\rangle$$

is in the superposition state

$$\begin{gathered} |\Psi_t\rangle_{\mathbf{n}} = c_1|s_1(t)\rangle + c_2|s_2(t)\rangle, \\ |c_1|^2 + |c_2|^2 = 1. \end{gathered} \tag{3.19}$$

Remark 3.5. Note that: (i) $|s_1(0)\rangle = \left|\text{undecayed nucleus at instant } t = 0\right\rangle =$

$\left|\ \alpha\text{-particle iside region } (0,l] \text{ at instant } t = 0\right\rangle$. (ii) Feynman propagator of α-particle inside region $(0,l]$ are cite: FeynmanHibbs05[14]:

$$K_2(x,t,x_0) = \left(\frac{m}{2\pi i\hbar t}\right)^{1/2} \exp\left\{\frac{i}{\hbar}[S(t,x,x_0)]\right\}, \tag{3.20}$$

where

$$S(t,x,x_0) = \frac{m(x-x_0)^2}{2t} + mt(U_0 - E). \tag{3.21}$$

Therefore from Eq.(2.11)-Eq.(2.12) and Eq.(3.20)-Eq.(3.21) we obtain

$$\begin{gathered}
|s_1(t)\rangle = \int_{-\infty}^{+\infty} \Psi^{\#}_{\mathbf{II}}(x,t)|x\rangle dx, \\
\Psi^{\#}_{\mathbf{II}}(x,t) = \int_0^l \Psi^{\#}_{\mathbf{II}}(x_0) K_2(x,t,x_0) dx_0 = \\
\left(\frac{m}{2\pi i\hbar t}\right)^{1/2} \int_0^l E(x_0,l)\Psi_{\mathbf{II}}(x_0)\theta_l(x_0) \exp\left\{\frac{i}{\hbar}[S(t,x,x_0)]\right\} dx_0,
\end{gathered} \tag{3.22}$$

where

$$\theta_l(x) = \begin{cases} 1 \text{ for } x \in [0,l] \\ 0 \text{ for } x \notin [0,l] \end{cases}$$

Remark 3.6.We assume for simplification now that $U_0 \simeq E$, i.e.,

$$k' = \frac{2\pi}{\hbar}\sqrt{2m(U_0 - E)} \ll 1. \tag{3.23}$$

Therefore oscillatory integral in RHS of Eq.(3.22) is calculated now directly using stationary phase approximation. The phase term $S(x,x_0)$ given by Eq.(3.21) is stationary when

$$\frac{\partial S(t,x,x_0)}{\partial x_0} = -\frac{m(x-x_0)}{t} = 0. \tag{3.24}$$

and thus stationary point $x_0(t,x)$ is

$$\begin{gathered}
-x + x_0 = 0 \\
x_0(t,x) = x.
\end{gathered} \tag{3.25}$$

Thus from Eq.(3.22) and Eq.(3.25) using stationary phase approximation we obtain

$$\Psi^{\#}_{\mathbf{II}}(x,t) =$$
$$E(x_0(t,x),l)\Psi_{\mathbf{II}}(x_0(t,x))\theta_l(x_0(t,x))\exp\left\{\frac{i}{\hbar}[S(t,x,x_0(t,x))]\right\} + O(\hbar) =$$
$$= E(x,l)\Psi_{\mathbf{II}}(x)\theta_l(x)\exp\left\{\frac{i}{\hbar}[mt(U_0 - E)]\right\} + O(\hbar) = \tag{3.26}$$
$$E(x,l)\theta_l(x)O(1)\exp\left\{\frac{i}{\hbar}[mt(U_0 - E)]\right\} + O(\hbar).$$

Therefore from Eq.(3.26) we obtain

$$|\Psi^{\#}_{\mathbf{II}}(x,t)|^2 = E^2(x,l)\theta_l(x)O(1) + O(\hbar). \tag{3.27}$$

Remark 3.7. Note that for each instant $t > 0$:

$$\left\{\overline{\mathbf{supp}(\Psi^{\#}_{\mathbf{II}}(x,t))}\right\} \cap \left\{\overline{\mathbf{supp}(\Psi^{\#}_{\mathbf{I}}(x,t))}\right\} = \varnothing.$$

Remark 3.8. Note that. From Eq.(3.11),Eq.(3.13), Eq.(3.19), Eq.(3.22)-Eq.(3.27) and Eq.(A.13) by Remark 3.7 we obtain

$${}_{\mathbf{n}}\langle\Psi_t|\hat{x}|\Psi_t\rangle_{\mathbf{n}} = |c_1|^2\langle s_1(t)|\hat{x}|s_1(t)\rangle + |c_2|^2\langle s_2(t)|\hat{x}|s_2(t)\rangle +$$
$$c_1c_2^*\langle s_2(t)|\hat{x}|s_1(t)\rangle + c_1^*c_2\langle s_2(t)|\hat{x}|s_1(t)\rangle^* = \tag{3.28}$$
$$|c_1|^2\langle s_1(t)|\hat{x}|s_1(t)\rangle + |c_2|^2\langle s_2(t)|\hat{x}|s_2(t)\rangle = |c_1|^2 l + |c_2|^2 L.$$

Consideration of the Schrödinger's cat paradox using canonical interpretation of the wave function and canonical von Neuman measurement postulate.

Proposition 3.2. (**i**) Suppose that a nucleus **n** is in the superposition state $|\Psi_t\rangle_{\mathbf{n}}$ ($|\Psi_t\rangle_{\mathbf{n}}$-particle) given by Eq.(3.19). Then the collapse: $|\text{live cat}\rangle \to |\text{death cat}\rangle$ arises at instant

$$T_{\text{col}} \approx \frac{L \pm l}{|c_2|^2\sqrt{8\pi^2 E/m}}. \tag{3.29}$$

with a probability $\mathbf{P}_{T_{\text{col}}}(|\text{death cat}\rangle)$ to observe a state $|\text{death cat}\rangle$ at instant T_{col} is $\mathbf{P}_{T_{\text{col}}}(|\text{death cat}\rangle) = |c_2|^2$.

(**ii**) Assume now that a Schrödinger's cat has performed the single measurement of $|\Psi_t\rangle_{\mathbf{n}}$-particle position with accuracy of $\delta x = l$ at instant $T = T_{\text{col}}$ (given by Eq.(3.29)) by internal monitor (see Pic.1.1) and the result $x \approx L \pm l$ is not observed by Schrödinger's cat. Then the collapse: $|\text{live cat}\rangle \to |\text{death cat}\rangle$ never arises at any instant $T > T_{\text{col}}$ and a probability $\mathbf{P}_{T>T_{\text{col}}}(|\text{death cat}\rangle)$ to observe a state $|\text{death cat}\rangle$ at instant $T > T_{\text{col}}$ is $\mathbf{P}_{T>T_{\text{col}}}(|\text{death cat}\rangle) = 0$.

Proof. (**i**) Note that for $t > 0$ the marginal density matrix $\rho(t)$ is diagonal

$$\rho(t) = \begin{pmatrix} |c_1|^2 \int |\Psi^{\#}_{\mathbf{II}}(x,t)|^2 dx & 0 \\ 0 & |c_2|^2 \int |\Psi^{\#}_{\mathbf{I}}(x,t)|^2 dx \end{pmatrix}$$

In this case a Schrödinger's cat in fact perform the single measurement of $|\Psi_t\rangle_{\mathbf{n}}$-particle position with accuracy of $\delta x = l$ at instant $t = T_{\text{col}}$ (given by Eq.(3.29)) by internal monitor (see Pic.1.1). The probability of getting a result L at instant $T \approx T_{\text{col}}$ with accuracy of $\delta x = l$

given by

$$\begin{aligned} &\int_{|L-x|\leq l/2} |\langle x|\Psi_T\rangle_{\mathbf{n}}|^2 dx = \int_{|L-x|\leq l/2} |\langle x|c_1|s_1(T)\rangle + \langle x|c_2|s_2(T)\rangle|^2 dx = \\ &\int_{|L-x|\leq l/2} |c_1\langle x|s_1(T) + c_2\langle x|s_2(T)|^2 dx = \\ &\int_{|L-x|\leq l/2} |c_1^2 \Psi^{\#2}_{\mathbf{II}}(x,T) + c_2^2 \Psi^{\#2}_{\mathbf{I}}(x,T) + 2c_1c_2\Psi^{\#}_{\mathbf{I}}(x,T)\Psi^{\#}_{\mathbf{II}}(x,T)|dx. \end{aligned} \tag{3.30}$$

From Eq.(3.30) by Remark 3.7 and Eq.(3.13) one obtains

$$\int_{|L-x|\leq l/2} |\langle x|\Psi_T\rangle_{\mathbf{n}}|^2 dx = \int_{|L-x|\leq l/2} |c_2^2 \Psi^{\#2}_{\mathbf{I}}(x,T)|dx = |c_2|^2 \int_{|L-x|\leq l/2} |\Psi^{\#}_{\mathbf{I}}(x,T)|^2 dx = |c_2|^2. \tag{3.31}$$

Note that. When Schrödinger's cat has permormed this measurement and the result $x \approx L \pm l$ is observed, then the immediate post measurement state of α-particle (by conventional von Neumann measurement postulate P.3.5) is

$$\begin{aligned} &|\breve{\Psi}_{T_{\text{col}}}\rangle_{\mathbf{n}} = \frac{\int_{|L-x|\leq l/2} |x\rangle\langle x|\Psi_{T_{\text{col}}}\rangle_{\mathbf{n}} dx}{\sqrt{\int_{|L-x|\leq l/2} |\langle x|\Psi_{T_{\text{col}}}\rangle_{\mathbf{n}}|^2 dx}} = \\ &\frac{\int_{|L-x|\leq l/2} |x\rangle\langle x|(c_1|s_1(T_{\text{col}})\rangle + c_2|s_2(T_{\text{col}})\rangle)dx}{\sqrt{\int_{|L-x|\leq l/2} |\langle x|\Psi_{T_{\text{col}}}\rangle_{\mathbf{n}}|^2 dx}} = \\ &\frac{c_1 \int_{|L-x|\leq l/2} |x\rangle\langle x|s_1(T_{\text{col}}) + c_2 \int_{|L-x|\leq l/2} |x\rangle\langle x|s_2(T_{\text{col}})dx}{\sqrt{\int_{|L-x|\leq l/2} |\langle x|\Psi_{T_{\text{col}}}\rangle_{\mathbf{n}}|^2 dx}} \in \mathbf{S}_{\Theta}, \Theta = \{x|L - x \leq l/2\}. \end{aligned} \tag{3.32}$$

From Eq.(3.32) by Eq.(3.31) and by Remark 3.7 one obtains

$$|\Psi_{T_{\mathrm{col}}}\rangle_{\mathbf{n}} = \frac{\int_{|L-x|\leq l/2} |x\rangle\langle x|\Psi_{T_{\mathrm{col}}\mathbf{n}} dx}{\sqrt{\int_{|L-x|\leq l/2} |\langle x|\Psi_{T_{\mathrm{col}}\mathbf{n}}|^2 dx}} = \frac{\int_{|L-x|\leq l/2} |x\rangle\langle x|(c_1|s_1(T_{\mathrm{col}})\rangle + c_2|s_2(T_{\mathrm{col}})\rangle)dx}{\sqrt{\int_{|L-x|\leq l/2} |\langle x|\Psi_{T_{\mathrm{col}}\mathbf{n}}|^2 dx}} =$$

$$= \frac{c_2}{|c_2|}\int_{|L-x|\leq l/2} |x\rangle\langle x|s_2(T_{\mathrm{col}})dx.$$

Obviously by Remark 3.4 the state $\left|\breve{\Psi}_{T_{\mathrm{col}}}\right\rangle_{\mathbf{n}}$ kills Schrödinger's cat with a probability $\mathbf{P}_{T_{\mathrm{col}}}\left(\left|\text{death cat}\right\rangle\right) = 1.$

(ii) The probability of getting a result L at any instant $T > T_{\mathrm{col}}$ with accuracy of $\delta x = l$ by

Eq.(3.31) and Eq.(3.13) given by formula

$$\int_{|L-x|\leq l/2} |\langle x|\Psi_{T_{\mathrm{col}}}\rangle_{\mathbf{n}}|^2 dx = \int_{|L-x|\leq l/2} |c_2^2 \Psi_{\mathbf{I}}^{\#2}(x,T)|dx = |c_2|^2 \int_{|L-x|\leq l/2} |\Psi_{\mathbf{I}}^{\#}(x,T)|^2 dx =$$

$$\simeq (\pi r_c^2)^{-1/2} \int_{|L-x|\leq l/2} dx\theta_1\left(x + \pi T\sqrt{8E/m}, l\right) \exp\left[-\frac{\left(x + \pi T\sqrt{8E/m}\right)^2}{r_c^2}\right] \simeq 0.$$

