The Universal Uncertainty Principle

RODOLFO A. FRINO – August 2014
Electronics Engineer – Degree from the National University of Mar del Plata - Argentina
rodolfo_frino@yahoo.com.ar

Abstract

This paper is concerned with a generalization of the Heisenberg's uncertainty principle which I developed in 2012 and that I called the universal uncertainty principle. This principle takes into account a) the quantized nature of space and b) the quantum fluctuations of the empty space. I have applied the simplified version of this principle to two different phenomena: a) black holes; where I explain both the temperature and the entropy of these cosmic objects and b) fundamental particles; where I calculated the approximate size of the electron. I have already published these two calculations in previous online articles so they are not included here. In this paper I propose a general form of the universal uncertainty principle which, unlike the simplified version, also includes the quantum fluctuations of the vacuum. All the laws of physics which are affected by this principle will need to be re-written as I have shown in the case of the temperature for the black hole.

Keywords*: quantum fluctuations, zero point momentum, entropy, Planck length.*

1. Introduction

In 1927 the German physicist Werner Heisenberg discovered a principle known as the Heisenberg uncertainty principle [1] which is normally written as

$$\Delta p \Delta x \geq \frac{h}{4\pi}$$

In the article entitled *"The Quantum Theory of Black Holes"* [2] published online I applied the *simplified universal uncertainty principle* to black holes to derive their thermodynamics properties: temperature and entropy. The surprising result of that research was that the equation for the black hole entropy was included into the equation for the temperature of the black hole. This meant that the black hole entropy equation emerged naturally as there was no need to introduce any additional physical concepts or postulates. In a second paper entitled *"The Size of Fundamental Particles",* I applied the same principle again to calculate the diameter of the electron. As a result the size of the electron turned out to be smaller than 10.166 times the Planck length [3].

Despite of not including the quantum fluctuations of the empty space, the simplified version of the *universal uncertainty principle* turned out to be an invaluable quantum mechanical tool.

This paper aims to generalize the previous formulation by including the fluctuations of the vacuum into the so called *general universal uncertainty principle*.

2. Abbrevations

In order to refer to these two principles I shall use the following abbreviations:

(a) GUUP or simply UUP: stands for *General Universal Uncertainty Principle*. This is the most general uncertainty principle, and

(b) SUUP or "simplified UUP": stands for *Simplified Universal Uncertainty Principle* [2]. This is a special or simplified version of the UUP principle.

3. Rationale

The UUP principle has to satisfy the following conditions

1) The principle must be quadratic in $\Delta p \Delta x$
2) When $P_Z = 0$ and $L_Z = 0$ the principle will reduce to $\Delta p \Delta x \geq h/4\pi$
3) When $\Delta p = 0$ and $\Delta x = 0$ the principle will reduce to $P_Z L_Z \geq h/4\pi$

Let's consider these three conditions separately

1) We shall adopt a second order uncertainty principle. We want this principle to be as general as possible to ensure an accurate description of nature for all phenomena including black holes. The entropy of the black hole should emerge naturally from this principle when applied to black holes without introducing additional conditions. This is not possible using a linear principle. Then the principle will be of the form

$$(\Delta p \Delta x)^2 \geq \left(\frac{h}{4\pi}\right)^2 + other\ terms \qquad (3\text{-}1)$$

Hence taking the square root on both sides

$$\Delta p \Delta x \geq \sqrt{\left(\frac{h}{4\pi}\right)^2 + other\ terms} \qquad (3\text{-}2)$$

2) When the effects of quantum fluctuations of space-time are neglected. (mathematically means that $P_Z = 0$ and $L_Z = 0$ **)**, the principle will be identical to the Heisenberg uncertainty principle (HUP). Thus under these conditions the principle will reduce to

$$\Delta p \Delta x \geq \frac{h}{4\pi} \qquad (3\text{-}3)$$

The reason of this is that a wave packet representing the wave function $\psi(x,y,z,t)$ of the particle is formed by the addition of a number of different wavelengths that produce interference (the superposition principle in quantum mechanics gives rise to interference). The more wavelengths we add the more localized the wave function will be and therefore the probability of finding the particle in a cubic box of volume $dV = dxdydz$ will be higher. This is so because the square of the wave function $|\psi(x,y,z,t)|^2$ is the probability density of a measurement of the finding the particle in the cubic volume dV. Thus the probability $P_{x1,x2,y1,y2,z1,z2}(t)$ of finding the particle in a cubic volume defined as

$$x \in [x1, x2] \; and \; y \in [y1, y2] \; and \; z \in [z1, z2])$$

$$\text{where } x1 < x2; \; y1 < y2; \; z1 < z2$$

at time t will be

$$P_{x1,x2,y1,y2,z1,z2}(t) = \int_{x_1}^{x_2} \int_{y_1}^{y_2} \int_{z_1}^{z_2} |\psi(x, y, z, t)|^2 \, dxdydz$$

This integral shows that the more localized the wave function the higher the probability of finding the particle in a given volume. However, this mechanism will make the momentum of the particle more uncertain. The reason is that, according to De Broglie, each individual wavelength has a momentum associated with it which is given by

$$p = \frac{h}{\lambda}$$

Because the wave function of the particle is composed of a large number of different wavelengths of different amplitudes (only the De Broglie relationships are shown here):

$$p_1 = \frac{h}{\lambda_1}; \quad p_2 = \frac{h}{\lambda_2}; \quad p_3 = \frac{h}{\lambda_3}; \quad p_4 = \frac{h}{\lambda_4}; \quad \ldots \quad ; \quad p_n = \frac{h}{\lambda_n}$$

the momentum of the particle becomes more uncertain (which is the momentum of the particle $p_1, p_2, p_3, p_4, \ldots or \; p_n$?)