Thus standard formalism of continuous quantum measurements

cite: BassiLochanSatinSinghUlbricht13, JacobsSteck06, Mensky93, Mensky00 [2, 3, 4, 5] leads to a definite but unpredictable measurement outcomes, either $|s_1(t)\rangle$ or $|s_2(t)\rangle$ and thus $|\Psi_t\rangle_{\mathbf{n}}$ "collapses" at unpredictable instant t' into the states $|s_i(t')\rangle, i = 1,2.$

5mm . Resolution of the Schrödinger's cat paradox using generalized von Neumann postulate 2mm

Proposition 3.3. Suppose that a nucleus **n** is in the superposition state given by Eq.(3.19), and therefore Schrödinger's cat in each instant $t < T_{\mathrm{col}}$ also is in the superposition state [6]:

$$\left|\text{Schrödinger's cat}\right\rangle = |c_1|^2 \left|\text{live cat}\right\rangle + |c_2|^2\left|\text{death cat}\right\rangle$$

Then the collapse:

$$\left|\text{Schrödinger's cat}\right\rangle = |c_1|^2 \left|\text{live cat}\right\rangle + |c_2|^2\left|\text{death cat}\right\rangle \to \left|\text{death cat}\right\rangle$$

arises at instant

$$T_{\mathrm{col}} = \frac{L}{|c_2|^2\sqrt{8\pi^2 E/m}}. \tag{3.33}$$

with a probability $\mathbf{P}_{T_{\mathrm{col}}}\left(\left|\text{death cat}\right\rangle\right)$ to observe a state $\left|\text{death cat}\right\rangle$ at instant T_{col} is

$$\mathbf{P}_{T_{\mathrm{col}}}\left(\left|\text{death cat}\right\rangle\right) = 1.$$

Proof. Let us consider now a state $|\Psi_t\rangle_{\mathbf{n}}$ given by Eq.(3.19). This state consists of a superposition of two wave packets $\Psi_{\mathbf{II}}^{\#}(x,t)$ and $\Psi_{\mathbf{I}}^{\#}(x,t)$. Wave packet $c_1\Psi_{\mathbf{II}}^{\#}(x,t)$ present an $\alpha_{\mathbf{II}}$-particle which lives in region **II** with a probability $|c_1|^2$ (see Pic. 2.1). Wave packet $\Psi_{\mathbf{I}}^{\#}(x,t)$ present an $\alpha_{\mathbf{I}}$-particle which lives in region **I** with a probability $|c_2|^2$ (see Pic. 2.1) and moves from the right to the left. Note that $\mathbf{I} \cap \mathbf{II} = \varnothing$. From Eq.(3.13) follows that $\alpha_{\mathbf{I}}$-particle at each instant $t \geq 0$ moves quasiclassically from right to left by the law

$$x(t) = -|c_2|^2 \pi t \sqrt{8E/m}, \tag{3.34}$$

From Eq.(3.34) one obtains

$$T = T_{\text{col}} \simeq \frac{L}{|c_2|^2 \sqrt{8\pi^2 E/m}}. \tag{3.35}$$

Note that. In this case Schrödinger's cat in fact permorm a single measurement of $|\Psi_t\rangle_{\mathbf{n}}$-particle position with accuracy of $\delta x = l$ at instant $t = T = T_{\text{col}}$ (given by Eq.(3.35)) by internal monitor (see Pic.1.1). The probability of getting the result L at instant $t = T_{\text{col}}$ with

accuracy of $\delta x = l$ by Remark 3.7 and by postulate P.4.2 (see subsection 4) (for complete explanation and motivation see cite: FoukzonPotapovMen'kovaPodosenov16 [15]) given by

$$\begin{aligned}
&\int_{|L-x|\leq l/2} \left[|\langle x|s_{1,c_1}(T_{\text{col}})\rangle|^2 * |\langle x|s_{2,c_2}(T_{\text{col}})\rangle|^2 \right] dx = \\
&\int_{|L-x|\leq l/2} \left[|\Psi^{\#}_{\mathbf{I},c_1}(x, T_{\text{col}})|^2 * |\Psi^{\#}_{\mathbf{II},c_2}(x, T_{\text{col}})|^2 \right] dx \\
&\int_{|L-x|\leq l/2} |c_2|^{-2}|c_1|^{-2} \left[|\Psi^{\#}_{\mathbf{I}}(x|c_2|^{-2}, T_{\text{col}})|^2 * |\Psi^{\#}_{\mathbf{II}}(x|c_1|^{-2}, T_{\text{col}})|^2 \right] dx = 1.
\end{aligned} \tag{3.36}$$

Note that. When Schrödinger's cat has permormed this measurement and the result $x \approx L \pm l$ is observed, then the immediate post measurement state $\left|\breve{\Psi}_{T_{\text{col}}}\right\rangle_{\mathbf{n}}$ of $|\Psi_t\rangle_{\mathbf{n}}$–particle (by generalized von Neumann postulate P.5.3,see section 4 Eq.(4.41) is

$$\begin{aligned}
\left|\breve{\Psi}_{T_{\text{col}}}\right\rangle_{\mathbf{n}} &\simeq \frac{\int_{|L-x|\leq l/2} |x\rangle\langle x|\Psi_{T_{\text{col}}}\rangle_{\mathbf{n}} dx}{\sqrt{\int_{|L-x|\leq l/2} \left[|\langle x|s_{1,c_1}(T_{\text{col}})\rangle|^2 * |\langle x|s_{2,c_2}(T_{\text{col}})\rangle|^2 \right] dx}} \simeq \\
&\frac{\int_{|L-x|\leq l/2} |x\rangle\langle x|s_{2,c_2}(T_{\text{col}})\rangle dx}{\sqrt{\int_{|L-x|\leq l/2} \left[|\langle x|s_{2,c_2}(T_{\text{col}})\rangle|^2 \right] dx}}.
\end{aligned} \tag{3.37}$$

The state $\left|\breve{\Psi}_{T_{\text{col}}}\right\rangle_{\mathbf{n}}$ again kills Schrödinger's cat with a probability $\mathbf{P}_{T\text{col}}\left(\left|\text{death cat}\right\rangle\right) = 1$.

Thus is the collapsed state of the cat always shows definite and predictable outcomes even if cat also consists of a superposition:

$$\left|\text{cat}\right\rangle = c_1\left|\text{live cat}\right\rangle + c_2\left|\text{death cat}\right\rangle.$$

Contrary to van Kampen's cite: van Kampen88[16] and some others' opinions, "looking" at the outcome changes nothing, beyond informing the observer of what has already happened.

We remain: there are widespread claims that Schrödinger's cat is not in a definite alive or dead state but is, instead, in a superposition of the two. van Kampen, for example, writes "The whole system is in a superposition of two states: one in which no decay has occurred and one in which it has occurred. Hence, the state of the cat also consists of a superposition: $\left|\text{cat}\right\rangle = c_1\left|\text{live cat}\right\rangle + c_2\left|\text{death cat}\right\rangle$. The state remains a superposition until an observer looks at the cat” cite: van Kampen88[16].

4.Generalized Postulates for Continuous Valued Observables.

P.1.Suppose we have an n-dimensional quantum system $\mathbf{Q}^s$ with continuous observables. **Then we claim the following**:
P.1.1.Any given n-dimensional quantum system is identified by a tuple $\mathbf{Q}^s$

$$\mathbf{Q}^s \triangleq \langle \mathbf{H}, \mathfrak{I}, \mathfrak{R}, \mathcal{L}_{2,1}, \mathbf{G}, |\psi_t\rangle\rangle \tag{4.1}$$

where:
(i) $\mathbf{H}$ that is some infinite-dimensional non projective complex Hilbert space,
(ii) $\mathfrak{I} = (\Omega, \mathcal{F}, \mathbf{P})$ that is complete probability space,
(iii) $\mathfrak{R} = (\mathbb{R}^n, \Sigma)$ that is measurable space ,
(iv) $\mathcal{L}_{2,1}(\Omega)$ that is complete space of random variables $X : \Omega \to \mathbb{R}^n$ such that

$$\int_\Omega \|X(\omega)\| d\mathbf{P} < \infty, \int_\Omega \|X(\omega)\|^2 d\mathbf{P} < \infty, \tag{4.2}$$

(v) $\mathbf{G} : \mathbf{\Delta}^*(\mathbf{H}) \times \mathbf{H} \to \mathcal{L}_{2,1}(\Omega)$ that is one to one correspondence such that the following conditions are satisfied

$$\begin{gathered}(1)\ |\psi\rangle \in \mathbf{H} \Rightarrow \left|\langle\psi|\widehat{Q}|\psi\rangle\right| = \int_\Omega \left(\mathbf{G}\left[\widehat{Q};|\psi\rangle\right](\omega)\right) d\mathbf{P} = \mathbf{E}_\Omega\left(\mathbf{G}\left[\widehat{Q};|\psi\rangle\right](\omega)\right),\\ (2)\ |\psi\rangle \in \mathbf{S}^\infty \Rightarrow \mathbf{G}\left[\widehat{\mathbf{1}};|\psi\rangle\right](\omega) = 1,\\ (3)\ |\psi\rangle \in \mathbf{S}^\infty \Rightarrow \left(\mathbf{G}\left[\widehat{Q};|\psi\rangle\right](\omega)\right) = \left(\mathbf{G}\left[\widehat{\mathbf{1}};\widehat{Q}|\psi\rangle\right](\omega)\right),\\ (4)\ |\psi\rangle \in \mathbf{S}^\infty, z \in \mathbb{C} \Rightarrow \left(\mathbf{G}\left[\widehat{Q};z|\psi\rangle\right](\omega)\right) = |z|^2\left(\mathbf{G}\left[\widehat{Q};|\psi\rangle\right](\omega)\right)\end{gathered} \tag{4.3}$$

for any Hermitian operator $\widehat{Q} : \mathbf{H} \to \mathbf{H}, \widehat{Q} \in \Delta^*(\mathbf{H}) \subsetneq C^*(\mathbf{H})$, where $C^*(\mathbf{H})$ that is C^* algebra of the Hermitian adjoint operators in the position representation in $\mathbf{H}$ and $\Delta^*(\mathbf{H})$ that is an commutative subalgebra of $C^*(\mathbf{H})$,
(vi) $|\psi_t\rangle$ is an continuous vector function $|\psi_t\rangle : \mathbb{R}_+ \to \mathbf{H}$ which representedthe evolution of the quantum system $\mathbf{Q}^s$.

Remark 4.1.(i) The *classical pure states* correspond to vectors $\mathbf{v} \in \mathbf{H}$ of norm $\|\mathbf{v}\| \equiv 1$. Thus the set of all classical pure states corresponds to the unit sphere $\mathbf{S}^\infty \subset \mathbf{H}$ in a Hilbert space $\mathbf{H}$.

(ii)We remind that the projective Hilbert space $P(\mathbf{H})$ of a complex Hilbert space $\mathbf{H}$ is the set of equivalence classes $[\mathbf{v}]$ of vectors $\mathbf{v}$ in $\mathbf{H}$, with $\mathbf{v} \neq \mathbf{0}$, for the equivalence relation given by $\mathbf{v} \sim_P \mathbf{w} \Leftrightarrow \mathbf{v} = \lambda\mathbf{w}$ for some non-zero complex number $\lambda \in \mathbb{C}$. The equivalence classes for the relation $\sim_P$ are also called rays or projective rays.

(iii) The physical significance of the projective Hilbert space $P(\mathbf{H})$ is that in canonical quantum theory, the states $|\psi\rangle$ and $\lambda|\psi\rangle$ represent the same physical state of the quantum system, for any $\lambda \neq 0$. It is conventional to choose a state $|\psi\rangle$ from the ray $[|\psi\rangle]$ so that it has unit norm $\sqrt{\langle\psi|\psi\rangle} = 1$.

(iv) In contrast with canonical quantum theory we have used instead $\sim_P$, contrary to $\sim_P$ equivalence relation $\sim_{\widehat{Q}}$, in a Hilbert space $\mathbf{H}$, see Definition 2.3.

P.1.2. We extend now the set of all classical pure states $\mathbf{CLPS}(\mathbf{H}) = \mathbf{S}^\infty$, corresponding to the unit sphere $\mathbf{S}^{\infty} \subset \mathbf{H}$ in a Hilbert space $\mathbf{H}$ from the set $\mathbf{S}^{\infty}$ to the set

$$\mathbf{W}^{\infty} = \bigcup_{z \in \mathbb{C}} \{z \cdot \mathbf{S}^{\infty}\} \supset \mathbf{S}^{\infty}. \quad (4.4)$$

The set of all non classical pure states $\mathbf{NCLPS}(\mathbf{H})$ is

$$\mathbf{NCLPS}(\mathbf{H}) = \mathbf{W}^{\infty} \backslash \mathbf{S}^{\infty}. \quad (4.5)$$

P.1.3. For any $|\psi_1\rangle, |\psi_2\rangle \in \mathbf{H}$ and for any Hermitian operator $\widehat{Q} : \mathbf{H} \to \mathbf{H}$ such that

$$\left\langle \psi_1 \middle| \widehat{Q} \middle| \psi_2 \right\rangle = \left\langle \psi_2 \middle| \widehat{Q} \middle| \psi_1 \right\rangle = 0, \quad (4.6)$$

the following conditions are satisfied

$$\mathbf{G}\left[\widehat{Q}; (|\psi_1\rangle + |\psi_2\rangle)\right](\omega) = \mathbf{G}\left[\widehat{Q}; |\psi_1\rangle\right](\omega) + \mathbf{G}\left[\widehat{Q}; |\psi_2\rangle\right](\omega). \quad (4.7)$$

P.1.4. Suppose that the evolution of the quantum system is represented by continuous vector function $|\psi_t\rangle : \mathbb{R}_+ \to \mathbf{H}$.

Then we claim the following:

(i) any process of continuous measurements on measuring continuous valued observable $\widehat{Q} \in \Delta^*(\mathbf{H})$ for the system $\mathbf{Q}^s$ in state $|\psi_t\rangle$ can be represented by an continuous $\mathbb{R}^n$-valued stochastic processes

$$X_t(\omega) = X_t\left(\omega; \widehat{Q}\right) = X_t\left(\omega; \widehat{Q}; |\psi_t\rangle\right) = \mathbf{G}\left[\widehat{Q}; |\psi_t\rangle\right](\omega) = \left(\mathbf{G}\left[\widehat{\mathbf{1}}; \widehat{Q}|\psi_t\rangle\right](\omega)\right), \quad (4.8)$$

given on probability space $\mathfrak{I} = (\Omega, \mathcal{F}, \mathbf{P})$ and a measurable space $\mathfrak{R} = (\mathbb{R}^n, \Sigma)$;

(ii) any continuous valued observable $\widehat{Q} \in \mathfrak{I}^*(\mathbf{H})$ for the system $\mathbf{Q}^s$ in state $|\psi\rangle \in \mathbf{W}^{\infty}$ is equivalent to continuous random variable $X : \Omega \to \mathbb{R}^n$

$$X(\omega) = X\left(\omega; \widehat{Q}\right) = \mathbf{G}\left[\widehat{Q}; |\psi\rangle\right](\omega) = \left(\mathbf{G}\left[\widehat{\mathbf{1}}; \widehat{Q}|\psi\rangle\right](\omega)\right), \quad (4.9)$$

given on probability space $\mathfrak{I} = (\Omega, \mathcal{F}, \mathbf{P})$ and a measurable space $\mathfrak{R} = (\mathbb{R}^n, \Sigma)$;

We assume now for a short but without loss of generality that $n = 1$.

Remark 4.2.Let $X(\omega)$ be a random variable $X(\omega) \in \mathcal{L}_{2,1}(\Omega)$

$$X(\omega) = X\left(\omega; \widehat{Q}\right) = \mathbf{G}\left[\widehat{Q}; |\psi\rangle\right](\omega) = \left(\mathbf{G}\left[\widehat{\mathbf{1}}; \widehat{Q}|\psi\rangle\right](\omega)\right), \quad (4.10)$$

given on probability space $(\Omega, \mathcal{F}, \mathbf{P})$ and a measurable space $(\mathbb{R}, \Sigma)$ and $|\psi\rangle \in \mathbf{W}^{\infty}$.

Then we denote such random variable by $X_{\widehat{Q}|\psi\rangle}(\omega)$. The probability density of the random variable $X_{\widehat{Q}|\psi\rangle}(\omega)$ we denote by $p_{\widehat{Q}|\psi\rangle}(q), q \in \mathbb{R}$.

Remark 4.3.Let $X_1(\omega)$ be a random variable $X_1(\omega) \in \mathcal{L}_{2,1}(\Omega)$

$$X_1(\omega) = X_1\left(\omega; \widehat{Q}\right) = \mathbf{G}\left[\widehat{Q}; |\psi_1\rangle\right](\omega) = \mathbf{G}\left[\widehat{\mathbf{1}}; \widehat{Q}|\psi_1\rangle\right](\omega), \quad (4.11)$$

given on probability space $(\Omega, \mathcal{F}, \mathbf{P})$ and a measurable space $(\mathbb{R}, \Sigma)$ and $|\psi_1\rangle = z|\psi\rangle \in \mathbf{W}^{\infty}$, where $|\psi\rangle \in \mathbf{S}^{\infty}$. Then from Eq.(2.10) and postulate **P.1.1** (v) follows that

$$X_1(\omega) = |z|^2 X(\omega). \quad (4.12)$$

Remark 4.4.The probability density of the random variables $X_{\widehat{Q}|\psi_1\rangle}(\omega) = X_1(\omega)$ and $X_{\widehat{Q}|\psi\rangle}(\omega) = X(\omega)$ we denote by $p_1(q) = p_{\widehat{Q}|\psi_1\rangle}(q)$ and by $p(q) = p_{\widehat{Q}|\psi\rangle}(q)$ correspondingly.

From Eq.(2.12) by change of variables we obtain

$$p_1(q) = |z|^{-2} p_1(|z|^{-2} q). \tag{4.13}$$

Remark 4.5.(1) Let $|\psi_1\rangle, |\psi_2\rangle \in \mathbf{S}^\infty$ and let $\widehat{Q}$ be Hermitian operator $\widehat{Q} : \mathbf{H} \to \mathbf{H}$ such that

$$\left\langle \psi_1 \middle| \widehat{Q} \middle| \psi_2 \right\rangle = \left\langle \psi_2 \middle| \widehat{Q} \middle| \psi_1 \right\rangle = 0, \tag{4.14}$$

and let $|\psi_{12}\rangle$ be the vector $|\psi_{12}\rangle = z_1|\psi_1\rangle + z_2|\psi_2\rangle, z_1, z_2 \in \mathbb{C}$. Then from postulate **P.1.1** (v) and from postulate **P.1.3** follows that

$$\begin{gathered}
\left|\langle \psi_{12} | \widehat{Q} | \psi_{12} \rangle\right| = |z_1|^2 \left\langle \psi_1 \middle| \widehat{Q} \middle| \psi_1 \right\rangle + |z_2|^2 \left\langle \psi_2 \middle| \widehat{Q} \middle| \psi_2 \right\rangle = \\
\int_\Omega \left(\mathbf{G}\left[\widehat{Q}; z_1 |\psi_1\rangle \right](\omega) \right) d\mathbf{P} + \int_\Omega \left(\mathbf{G}\left[\widehat{Q}; z_2 |\psi_2\rangle \right](\omega) \right) \mathbf{dP} = \\
|z_1|^2 \int_\Omega \left(\mathbf{G}\left[\widehat{Q}; |\psi_1\rangle \right](\omega) \right) d\mathbf{P} + |z_2|^2 \int_\Omega \left(\mathbf{G}\left[\widehat{Q}; |\psi_2\rangle \right](\omega) \right) \mathbf{dP} = \\
|z_1|^2 \int_\Omega X_{\widehat{Q}|\psi_1\rangle}(\omega) d\mathbf{P} + |z_2|^2 \int_\Omega X_{\widehat{Q}|\psi_2\rangle}(\omega) d\mathbf{P} \\
|z_1|^2 \int_\Omega X_1(\omega) d\mathbf{P} + |z_2|^2 \int_\Omega X_2(\omega) d\mathbf{P}, \\
|z_1|^2 \int_{-\infty}^{\infty} p_1(q) q dq + |z_2|^2 \int_{-\infty}^{\infty} p_2(q) q dq = \\
\int_{-\infty}^{\infty} [|z_1|^2 p_1(q) + |z_2|^2 p_2(q)] q dq = \\
= \int_{-\infty}^{\infty} [p_1^\#(q) + p_2^\#(q)] q dq, \\
p_1^\#(q) = |z_1|^{-2} p_1(|z_1|^{-2} q), p_2^\#(q) = |z_2|^{-2} p_2(|z_2|^{-2} q),
\end{gathered} \tag{4.15}$$

where $X_1(\omega) = X_{\widehat{Q}|\psi_1\rangle}(\omega)$ and $X_2(\omega) = X_{\widehat{Q}|\psi_2\rangle}(\omega)$ and $p_1(q)$ and $p_2(q)$ are probability density
of the random variables $X_1(\omega)$ and $X_2(\omega)$ correspondingly.

(2) Assume that random variables $X_1(\omega)$ and $X_2(\omega)$ are independent. Then from Eq.(2.15) we obtain

$$\begin{gathered}
\left|\langle \psi_{12} | \widehat{Q} | \psi_{12} \rangle\right| = |z_1|^2 \left\langle \psi_1 \middle| \widehat{Q} \middle| \psi_1 \right\rangle + |z_2|^2 \left\langle \psi_2 \middle| \widehat{Q} \middle| \psi_2 \right\rangle = \\
\int_{-\infty}^{\infty} [p_1^\#(q) + p_2^\#(q)] q dq = \\
\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} p_1^\#(q - q_1) p_2^\#(q_1) q dq_1 dq = \int_{-\infty}^{\infty} [p_1^\#(q) * p_2^\#(q)] q dq, \\
p_1^\#(q) = |z_1|^{-2} p_1(|z_1|^{-2} q), p_2^\#(q) = |z_2|^{-2} p_2(|z_2|^{-2} q).
\end{gathered} \tag{4.16}$$

P.2. Suppose we have an observable $\widehat{Q}_{\mathcal{F}}$ of a quantum system that is found through an exhaustive series of measurements, to have a set $\mathcal{F}$ of values $q \in \mathcal{F}$ such that

$$\mathcal{F} = \cup_{i=1}^{m} \Theta_i = \cup_{i=1}^{m} (\theta_1^i, \theta_2^i), m \geq 1, (\theta_1^i, \theta_2^i) \cap \left(\theta_1^j, \theta_2^j\right) = \varnothing, i \neq j.$$

Note that in practice any observable $\widehat{Q}_{\mathcal{F}}$ is measured to an accuracy δq determined by the measuring device. We represent now by $|q\rangle$ the idealized state of the system in the limit $\delta q \to 0,$ for which the observable definitely has the value q.

Then we claim the following:

P.2.1. The states $\{|q\rangle : q \in \mathcal{F}\}$ form a complete set of δ-function normalized basis states for the state space $\mathbf{H}_{\mathcal{F}} \supset \mathbf{S}_{\mathcal{F}}^{\infty}$ of the system.

That the states $\{|q\rangle : q \in \mathcal{F}\}$ form a complete set of basis states means that any state $|\psi[\mathcal{F}]\rangle \in \mathbf{S}_{\mathcal{F}}^{\infty}$ of the system can be expressed as:

$$|\psi[\mathcal{F}]\rangle = \int_{\mathcal{F}} c_{\psi[\mathcal{F}]}(q)|q\rangle dq, \tag{4.17}$$

where $\operatorname{supp}(c_{\psi[\mathcal{F}]}(q)) \subseteq \mathcal{F}$ and while δ-function normalized means that $\langle q|q'\rangle = \delta(q - q')$ from which follows

$$c_{\psi[\mathcal{F}]}(q) = \langle q|\psi[\mathcal{F}]\rangle \triangleq \psi(q;\mathcal{F}) \tag{4.18}$$

so that $|\psi[\mathcal{F}]\rangle = \int_{\mathcal{F}} |q\rangle\langle q|\psi[\mathcal{F}]\rangle dq.$ The completeness condition can then be written as $\int_{\mathcal{F}} |q\rangle\langle q| dq = \widehat{\mathbf{1}}_{\mathbf{H}_{\mathcal{F}}}.$

P.2.2.For the system in the state $|\psi[\mathcal{F}]\rangle \in \mathbf{S}_{\mathcal{F}}^{\infty},$ the probability $P(q, q + dq; |\psi[\mathcal{F}]\rangle)$ of obtaining the result $q \in \mathcal{F}$ lying in the range $(q, q + dq) \subset \mathcal{F}$ on measuring observable $\widehat{Q}$ is given by

$$P(q, q + dq; |\psi[\mathcal{F}]\rangle) = p_{\left|\widehat{Q}\psi[\mathcal{F}]\right\rangle}(q) dq \tag{4.19}$$

for any $|\psi[\mathcal{F}]\rangle \in \mathbf{H}_{\mathcal{F}},$ and where $p_{\left|\widehat{Q}\psi[\mathcal{F}]\right\rangle}$ is a probability density of the random variable given by formula

$$\mathbf{G}\left[\widehat{Q}; |\psi[\mathcal{F}]\rangle\right](\omega) = \mathbf{G}\left[\widehat{\mathbf{1}}; \widehat{Q}|\psi[\mathcal{F}]\rangle\right](\omega) \tag{4.20}$$

Remark 4.6. Note that in general case

$$p_{\left|\widehat{Q}\psi[\mathcal{F}]\right\rangle}(q) \neq |c_{\psi[\mathcal{F}]}(q)|^2. \tag{4.21}$$

It will be shown later.

P.2.3.The observable $\widehat{Q}_{\mathcal{F}}$ is represented by a Hermitian operator $\widehat{Q}_{\mathcal{F}} : \mathbf{H}_{\mathcal{F}} \to \mathbf{H}_{\mathcal{F}}$ whose eigenvalues are the possible results $\{q : q \in \mathcal{F}\},$ of a measurement of $\widehat{Q}_{\mathcal{F}},$ and the associated eigenstates are the states $\{|q\rangle : q \in \mathcal{F}\},$ i.e. $\widehat{Q}_{\mathcal{F}}|q\rangle = q|q\rangle, q \in \mathcal{F}.$

Remark 4.7. Note that the spectral decomposition of the operator $\widehat{Q}_{\mathcal{F}}$ is then

$$\widehat{Q}_{\mathcal{F}} = \int_{\mathcal{F}} q|q\rangle\langle q| dq. \tag{4.22}$$

Definition 4.1. A connected set in $\mathbb{R}$ is a set $X \subset \mathbb{R}$ that cannot be partitioned into two nonempty subsets which are open in the relative topology induced on the set. Equivalently, it is a set which cannot be partitioned into two nonempty subsets such that each subset has no points in common with the set closure of the other.

Definition 4.2.*Well localized pure states* $|\psi[\Theta]\rangle$ with a support $\Theta = (\theta_1, \theta_2)$ corresponds to vectors in $\mathbf{S}_{\Theta}^{\infty}$ or to vectors in $\mathbf{W}_{\Theta}^{\infty} \supset \mathbf{S}_{\Theta}^{\infty}$ and such that a set: $\operatorname{supp}(c_{\psi[\Theta]}(q)) = \Theta$ is a connected set in $\mathbb{R}$ Thus the set of all well localized pure states $\mathbf{W}_{\Theta}^{\infty} \supset \mathbf{S}_{\Theta}^{\infty}$ with a support Θ lies in the Hilbert space $\mathbf{H}_{\Theta} \subsetneqq \mathbf{H}$.

P.3.Suppose we have an observable $\widehat{Q}_{\Theta}$ of a system that is found through an exhaustive series of measurements, to have a continuous range of values $q : \theta_1 < q < \theta_2$.

Then we claim the following:

P.3.1.For the system in well localized pure statestate $|\psi[\Theta]\rangle$ such that:

(i) $|\psi[\Theta]\rangle \in \mathbf{S}_{\Theta}^{\infty}$ and

(ii) $\operatorname{supp}(c_{\psi[\Theta]}(q)) \triangleq \{q|c_{\psi[\Theta]}(q) \neq 0\}$ is a connected set in $\mathbb{R}$, then the probability $P(q, q+dq; |\psi[\Theta]\rangle)$ of obtaining the result q lying in the range $(q, q+dq) \subset \Theta$ on measuring

observable Q_{Θ} is given by

$$P(q, q+dq; |\psi[\Theta]\rangle) = |\langle q|\psi[\Theta]\rangle|^2 dq = |c_{\psi[\Theta]}(q)|^2 dq. \tag{4.23}$$

P.3.2.

$$p_{\left|\widehat{Q}_{\Theta}\psi[\Theta]\right\rangle}(q)dq = |\langle q|\psi[\Theta]\rangle|^2 dq = |c_{\psi[\Theta]}(q)|^2 dq. \tag{4.24}$$

P.3.3.Let $|\psi_1[\Theta_1]\rangle \in \mathbf{S}_{\Theta_1}^{\infty}$ and $|\psi_2[\Theta_2]\rangle \in \mathbf{S}_{\Theta_2}^{\infty}$ be well localized pure states with $\Theta_1 = (\theta_1^1, \theta_2^1)$, and $\Theta_2 = (\theta_1^2, \theta_2^2)$ correspondingly. Let

$$X_1(\omega) = X_{\left|\widehat{Q}_{\Theta_1}\psi_1[\Theta_1]\right\rangle}(\omega) \text{ and } X_2(\omega) = X_{\left|\widehat{Q}_{\Theta_2}\psi_2[\Theta_2]\right\rangle}(\omega)$$

correspondingly. Assume that:

(i) $\overline{\Theta}_1 \cap \overline{\Theta}_2 = \varnothing$ (here the closure of $\Theta_i, i = 1,2$ is denoted by $\overline{\Theta}_i, i = 1,2$),

(ii) the result a measurement of $\widehat{Q}_{\Theta_1}$ at instant t in any point $\mathbf{A} \in \Theta_1$ does not perfectly predict the result a measurement of $\widehat{Q}_{\Theta_2}$ at instant t in any point $\mathbf{B} \in \Theta_2$,i.e.states is not entangled.

Remark 4.8.Notice that on performing a measurement of $\widehat{Q}_{\Theta_1}$ in point $\mathbf{A} \in \Theta_1$ and performing a measurement of $\widehat{Q}_{\Theta_2}$in point $\mathbf{B} \in \Theta_2$ simultaneously, i.e. in spacelike configurations (see Pic.2.1) due to measurements made on system in point $\mathbf{A}$, without the slightest possibility of system in point $\mathbf{B}$ being physically disturbed. Therefore by postulate **P.1.4**(ii) random variables $X_1(\omega)$ and $X_2(\omega)$ are independent.

Pic.2.1.Spacetime diagrams for spacelike
(i) $c^2t^2 - q^2 < 0$ and timelike (ii) $c^2t^2 - q^2 > 0$
configurations. $\mathbf{A} \in \Theta_1$ and $\mathbf{B} \in \Theta_2$
represent the locations of the detectors.
$\mathbf{D_A}$and $\mathbf{D_B}$ represent the detection events.

P.3.4. If the system is in well localized pure state $|\psi[\Theta]\rangle \in \mathbf{S}_{\Theta}^{\infty}$ the state $|\psi[\Theta]\rangle$ described by a wave function $\psi(q;\Theta) = \langle q|\psi[\Theta]\rangle \triangleq \psi(q;\Theta)$ and the value of observable Q_Θ is measured once each on many identically prepared system, the average value of all the measurements will be

$$\langle Q_\Theta \rangle = \frac{\int\limits_{\Theta} q|\psi(q;\Theta)|^2 dq}{\int\limits_{\Theta} |\psi(q;\Theta)|^2 dq}. \tag{4.25}$$

The completeness condition can then be written as $\int_{\Theta} |q\rangle\langle q| dq = \widehat{\mathbf{1}}_{\mathbf{H}_\Theta}$. Completeness means that for any state $|\psi[\Theta]\rangle \in \mathbf{S}_{\Theta}^{\infty}$ it must be the case that $\int_{\Theta} |\langle q|\psi[\Theta]\rangle|^2 dq \neq 0$, i.e. there must be a non-zero probability to get some result on measuring observable Q_Θ.

P.3.5.(**von Neumann measurement postulate**) Assume that
(i) $|\psi[\Theta]\rangle \in \mathbf{S}_{\Theta}^{\infty}$ and (ii) $\mathrm{supp}(c_\psi(q)) = \Theta$ is a connected set in $\mathbb{R}$. Assume that the particle is initially in the state $|\psi[\Theta]\rangle$. If on performing a measurement of Q_Θ with an accuracy δq, the result $q \in \Theta$ is obtained in the range $(q - \frac{1}{2}\delta q, q + \frac{1}{2}\delta q) \subset \Theta$, then the system will end up in the state

$$\frac{\widehat{P}(q,\delta q)|\psi[\Theta]\rangle}{\sqrt{\langle\psi|\widehat{P}(q,\delta q)|\psi[\Theta]\rangle}} = \frac{\int_{|q-q'|\leq\delta q/2}|q'\rangle\langle q'|\psi[\Theta]\rangle dq'}{\sqrt{\int_{|q-q'|\leq\delta q/2}|\langle q'|\psi[\Theta]\rangle|^2 dq'}}. \tag{4.26}$$

where the term in the denominator is there to guarantee that the state after the measurement is normalized to unity.

P.4. Let $\mathbf{Q}^s$ be the system in an non classical pure state $|\psi^a[\Theta]\rangle = a|\psi[\Theta]\rangle \in \mathbf{W}_\Theta^\infty \subsetneqq \mathbf{H}_\Theta$, where:

(i) $|\psi[\Theta]\rangle \in \mathbf{S}_\Theta^\infty, |a| \neq 1, \Theta = (\theta_1,\theta_2)$,

(ii) $\mathrm{supp}(c_{\psi[\Theta]}(q))$ is a connected set in $\mathbb{R}$ and

(iii) $|\psi[\Theta]\rangle = \int_\Theta c_{\psi[\Theta]}(q)|q\rangle dq = \int_{\theta_1}^{\theta_2} c_{\psi[\Theta]}(q)|q\rangle dq$.

Then we claim the following:

P.4.1.

$$\mathbf{G}\left[\widehat{Q}_\Theta, |\psi^a[\Theta]\rangle\right] = |a|^2\mathbf{G}\left[\widehat{Q}_\Theta, |\psi[\Theta]\rangle\right]. \tag{4.27}$$

P.4.2.The system in non classical state $|\psi^a[\Theta]\rangle = a|\psi[\Theta]\rangle \in \mathbf{W}_\Theta^\infty$ described by a wave function

$$\psi^a(q;\Theta) = a^{-1}\left\langle \frac{q}{|a|^2} \middle| \psi[\Theta]\right\rangle, \tag{4.28}$$

and the value of observable Q_Θ is measured once each on many identically prepared system, the average value of all the measurements will be

$$\begin{gathered}\langle Q_\Theta\rangle = |a|^2\int\limits_\Theta q|c_{\psi[\Theta]}(q)|^2 dq = |a|^{-2}\int\limits_{|a|^2\Theta} q'\left|c_{\psi[\Theta]}\left(\frac{q'}{|a|^2}\right)\right|^2 dq' = \\ = \int\limits_{|a|^2\theta_1}^{|a|^2\theta_2} q'|\psi^a(q';\Theta)|^2 dq'.\end{gathered} \tag{4.29}$$

The probability $P(q,q+dq;|\psi^a[\Theta]\rangle)dq$ of obtaining the result q lying in the range $(q,q+dq) \subset \Theta$ on measuring Q_Θ is

$$P(q,q+dq;|\psi^a[\Theta]\rangle) = |a|^{-2}|c_{\psi[\Theta]}(q|a|^{-2})|^2 dq = |\psi^a(q;\Theta)|^2 dq \tag{4.30}$$

Remark 2.9.Notice that Eq.(2.28) follows from Eq.(2.13),Eq.(2.22) and Eq.(2.25).

Definition 4.3.Let $|\psi^a[\Theta]\rangle$ be a state $|\psi^a[\Theta]\rangle = a|\psi[\Theta]\rangle \in \mathbf{W}_\Theta^\infty$, where $|\psi[\Theta]\rangle \in \mathbf{S}_\Theta^\infty$, $a \in \mathbb{C}, |a| \neq 1$ and $|\psi[\Theta]\rangle = \int_{\theta_1}^{\theta_2} c_{\psi[\Theta]}(q)|q\rangle dq$. Let $|\psi_a\rangle$ be an state such that $|\psi_a[\Theta]\rangle \in \mathbf{S}_\Theta^\infty$. States $|\psi^a[\Theta]\rangle$ and $|\psi_a[\Theta]\rangle$ is a $\widehat{Q}_\Theta$-equivalent: $|\psi^a[\Theta]\rangle \sim_{\widehat{Q}_\Theta} |\psi_a[\Theta]\rangle$ iff

$$P(q,q+dq;|\psi^a[\Theta]\rangle) = |a|^{-2}|c_\psi(q|a|^{-2})|^2 dq = P(q,q+dq;|\psi_a[\Theta]\rangle). \tag{4.31}$$

It is clear that

$$|\psi^a[\Theta]\rangle \sim_{\widehat{Q}_\Theta} |\psi_a[\Theta]\rangle = \int_{|a|^2\theta_1}^{|a|^2\theta_2} \psi^a(q;\Theta)|q\rangle dq. \tag{4.32}$$

We set now $|\psi^a[\Theta]\rangle \sim_{\widehat{Q}_\Theta} |\psi_1^b[\Theta]\rangle$ iff $|\psi^a[\Theta]\rangle \sim_{\widehat{Q}_\Theta} |\psi_a[\Theta]\rangle$ and $|\psi^a[\Theta]\rangle \sim_{\widehat{Q}_\Theta} |\psi_1^b[\Theta]\rangle$, i.e.

$$|\psi^a[\Theta]\rangle \sim_{\widehat{Q}_\Theta} |\psi_1^b[\Theta]\rangle \Leftrightarrow |a|^{-2}|c_\psi(q|a|^{-2})|^2 = |b|^{-2}\left|c_{\psi_1}(q|b|^{-2})\right|^2$$

Remark 4.10.(i) Notice that the equivalence relation $\sim_{\widehat{Q}_\Theta}$ devided a set $\mathbf{W}_\Theta^\infty$ by classes

$$[|\psi^a[\Theta]\rangle]_{\widehat{Q}_\Theta} \triangleq \left\{|\Psi[\Theta]\rangle \in \mathbf{W}_\Theta^\infty \| \Psi[\Theta]\rangle \sim_{\widehat{Q}_\Theta} |\psi_a[\Theta]\rangle\right\}. \tag{4.33}$$

(ii)The physical significance of the factor set $\mathbf{W}_\Theta^\infty/_{\sim_{\widehat{Q}}}$ is that the non classical pure states $|\psi^a[\Theta]\rangle$ and $|\psi_1^b[\Theta]\rangle$ represent the same physical state of the quantum system,described by a wave function given by Eq.(2.28) and therefore such states equivalent an classical pure state $|\psi_a[\Theta]\rangle \in \mathbf{S}_\Theta^\infty$.

P.4.3. (**Generalized von Neumann measurement postulate I**) Assume that the particle is initially in the state $|\psi^a[\Theta]\rangle \in \mathbf{W}_\Theta^\infty$ If on performing a measurement of observable Q_Θ with an accuracy δq, and the result $q \in |a|^2\Theta$ is obtained in the range $(q - \frac{1}{2}\delta q, q + \frac{1}{2}\delta q) \subset |a|^2\Theta$, then the system immediately after measurement will end up in the state

$$\frac{\widehat{P}(q,\delta q)|\psi^a[\Theta]\rangle}{\sqrt{\langle\psi|\widehat{P}(q,\delta q)|\psi_a[\Theta]\rangle}} = \frac{\int_{|q-q'|\leq\delta q/2} |q'\rangle\langle q'|\psi_a[\Theta]\rangle dq'}{\sqrt{\int_{|q-q'|\leq\delta q/2} |\langle q'|\psi_a[\Theta]\rangle|^2 dq'}}. \tag{4.34}$$

P.5. Let $|\Psi^{a_1,a_2}[\Theta_1,\Theta_2]\rangle = |\psi_1^{a_1}[\Theta_1]\rangle + |\psi_2^{a_2}[\Theta_2]\rangle \in \mathbf{W}_{1,2}^\infty \triangleq \mathbf{W}_{\Theta_1}^\infty \oplus \mathbf{W}_{\Theta_2}^\infty$, where

(i) $|\psi_i^{a_i}[\Theta_i]\rangle = a_i|\psi_i[\Theta_i]\rangle \in \mathbf{W}_{\Theta_i}^\infty, |\psi_i\rangle = |\psi_i[\Theta_i]\rangle \in \mathbf{S}_{\Theta_i}^\infty, \Theta_i = (\theta_1^i,\theta_2^i), a_i \in \mathbb{C}, a_i \neq 0, i = 1,2$;

(ii) $\operatorname{supp}(c_{\psi_i[\Theta_i]}(q)), i = 1,2$ is a connected sets in $\mathbb{R}$;

(iii) $\{\Theta_1(a_1)\} \cap \{\Theta_2(a_2)\} = \varnothing$, where

$$\Theta_1(a_1) = |a_1|^2 \cdot \left(\overline{\operatorname{supp}(c_{\psi_1[\Theta_1]}(q))}\right), \Theta_1(a_1) = |a_1|^2 \cdot \left(\overline{\operatorname{supp}(c_{\psi_1[\Theta_1]}(q))}\right) \tag{4.35}$$

and

(iv) $|\psi_i[\Theta_i]\rangle = \int_{\theta_1^i}^{\theta_2^i} c_{\psi_i[\Theta_i]}(q)|q\rangle dq, i = 1,2.$

Then we claim the following:

P.5.1. The system in a state $|\Psi^{a_1,a_2}[\Theta_1,\Theta_2]\rangle$ described by a wave function

$$\Psi^{a_1,a_2}(q;\Theta_1,\Theta_2) = \Psi_1^{a_1}(q;\Theta_1) + \Psi_2^{a_2}(q;\Theta_2), \tag{4.36}$$

where

$$\Psi_1^{a_1}(q;\Theta_1) = (a_1^*)^{-1}\left\langle \frac{q}{|a_1|^2} \middle| \psi_1[\Theta_1] \right\rangle,$$
$$\Psi_2^{a_2}(q;\Theta_2) = (a_2^*)^{-1}\left\langle \frac{q}{|a_2|^2} \middle| \psi_2[\Theta_2] \right\rangle, \tag{4.37}$$

and the value of observable $\widehat{Q}$ is measured once each on many identically prepared system, the average value of all the measurements will be

$$\langle Q \rangle = |a_1|^2 \int_{\Theta_1} |c_{\psi_1[\Theta_1]}(q)|^2 q dq + |a_2|^2 \int_{\Theta_1} |c_{\psi_2[\Theta_2]}(q)|^2 q dq =$$
$$\int_{|a_1|^2\Theta_1} |\Psi_1^q(q;\Theta_1)|^2 q dq + \int_{|a_2|^2\Theta_2} |\Psi_2^q(q;\Theta_2)|^2 q dq = \tag{4.38}$$
$$\int_{\Theta_1 \cup \Theta_2} q|\Psi^{a_1,a_2}(q;\Theta_1,\Theta_2)|^2 dq$$

P.5.2. The probability of getting the result $q \in \Theta_{1,2} = \Theta_1(a_1) + \Theta_2(a_2)$ with an accuracy δq such that $(q - \frac{1}{2}\delta q, q + \frac{1}{2}\delta q) \subset \operatorname{supp}\left(|\Psi_1^{a_1}(q';\Theta_1)|^2 * \Psi_2^{a_2}(q';\Theta_2)\right) = \Theta_{1,2} = \Theta_1(a_1) + \Theta_2(a_2)$, given by formula

$$\int_{|q-q'|\leq\delta q/2} \left[\left(|\Psi_1^{a_1}(q';\Theta_1)|^2\right) * \left(|\Psi_2^{a_2}(q';\Theta_2)|^2\right) \right] dq'.$$
$$\int_{|q-q'|\leq\delta q/2} \left[\left(|\langle q'|\Psi_{1,a_1}(\Theta_1)\rangle|^2\right) * \left(|\langle q'|\Psi_2^{a_2}(\Theta_2)\rangle|^2\right) \right] dq'. \tag{4.39}$$

where $\Psi_1^{a_1}(q';\Theta_1)$ and $\Psi_2^{a_2}(q';\Theta_2)$ given by Eq.(2.37) and

$$|\Psi_{1,a_1}(\Theta_1)\rangle = \int_{|a|^2\theta_1^1}^{|a|^2\theta_2^1} \Psi_1^{a_1}(q';\Theta_1)|q'\rangle dq',$$
$$|\Psi_{2,a_2}(\Theta_2)\rangle = \int_{|a|^2\theta_1^2}^{|a|^2\theta_2^2} \Psi_2^{a_2}(q';\Theta_2)|q'\rangle dq'. \tag{4.40}$$

P.5.3. (**Generalized von Neumann measurement postulate II**)Assume that the system is initially in state $|\Psi^{a_1,a_2}[\Theta_1,\Theta_2]\rangle$. If on performing a measurement of $\widehat{Q}$ with an accuracy δq, the result $q = q_1 + q_2 \in \Theta_{1,2} = \Theta_1(a_1) + \Theta_2(a_2)$, where $q_1 \in \Theta_1(a_1), q_2 \in \Theta_2(a_2)$ is obtained in the range $(q - \frac{1}{2}\delta q, q + \frac{1}{2}\delta q) \subset \Theta_{1,2}$, then the state of the system immediately after measurement given by

$$\frac{\widehat{P}(q,\delta q)|\Psi^{a_1,a_2}[\Theta_1,\Theta_2]\rangle}{\sqrt{\langle\psi|\widehat{P}(q,\delta q)|\psi\rangle}} =$$

$$\frac{\int_{|q_1-q'|\leq\delta q/2}|q'\rangle\langle q'|\Psi_{1,a_1}[\Theta_1]\rangle dq' + \int_{|q_2-q'|\leq\delta q/2}|q'\rangle\langle q'|\Psi_{2,a_2}[\Theta_2]\rangle dq'}{\sqrt{\int_{|q-q'|\leq\delta q/2}\left[\left(|\langle q'|\Psi_{1,a_1}[\Theta_1]\rangle|^2\right) * \left(|\langle q'|\Psi_{2,a_2}[\Theta_2]\rangle|^2\right)\right]dq'}}. \tag{4.41}$$

Definition 4.4.A measure algebra $\mathcal{F} = (\mathbf{B},\mathbf{P})$ with a probability measure $\mathbf{P}$, is a Boolean algebra $\mathbf{B}$ with a countably additive probability measure.

Definition 4.5.(i) A measure algebra of physical events $\mathcal{F}^{ph} = (\mathbf{B}^{ph},\mathbf{P})$ with a probability measure $\mathbf{P}$, is an Boolean algebra of physical events $\mathbf{B}^{ph}$ with an countably additive probability measure. We assume that such physical events corresponds to performing a measurements of $\widehat{Q}$ with an accuracy δq, and $\widehat{Q} \in \Delta^*(\mathbf{H})$.

(ii) A Boolean algebra of physical events $\mathbf{B}^{ph}$ can be formally defined as a set $\mathbf{B}^{ph}$ of elements $a,b,\ldots$ with the following properties:

1. $\mathbf{B}^{ph}$ has two binary operations, $\wedge$ (logical AND, or "wedge") and $\vee$ (logical OR, or "vee"), which satisfy:

the idempotent laws:

(1) $a \wedge a = a \vee a = a$,

the commutative laws:

(2) $a \wedge b = b \wedge a$,

(3) $a \vee b = b \vee a$,

and the associative laws:

(4) $a \wedge (b \wedge c) = (a \wedge b) \wedge c$,

(5) $a \vee (b \vee c) = (a \vee b) \vee c$.

2. The operations satisfy the absorption law:

(6) $a \wedge (a \vee b) = a \vee (a \wedge b) = a$.

3. The operations are mutually distributive

(7) $a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$,

(8) $a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$.

4. $\mathbf{B}^{ph}$ contains universal bounds $\mathbf{0}$ and $\mathbf{1}$ which satisfy

(9) $\mathbf{0} \wedge a = \mathbf{0}$,

(10) $\mathbf{0} \vee a = a$,

(11) $\mathbf{1} \wedge a = a$,

(12) $\mathbf{1} \vee a = \mathbf{1}$.

5. $\mathbf{B}^{ph}$ has a unary operation $\neg a$ (or a') of complementation (logical negation), which obeys the laws:

(13) $a \wedge \neg a = \mathbf{0}$,

(14) $a \vee \neg a = \mathbf{1}$.

All properties of negation including the laws below follow from the above two laws alone.

6. Double negation law: $\neg(\neg a) = a$.
7.De Morgan's laws: (i) $\neg a \wedge \neg b = \neg(a \vee b)$, (ii) $\neg a \vee \neg b = \neg(a \wedge b)$.
8.Operations composed from the basic operations include the following importent examples:
The first operation, $a \rightarrow b$ (logical material implication):
(i) $a \rightarrow b \triangleq \neg a \vee b$.
The second operation, $a \oplus b$, is called exclusive. It excludes the possibility of both a and b
The third operation, the complement of exclusive or, is equivalence or Boolean equality:

(iii) $a \equiv b \triangleq \neg(a \oplus b)$
9. $\mathbf{B}^{ph}$ has a unary predicate $\mathbf{Occ}(a)$, which meant that event a has occurred, and which obeys the laws:
(i) $\mathbf{Occ}(a \wedge b) \Leftrightarrow \mathbf{Occ}(a) \wedge \mathbf{Occ}(b)$,
(ii) $\mathbf{Occ}(a \vee b) \Leftrightarrow \mathbf{Occ}(a) \vee \mathbf{Occ}(b)$,
(iii) $\mathbf{Occ}(\neg a) \Leftrightarrow \neg\mathbf{Occ}(a)$.
Definition 4.6.(i) Let $\mathbf{B}$ be a Boolean algebra of physical events. A Boolean algebra $\mathbf{B}_{M_4}^{ph}$ of physical events in Minkowski spacetime $M_4 = \mathbb{R}^{1,3}$ that is cartesian product $\mathbf{B}_{M_4} = \mathbf{B}^{ph} \times M_4$.
(ii) Let $\mathbf{B}_{M_4}^{ph}$ be a Boolean algebra of physical events in Minkowski spacetime. A measure algebra of physical events $\mathcal{F}_{M_4}^{ph} = \left(\mathbf{B}_{M_4}^{ph}, \mathbf{P}\right)$ in Minkowski spacetime that is a Boolean algebra $\mathbf{B}_{M_4}^{ph}$ with a probability measure $\mathbf{P}$.
(iii) Let $\mathbf{B}_{M_4}$ be Boolean algebra of the all physical events in Minkowski spacetime and let $\mathcal{F}_{M_4}^{ph}$ be an measure algebra $\mathcal{F}_{M_4}^{ph} = \left(\mathbf{B}_{M_4}^{ph}, \mathbf{P}\right)$ with a probability measure $\mathbf{P}$. We denote such physical events by $A(\mathbf{x}), B(\mathbf{x}), \ldots$ etc.,where $\mathbf{x} = (t, x_1, x_2, x_3) \in M_4$ or $A, B, \ldots$ etc.
(iv) We will be write for a short $A^{Oc}(\mathbf{x}), B^{Oc}(\mathbf{x}), \ldots$ etc., instead $\mathbf{Occ}(A(\mathbf{x})), \mathbf{Occ}(B(\mathbf{x})), \ldots$ etc.
Definition 4.7. Let $\mathbf{Aut}_{\mathbf{P}}\left(\mathbf{B}_{M_4}^{ph}\right)$ be a set of the all measure-preserving automorphism of $\mathbf{B}_{M_4}^{ph}$. This is a group,being a subgroup of the group $\mathbf{Aut}\left(\mathbf{B}_{M_4}^{ph}\right)$ of all Boolean automorphism of $\mathbf{B}_{M_4}^{ph}$. Let $P_{+}^{\uparrow}$ be Poincaré group.We assume that any element $\Theta = (\Lambda, a) \in P_{+}^{\uparrow}$ induced an corresponding element $\widetilde{\Theta} \in \mathbf{Aut}_{\mathbf{P}}\left(\mathbf{B}_{M_4}^{ph}\right)$ by formula $\widetilde{\Theta}[A(\mathbf{x})] = A(\Lambda\mathbf{x} + \mathbf{a}) \in \mathbf{B}_{M_4}^{ph}$.
Definition 4.8.Events $A_1, A_2, \ldots, A_n \in \mathcal{F}_{M_4}^{ph} = \left(\mathbf{B}_{M_4}^{ph}, \mathbf{P}\right)$ are said to be exactly mutually exclusive if the occurrence of any one of them implies the non-occurrence of the remaining $n-1$ events. Therefore, two mutually exclusive events cannot both occur. Formally said, the conjunction of each two of them is $\mathbf{0}$ (the null event): $A \wedge B = \mathbf{0}$. In consequence, exactly mutually exclusive events A and B have the property:

$$\mathbf{P}(A \wedge B) = 0. \tag{4.42}$$

Then we claim the following:

P.6. There exist isomorphism $\lambda : \mathcal{F}_{M_4}^{ph} \mapsto \mathfrak{I} \times M_4 = (\Omega, \Sigma, \mathbf{P}) \times M_4$, where $\mathfrak{I}$ is a

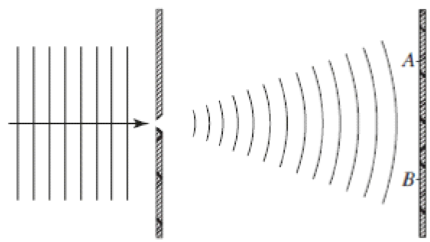
probability measure algebra $\mathfrak{I} = (\Omega, \Sigma, \mathbf{P})$ corresponding to quantum system $\mathbf{Q}^s$ by postulate **P.1** such that for any event $A(\mathbf{x}) \in \mathcal{F}_{M_4}^{ph}, \mathbf{x} = (t, x_1, x_2, x_3) \in M_4$

$$\lambda[A(\mathbf{x})] = \lambda[A](\mathbf{x}),$$
$$\mathbf{P}(A(\mathbf{x})) = \mathbf{P}(\lambda[A](\mathbf{x})) \triangleq \mathbf{P}(A_\lambda(\mathbf{x})). \tag{4.43}$$

Definition 4.9.Let $A(\mathbf{x}_1) = A(t_1,\mathbf{r}_1)$ and $B(\mathbf{x}_2) = B(t_2,\mathbf{r}_2)$ be an events $A(\mathbf{x}_1) \in \mathcal{F}^{\#}_{M_4}$ which occurs at instant t_1 and $B(\mathbf{x}_2) \in \mathcal{F}^{\#}_{M_4}$ at instant t_2 correspondingly.
Let $\mathbf{x}_{1,2}$ be a vector: $\mathbf{x}_{1,2} = \{c(t_1 - t_2),\mathbf{r}_1 - \mathbf{r}_2\} = (ct_{1,2},\mathbf{r}_{1,2}), t_{1,2} = t_1 - t_2, \mathbf{r}_{\mathbf{1,2}} = \mathbf{r}_1 - \mathbf{r}_2$.
Vectors $\mathbf{x}_{1,2} = (ct_{1,2},\mathbf{r}_{1,2})$ are classified according to the sign of $c^2t^2_{1,2} - \mathbf{r}^2_{1,2}$. A vector $\mathbf{x}_{1,2}$ is (i) timelike if $c^2t^2_{1,2} > \mathbf{r}^2_{\mathbf{1,2}}$, (ii) spacelike if $c^2t^2_{1,2} < \mathbf{r}^2_{\mathbf{1,2}}$, and null or lightlike if (iii) $c^2t^2_{1,2} = \mathbf{r}^2_{\mathbf{1,2}}$.
Pairs of events $\{A(t_1,\mathbf{r}_1),B(t_2,\mathbf{r}_2)\} \in \mathcal{F}^{\#}_{M_4} \times \mathcal{F}^{\#}_{M_4}$ are classified according to the sign of $c^2t^2_{1,2} - \mathbf{r}^2_{1,2}$:
(i) a pair $\{A(t_1,\mathbf{r}_1),B(t_2,\mathbf{r}_2)\}$ is timelike separated if $c^2t^2_{1,2} > \mathbf{r}^2_{\mathbf{1,2}}$,
and we denoted such pairs by $\{A(t_1,\mathbf{r}_1),B(t_2,\mathbf{r}_2)\}_{\mathbf{t.l.s.}}$
(ii) a pair $\{A(t_1,\mathbf{r}_1),B(t_2,\mathbf{r}_2)\}$ is spacelike separated if $c^2t^2_{1,2} < \mathbf{r}^2_{\mathbf{1,2}}$,
and we denoted such pairs by $\{A(t_1,\mathbf{r}_1),B(t_2,\mathbf{r}_2)\}_{\mathbf{s.l.s.}}$
(iii) a pair $\{A(t_1,\mathbf{r}_1),B(t_2,\mathbf{r}_2)\}$ is null or lightlike separated if $c^2t^2_{1,2} = \mathbf{r}^2_{\mathbf{1,2}}$.
and we denoted such pairs by $\{A(t_1,\mathbf{r}_1),B(t_2,\mathbf{r}_2)\}_{\mathbf{l.l.s.}}$
Definition 4.10. Let $A^{t_1} \triangleq A(\mathbf{x}_1) = A(t_1,x_A)$ and $B^{t_2} \triangleq B(\mathbf{x}_2) = B(t_2,x_B)$ be a symbols such that A^{t_1} and B^{t_2} represent there is detection events $A(\mathbf{x}_1) \in \mathcal{F}^{\#}_{M_4}$ at instant t_1 and $B(\mathbf{x}_2) \in \mathcal{F}^{\#}_{M_4}$ at instant t_2 correspondingly, where symbols x_A and x_B represent the locations of the detectors A and B correspondingly (see Pic. 4.2).

Einstein's 1927 gedanken experiment revisited

During the famous 5-th Solvay conference in 1927, Einstein considered a single particle which, after diffraction in a pin-hole encounters a “detection plate” (e.g. in the case of photons, a photographic plate). We simplify this thought experiment, though keeping the essence, by replacing the “detection plate” by two detectors. Einstein noted that there is no question, that only one of them can detect the particle, otherwise energy would not be conserved. However, he was deeply concerned about the situation in which the two detectors are space-like separated, as this prevents - according to relativity - any possible coordination among the detectors: “It seems to me,” Einstein continued, “that this difficulty cannot be overcome unless the description of the process in terms of the Schrödinger wave is supplemented by some detailed specification of the localization of the particle during its propagation. I think M. de Broglie is right in searching in this direction.”

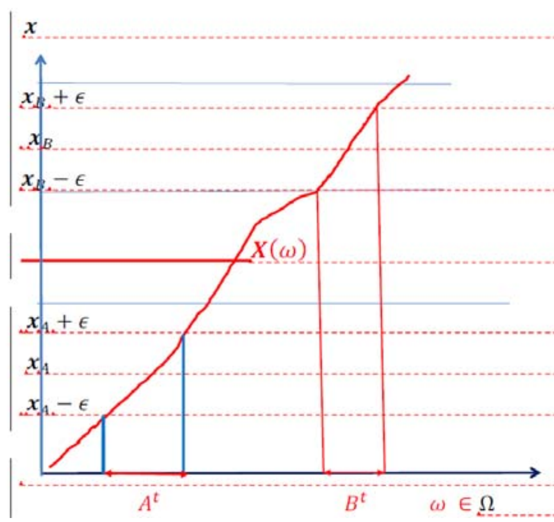


Pic. 4.2.

A and B are points on the photographic plate, for which the events of detection can be spacelike separated from each other.

Let $|\psi(x)\rangle$ be a state vector of the particle immediately after diffraction. From postulates **P.1.1** follows that there exist unique random variable $X(\omega)$ given on a probability space $\Im = (\Omega, \mathcal{F}, \mathbf{P})$ and a measurable space $(\mathbb{R}^n, \Sigma)$ by formula

$$X(\omega) \triangleq X_{|\hat{x}\psi(x)\rangle}(\omega) = \mathbf{G}[\hat{x}; |\psi(x)\rangle]. \tag{4.44}$$



Pic. 4.3.The plot of the random variable

$$X(\omega) = X_{|\hat{x}\psi^{\mathbf{ph}}\rangle}(\omega).$$

$$A^t = A^{ph}_{\lambda}(t, x_A), B^t = B^{ph}_{\lambda}(t, x_B), A^t \cap B^t = \varnothing.$$

Now we go to explain Einstein's 1927 gedanken experiment. Let $A^{ph}(t, x_A)$ and $B^{ph}(t, x_A)$ be events such that detectors A and B detect photon at an instant t correspondingly. By properties (4.31) we obtain

$$
\begin{aligned}
\mathbf{P}\Big(A_{\lambda}^{ph}(t,x_A)\Big) &\triangleq \mathbf{P}(\lambda[A^{ph}](t,x_A)) = \widetilde{\mathbf{P}}(A^{ph}(t,x_A)),\\
\mathbf{P}\Big(B_{\lambda}^{ph}(t,x_B)\Big) &\triangleq \mathbf{P}(\lambda[B^{ph}](t,x_B)) = \widetilde{\mathbf{P}}(B^{ph}(t,x_B)).
\end{aligned} \tag{4.45}
$$

Note that by definition of random variable we obtain

$$
\begin{gathered}
A^t \triangleq A_{\lambda}^{ph}(t,x_A) = \{\omega | x_A - \epsilon \leq X_{|\hat{x}\psi(x)\rangle}(\omega) \leq x_A - \epsilon\},\\
B^t \triangleq B_{\lambda}^{ph}(t,x_B) = \{\omega | x_B - \epsilon \leq X_{|\hat{x}\psi(x)\rangle}(\omega) \leq x_B - \epsilon\},\\
\epsilon \in (0,\gamma], \gamma \ll 1,
\end{gathered} \tag{4.46}
$$

(see Pic. 4.3), where a small parameter $\epsilon \ll |x_A - x_B|$ dependent on measuring device. Thus we obtain

$$
A_{\lambda}^{ph}(t,x_A) \cap B_{\lambda}^{ph}(t,x_B) = \varnothing \tag{4.47}
$$

and therefore

$$
\mathbf{P}\Big(A_{\lambda}^{ph}(t,x_A) \cap B_{\lambda}^{ph}(t,x_B)\Big) = 0. \tag{4.48}
$$

6mm . **Conclusion** 3mm

A new quantum mechanical formalism based on the probability representation of quantum states is proposed cite: FoukzonPotapovMen'kovaPodosenov16[15]. This paper in particular deals with the special case of the measurement problem, known as Schrödinger’s cat paradox. We pointed out that Schrödinger’s cat demands to reconcile Born’s rule. Using new quantum mechanical formalism we find the collapsed state of the Schrödinger’s cat always shows definite and predictable outcomes even if cat also consists of a superposition (see cite: FoukzonPotapovMen'kovaPodosenov16 [15], cite: FoukzonPotapovPodosenov14[17], cite: FoukzonPotapovPodosenov15[18])

$$
\begin{gathered}
\big|\text{cat}\big\rangle = c_1\big|\text{live cat}\big\rangle + c_2\big|\text{death cat}\big\rangle\\
|c_1|^2 + |c_2|^2 = 1.
\end{gathered}
$$

Using new quantum mechanical formalism the EPRB-paradox is considered successfully. We find that the EPRB-paradox can be resolved by nonprincipal and convenient relaxing of the

Einstein's locality principle.

6mm . **Appendix** 3mm

5mm . **Appendix A ref: lem:conv_K-M_ESS** 2mm

The time-dependent Schrodinger equation governs the time evolution of a quantum mechanical system:

$$i\hbar\frac{\partial\Psi(\mathbf{x},t)}{\partial t} = \widehat{\mathbf{H}}\Psi(\mathbf{x},t). \tag{A.1}$$

The average, or expectation, value $\langle x_i\rangle$ of an observable x_i corresponding to a quantum mechanical operator $\widehat{x}_i$ is given by:

$$\langle x_i\rangle(t,\mathbf{x}_0,t_0;\hbar) = \frac{\int\limits_{\mathbb{R}^d} x_i|\Psi(\mathbf{x},t,\mathbf{x}_0,t_0;\hbar)|^2 d^d x}{\int\limits_{\mathbb{R}^d}|\Psi(\mathbf{x},t,\mathbf{x}_0,t_0;\hbar)|^2 d^d x}. \tag{A.2}$$

$$i = 1,\ldots,d.$$

Remark A.1. We assume now that: the solution $\Psi(\mathbf{x},t,\mathbf{x}_0,t_0;\hbar)$ of the time-dependent

Schrödinger equation (A.1) has a good approximation by a delta function such that

$$|\Psi(\mathbf{x},t,\mathbf{x}_0,t_0;\hbar)|^2 \simeq \prod_{i=1}^{d}\delta(x_i - x_i(t,\mathbf{x}_0,t_0)),$$
$$x_i(t,\mathbf{x}_0,t_0) = x_{i,0}, \tag{A.3}$$
$$i = 1,\ldots,d.$$

Remark A.2. Note that under conditions given by Eq.(A.3) QM-system which governed by

Schrödinger equation Eq.(A.1) completely evolve quasiclassically i.e. estimating the position $\{x_i(t,\mathbf{x}_0,t_0;\hbar)\}_{i=1}^{d}$ at each instant t with final error δ gives $|\langle x_i\rangle(t,\mathbf{x}_0,t_0;\hbar) - x_i(t,\mathbf{x}_0,t_0)| \leq \delta, i = 1,\ldots,d$ with a probability

$$\mathbf{P}\{|\langle x_i\rangle(t,\mathbf{x}_0,t_0;\hbar) - x_i(t,\mathbf{x}_0,t_0)| \leq \delta\} \simeq 1.$$

Thus from Eq.(A.2) and Eq.(A.3) we obtain

$$\langle x_i \rangle(t, \mathbf{x}_0, t_0; \hbar) \simeq$$

$$\simeq \frac{\int_{\mathbb{R}^d} x_i \prod_{i=1}^{d=1} \delta(x_i - x_i(t, \mathbf{x}_0, t_0)) d^d x}{\int_{\mathbb{R}^d} \prod_{i=1}^{d=1} \delta(x_i - x_i(t, \mathbf{x}_0, t_0)) d^d x} = x_i(t, \mathbf{x}_0, t_0). \quad (A.4)$$

$$i = 1, \ldots, d.$$

Thus under condition given by Eq.(A.3) one obtains

$$\langle x_{i,t} \rangle(t, \mathbf{x}_0, t_0; \hbar) \simeq x_i(t, \mathbf{x}_0, t_0), \quad (A.5)$$
$$i = 1, \ldots, d.$$

Remark A.3.Let $\Psi_i(\mathbf{x}, t, \mathbf{x}_0, t_0), i = 1,2$ be the solutions of the time-dependent Schrödinger equation (A.1). We assume now that $\Phi(\mathbf{x}, t, \mathbf{x}_0, \mathbf{y}_0, t_0)$ is a linear superposition such that

$$\Phi(\mathbf{x}, t, \mathbf{x}_0, \mathbf{y}_0, t_0) = c_1 \Psi_1(\mathbf{x}, t, \mathbf{x}_0, t_0) + c_2 \Psi_2(\mathbf{x}, t, \mathbf{y}_0, t_0). \quad (A.6)$$
$$|c_1|^2 + |c_2|^2 = 1.$$

Then we obtain

$$|\Phi(\mathbf{x}, t, \mathbf{x}_0, \mathbf{y}_0, t_0)|^2 = (\Phi(\mathbf{x}, t, \mathbf{x}_0, \mathbf{y}_0, t_0) \Phi^*(\mathbf{x}, t, \mathbf{x}_0, \mathbf{y}_0, t_0)) =$$

$$= ([c_1 \Psi_1(\mathbf{x}, t, \mathbf{x}_0, t_0) + c_2 \Psi_2(\mathbf{x}, t, \mathbf{y}_0, t_0)]) \times$$

$$\times ([c_1^* \Psi_1^*(\mathbf{x}, t, \mathbf{x}_0, t_0) + c_2^* \Psi_2^*(\mathbf{x}, t, \mathbf{x}_0, \mathbf{y}_0, t_0)]) = \quad (A.7)$$

$$= |c_1|^2 \left(|\Psi_1(\mathbf{x}, t, \mathbf{x}_0, t_0)|^2 \right) + c_1^* c_2 (\Psi_1^*(\mathbf{x}, t, \mathbf{x}_0) \Psi_2(\mathbf{x}, t, \mathbf{y}_0, t_0)) +$$

$$|c_2|^2 \left(|\Psi_2(\mathbf{x}, t, \mathbf{y}_0, t_0)|^2 \right) + c_1 c_2^* (\Psi_1(\mathbf{x}, t, \mathbf{x}_0) \Psi_2^*(\mathbf{x}, t, \mathbf{y}_0, t_0)).$$

Definition A.1. Let $\langle \mathbf{x} \rangle(t, \mathbf{x}_0, \mathbf{y}_0, t_0)$ be a vector-function

$$\langle \mathbf{x} \rangle(t, \mathbf{x}_0, \mathbf{y}_0, t_0) \,:\, [0,T] \times \mathbb{R}^d \times \mathbb{R}^d \times [0,T] \to \mathbb{R}^d$$

$$\langle \mathbf{x} \rangle(t, \mathbf{x}_0, \mathbf{y}_0, t_0) = \{\langle x_1 \rangle(t, \mathbf{x}_0, \mathbf{y}_0, t_0), \ldots, \langle x_d \rangle(t, \mathbf{x}_0, \mathbf{y}_0, t_0)\}, \qquad (A.8)$$

where

$$\langle x_i \rangle(t, \mathbf{x}_0, \mathbf{y}_0, t_0) = \int\limits_{\mathbb{R}^d} x_i |\Phi(\mathbf{x}, t, \mathbf{x}_0, \mathbf{y}_0, t_0)|^2 d^d x =$$

$$= |c_1|^2 \int\limits_{\mathbb{R}^d} x_i |\Psi_1(\mathbf{x}, t, \mathbf{x}_0, t_0)|^2 d^d x +$$

$$+ |c_2|^2 \int\limits_{\mathbb{R}^d} x_i |\Psi_2(\mathbf{x}, t, \mathbf{y}_0, t_0)|^2 d^d x + \qquad (A.9)$$

$$+ c_1^* c_2 \int\limits_{\mathbb{R}^d} x_i \Psi_1^*(\mathbf{x}, t, \mathbf{x}_0, t_0) \Psi_2(\mathbf{x}, t, \mathbf{y}_0, t_0) d^d x +$$

$$+ c_1 c_2^* \int\limits_{\mathbb{R}^d} x_i \Psi_1(\mathbf{x}, t, \mathbf{x}_0, t_0) \Psi_2^*(\mathbf{x}, t, \mathbf{y}_0, t_0) d^d x.$$

Definition A.2. Let $\Delta(t, \mathbf{x}_0, \mathbf{y}_0, t_0)$ be a vector-function

$$\Delta(t, \mathbf{x}_0, \mathbf{y}_0, t_0) \,:\, [0,T] \times \mathbb{R}^d \times \mathbb{R}^d \to \mathbb{R}^d$$

$$(\Delta(t, \mathbf{x}_0, \mathbf{y}_0, t_0)) = \{\delta_1(t, \mathbf{x}_0, \mathbf{y}_0, t_0), \ldots, \delta_d(t, \mathbf{x}_0, \mathbf{y}_0, t_0)\}, \qquad (A.10)$$

where

$$\delta_i(t,\mathbf{x}_0,\mathbf{y}_0,t_0) = \delta[x_i(t,\mathbf{x}_0,\mathbf{y}_0,t_0)] =$$

$$= c_1^* c_2 \int\limits_{\mathbb{R}^d} x_i \Psi_1^*(\mathbf{x},t,\mathbf{x}_0,t_0)\Psi_2(\mathbf{x},t,\mathbf{y}_0,t_0)d^d x + \qquad (A.11)$$

$$+ c_1 c_2^* \int\limits_{\mathbb{R}^d} x_i \Psi_1(\mathbf{x},t,\mathbf{x}_0,t_0)\Psi_2^*(\mathbf{x},t,\mathbf{y}_0,t_0)d^d x.$$

Substituting Eqs.(A.11) into Eqs.(A.9) gives

$$\langle x_i \rangle(t,\mathbf{x}_0,\mathbf{y}_0,t_0) = \int\limits_{\mathbb{R}^d} x_i |\Phi(\mathbf{x},t,\mathbf{x}_0,\mathbf{y}_0,t_0)|^2 d^d x =$$

$$= |c_1|^2 \int\limits_{\mathbb{R}^d} x_i |\Psi_1(\mathbf{x},t,\mathbf{x}_0,t_0)|^2 d^d x + \qquad (A.12)$$

$$+ |c_2|^2 \int\limits_{\mathbb{R}^d} x_i |\Psi_2(\mathbf{x},t,\mathbf{y}_0,t_0)|^2 d^d x + \delta_i(t,\mathbf{x}_0,\mathbf{y}_0,t_0) =$$

$$= |c_1|^2 \langle x_i \rangle(t,\mathbf{x}_0,t_0) + |c_2|^2 \langle x_i \rangle(t,\mathbf{y}_0,t_0) + \delta_i(t,\mathbf{x}_0,\mathbf{y}_0,t_0).$$

Substitution equations (A.5) into equations (A.12) gives

$$\langle x_i \rangle(t, \mathbf{x}_0, \mathbf{y}_0, t_0) = \int\limits_{\mathbb{R}^d} x_i |\Phi(\mathbf{x}, t, \mathbf{x}_0, \mathbf{y}_0, t_0)|^2 d^d x =$$

$$= |c_1|^2 \langle x_i \rangle(t, \mathbf{x}_0, t_0) + |c_2|^2 \langle x_i \rangle(t, \mathbf{y}_0, t_0) + \delta_i(t, \mathbf{x}_0, \mathbf{y}_0, t_0) \quad (A.13)$$

$$\simeq |c_1|^2 x_i(t, \mathbf{x}_0, t_0) + |c_2|^2 x_i(t, \mathbf{y}_0, t_0) + \delta_i(t, \mathbf{x}_0, \mathbf{y}_0, t_0).$$

5mm . **Appendix B ref: thm:conv_lambda0mle** 2mm

The Schrödinger equation (2.1) in region $\mathbf{I} = \{x | x < 0\}$ has the folloving form

$$\hbar^2 \frac{\partial^2 \Psi_{\mathbf{I}}(x)}{\partial x^2} + 2mE\Psi_{\mathbf{I}}(x) = 0. \quad (B.1)$$

From Schrödinger equation (B.1) it follows

$$\hbar^2 \int\limits_{-\infty}^{0} \frac{\partial^2 \Psi_{\mathbf{I}}(x)}{\partial x^2} dx + 2mE \int\limits_{-\infty}^{0} \Psi_{\mathbf{I}}(x) dx = 0. \quad (B.2)$$

Let $\Psi_{\mathbf{I}}^{\#}(x)$ be a function

$$\Psi_{\mathbf{I}}^{\#}(x) = \phi(x)\Psi_{\mathbf{I}}(x), \quad (B.3)$$

where

$$\phi(x) = (\pi r_c^2)^{-1/4} \exp\left(\frac{x^2}{2r_c^2}\right) \quad (B.4)$$

see Eq.(2.9). Note that

$$\frac{\partial^2[\phi(x)\Psi_{\mathbf{I}}(x)]}{\partial x^2} = \frac{\partial}{\partial x}\left[\Psi_{\mathbf{I}}(x)\frac{\partial\phi(x)}{\partial x} + \phi(x)\frac{\partial\Psi_{\mathbf{I}}(x)}{\partial x}\right] =$$

$$2\frac{\partial\Psi_{\mathbf{I}}(x)}{\partial x}\frac{\partial\phi(x)}{\partial x} + \Psi_{\mathbf{I}}(x)\frac{\partial^2\phi(x)}{\partial x^2} + \phi(x)\frac{\partial^2\Psi_{\mathbf{I}}(x)}{\partial x^2}. \qquad (B.5)$$

Therefore substitution (B.2) into LHS of the Schrödinger equation (B.1) gives

$$\hbar^2 \int_{-\infty}^{0} \frac{\partial^2\Psi^{\#}_{\mathbf{I}}(x)}{\partial x^2}dx + 2mE\int_{-\infty}^{0} \Psi^{\#}_{\mathbf{I}}(x)dx =$$

$$\hbar^2 \int_{-\infty}^{0} \frac{\partial^2\phi(x)\Psi_{\mathbf{I}}(x)}{\partial x^2}dx + 2Em\int_{-\infty}^{0} \phi(x)\Psi_{\mathbf{I}}(x)dx = \qquad (B.6)$$

$$2\hbar^2 \int_{-\infty}^{0} \frac{\partial\Psi_{\mathbf{I}}(x)}{\partial x}\frac{\partial\phi(x)}{\partial x}dx + \hbar^2\int_{-\infty}^{0} \Psi_{\mathbf{I}}(x)\frac{\partial^2\phi(x)}{\partial x^2}dx +$$

$$+\int_{-\infty}^{0} \phi(x)\left\{\hbar^2\frac{\partial^2\Psi_{\mathbf{I}}(x)}{\partial x^2} + 2Em\int_{-\infty}^{0} \Psi_{\mathbf{I}}(x)\right\}dx.$$

Note that

$$\int_{-\infty}^{0} \phi(x)\left\{\hbar^2\frac{\partial^2\Psi_{\mathbf{I}}(x)}{\partial x^2} + 2Em\int_{-\infty}^{0} \Psi_{\mathbf{I}}(x)\right\}dx = 0. \qquad (B.7)$$

Therefore from Eq.(B.6) and Eq.(2.3)-Eq.(2.4) one obtains

$$\hbar^2 \int\limits_{-\infty}^{0} \frac{\partial^2 \Psi_{\mathbf{I}}^{\#}(x)}{\partial x^2} dx + 2mE \int\limits_{-\infty}^{0} \Psi_{\mathbf{I}}^{\#}(x) dx =$$

$$\hbar^2 \int\limits_{-\infty}^{0} \frac{\partial^2 \phi(x) \Psi_{\mathbf{I}}(x)}{\partial x^2} dx + 2Em \int\limits_{-\infty}^{0} \phi(x) \Psi_{\mathbf{I}}(x) dx = \qquad (B.8)$$

$$= 2\hbar^2 \int\limits_{l}^{\infty} \frac{\partial \Psi_{\mathbf{I}}(x)}{\partial x} \frac{\partial \phi(x)}{\partial x} dx + \hbar^2 \int\limits_{l}^{\infty} \Psi_{\mathbf{I}}(x) \frac{\partial^2 \phi(x)}{\partial x^2} dx.$$

From Eq.(B.6) one obtains

$$\frac{\partial \phi(x)}{\partial x} = (\pi r_c^2)^{-1/4} \frac{\partial}{\partial x} \exp\left[-\frac{x^2}{2r_c^2} \right] = -(\pi r_c^2)^{-1/4} r_c^{-2} x \exp\left[-\frac{x^2}{2r_c^2} \right],$$

$$\frac{\partial^2 \phi(x)}{\partial x^2} = -(\pi r_c^2)^{-1/4} r_c^{-2} \exp\left[-\frac{x^2}{2r_c^2} \right] + \qquad (B.9)$$

$$+(\pi r_c^2)^{-1/4} r_c^{-4} x^2 \exp\left[-\frac{x^2}{2r_c^2} \right].$$

From Eq.(B.9) and Eq.(2.3)-Eq.(2.4) one obtains

$$\hbar^2 \int_{-\infty}^{0} \frac{\partial \Psi_{\mathbf{I}}(x)}{\partial x} \frac{\partial \phi(x)}{\partial x} dx =$$

$$-\frac{\hbar^2}{(\pi r_c^2)^{1/4} r_c^2} \int_{-\infty}^{0} \frac{\partial \exp(ikx)}{\partial x} x \exp\left[-\frac{x^2}{2r_c^2}\right] dx = \qquad (B.10)$$

$$-\frac{2\pi\sqrt{2mE}\,\hbar}{(\pi r_c^2)^{1/4} r_c^2} \int_{-\infty}^{0} x \exp\left(i\frac{2\pi\sqrt{2mE}}{\hbar}x\right) \exp\left[-\frac{x^2}{2r_c^2}\right] dx,$$

$$k = \frac{2\pi}{\hbar}\sqrt{2mE}\,.$$

and

$$\hbar^2 \int_{-\infty}^{0} \Psi_{\mathbf{I}}(x) \frac{\partial^2 \phi(x)}{\partial x^2} dx = -\frac{\hbar^2}{(\pi r_c^2)^{3/4} r_c^2} \int_{-\infty}^{0} \exp(ikx) \exp\left[-\frac{x^2}{2r_c^2}\right] dx + \qquad (B.11)$$

$$+\frac{\hbar^2}{(\pi r_c^2)^{1/4} r_c^2} \int_{-\infty}^{0} x^2 \exp(ikx) \exp\left[-\frac{x^2}{2r_c^2}\right] dx.$$

6mm

Acknowledgments

3mm

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bibitem

Weinberg12 [1] S. Weinberg. *Phys. Rev. A.*, 85: 062116, 2012.
BassiLochanSatinSinghUlbricht13 [2] A. Bassi, K. Lochan, S. Satin, T. P. Singh, and H. Ulbricht. Models of wave function Collapse, Underlying Theories, and Experimental Tests.*Rev. Mod. Phys.*,85: 471-527, 2013. http://arxiv.org/abs/1204.4325
JacobsSteck06 [3] K. Jacobs and D. A. Steck. A Straightforward Introduction to Continuous Quantum Measurement. *Contemporary Physics*, 47: 279, 2006. DOI:10.1080/00107510601101934
Mensky93 [4] M. B. Mensky. . Continuous Quantum Measurements and Path Integrals. Hardcover: 188 pp. Publisher CRC Press., January 1, 1993. ISBN-10: 0750302283 ISBN-13: 978-0750302289
Mensky00 [5] M. B. Mensky. Quantum Measurements and Decoherence: Models and Phenomenology. *Series: Fundamental Theories of Physics*, 110: XVI, 231 pp., 2000. ISBN 978-94-015-9566-7
Leggett84 [6] A.J. Leggett. Schrödinger's Cat and Her Laboratory Cousins. *Contemp. Phys. Rev.*, 25(6): 583-598, 1984.
Hobson13 [7] A. Hobson. Two-photon interferometry and quantum state collapse. *Phys. Rev. A* ,88: 022105, Published 8 August 2013.
Schrodinger35 8[11] E. Schrödinger. Die gegenwärtige Situation in der Quantenmechanik (The present situation in quantum mechanics). *Naturwissenschaften*, 23 (49): 807–812, 1935. doi:10.1007/BF0491891.
Wimmel92 [9] H. Wimmel. Quantum physics & observed reality: a critical interpretation of quantum mechanics.World Scientific, 1992. p. 2. ISBN 978-981-02-1010-6.
Faye08 [10] J. Faye. Copenhagen Interpretation of Quantum Mechanics.Stanford Encyclopedia of Philosophy. The Metaphysics Research Lab Center for the Study of Language and Information, Stanford University, 2008-01-24.
CarpenterAnderson06 [11] R. H. S. Carpenter and A. J. Anderson. The death of Schroedinger's cat and of consciousness-based wave-function collapse. *Annales de la Fondation Louis de Broglie*, 31 (1): 45–52, 2006. Archived from the original (PDF) on 2006-11-30.
Omnes99 [12] R. Omnès. . Quantum Philosophy. Understanding and Interpreting Contemporary Science. English Edition - Princeton University Press, 1999.
Gamov28 [13] G. Gamow. Zur Quantentheorie des Atomkernes.*Z. Physik* , 51: 204, 1928.
FeynmanHibbs05 [14] R. Feynman and A. Hibbs. . Quantum Mechanics and Path Integrals, Dover (Emended Edition), 2005.
FoukzonPotapovMen'kovaPodosenov16 [15] J. Foukzon, A. A. Potapov, E. R. Menkova, and S .A. Podosenov. A New Quantum Mechanical Formalism Based on the Probability Representation of Quantum States. 179 Pages. http://vixra.org/abs/1612.0298 Submission history[v1] 2016-12-18 17:42:02
van Kampen88 [16] N. G. van Kampen. *Physica A*, 153(97), 1988.
FoukzonPotapovPodosenov14 [17] J. Foukzon, A. A. Potapov, and S. A. Podosenov. Schrödinger's Cat Paradox Resolution Using GRW Collapse Model. *Jnternational Journal of Recent Advances in Physics (IJRAP)*, 3(3): August 2014. http://wireilla.com/physics/ijrap/papers/3314ijrap02.pdf
FoukzonPotapovPodosenov15 [18] J. Foukzon, A. A. Potapov, and S. A. Podosenov. Exact quasiclassical asymptotics beyond Maslov canonical operator. http://arxiv.org/abs/1110.0098v5 (Submitted on 1 Oct 2011 (v1), last revised 26 Dec

2015 (this version, v5)). *Jnternational Journal of Recent Advances in Physics (IJRAP)*, 3 (4): November 2014. DOI: 10.14810/ijrap.2014.3401