From approximate analysis we see that the HUP relates to the wave nature of the wave packet and not to the quantum fluctuations of the vacuum.

3) When the effects of the uncertainties due to the wave nature of the wave packet describing the particle are neglected (mathematically means that $\Delta p = 0$ and $\Delta x = 0$**),** the principle will reduce to

$$P_Z L_Z \geq \frac{h}{4\pi} \tag{3-4}$$

4. The General Universal Uncertainty Principle

The only way of satisfying all three conditions given in the previous section simultaneously is to define the GUUP principle as follows:

$$\Delta p \Delta x \geq \sqrt{\left(\frac{h}{4\pi}\right)^2 - \frac{h}{8\pi} P_Z(\Delta x + L_Z) - \frac{h}{8\pi}(\Delta p + P_Z) L_Z} \qquad (4\text{-}1)$$

Where

$\Delta p =$ Uncertainty in the momentum of a particle due to its wave nature (wave packet representing the particle). This uncertainty does not include the uncertainty in the zero point momentum, P_Z, due to the quantum fluctuations of space-time.

$\Delta x =$ Uncertainty in the position of the particle due to the wave packet representing the particle. This uncertainty does not include the uncertainty in the position, L_Z, due to the quantum fluctuations of space-time.

$P_Z =$ Uncertainty in the momentum of the particle in the direction of the movement of the wave packet representing the particle. This momentum is due to the quantum fluctuations of space-time and does not include the uncertainty in the momentum, Δp, due to the wave nature of the wave packet. This momentum, P_Z, is also know as the zero point momentum.

$L_Z =$ Uncertainty in the position of the particle due to the quantum fluctuations of space-time. This uncertainty does not include the uncertainty in the position, Δx, due to the wave nature of the wave packet representing the particle. It is worthy to remark that the minimum value of this uncertainty cannot be measured experimentally with the present technology. It seems logical to assume that this uncertainty is identical to the Planck length, L_P. However, these two lengths could be slightly different.

5. Verification

(a) $P_Z = 0$ and $L_Z = 0$

$$\Delta p \Delta x \geq \sqrt{\left(\frac{h}{4\pi}\right)^2 - 0 - 0}$$

$$\Delta p \Delta x \geq \frac{h}{4\pi}$$

(b) $\Delta p = 0$ and $\Delta x = 0$

$$0 \geq \sqrt{\left(\frac{h}{4\pi}\right)^2 - \frac{h}{8\pi} P_Z(0 + L_Z) - \frac{h}{8\pi}(0 + P_Z) L_L}$$

$$0 \geq \sqrt{\left(\frac{h}{4\pi}\right)^2 - \frac{h}{8\pi} P_Z L_Z - \frac{h}{8\pi} P_Z L_L}$$

$$0 \geq \sqrt{\left(\frac{h}{4\pi}\right)^2 - \frac{h}{4\pi} P_Z L_Z}$$

$$0 \geq \left(\frac{h}{4\pi}\right)^2 - \frac{h}{4\pi} P_Z L_Z$$

$$\frac{h}{4\pi} P_Z L_Z \geq \left(\frac{h}{4\pi}\right)^2$$

$$P_Z L_Z \geq \frac{h}{4\pi}$$

6. The Simplified Universal Uncertainty Principle

The expression of the simplified or special universal uncertainty principle (SUUP) as I introduced it in the article mentioned above (*"The Quantum Theory of Black Holes"*) is

$$\Delta p \Delta x \geq \sqrt{\left(\frac{h}{4\pi}\right)^2 - \frac{h}{4\pi} \Delta p L_Z} \qquad (6\text{-}1)$$

It is worthy to remark that making $P_Z = 0$ in inequality (3-1), yields

$$\Delta p \Delta x \geq \sqrt{\left(\frac{h}{4\pi}\right)^2 - \frac{h}{8\pi} 0 \left(\Delta x + L_Z\right) - \frac{h}{8\pi} \left(\Delta p + 0\right) L_Z} \qquad (6\text{-}2)$$

$$\Delta p \Delta x \geq \sqrt{\left(\frac{h}{4\pi}\right)^2 - \frac{h}{8\pi} \Delta p L_Z} \qquad (6\text{-}3)$$

Which is different to the simplified universal uncertainty principle (6-1). The simplified universal uncertainty principle takes into account the fact that P_Z was neglected and it compensates for this simplification since in reality P_Z is not zero.

The other relationship (6-3), on the other hand, also assumes that P_Z is zero but it does not compensate for the fact that P_Z is not zero.

7. Conclusions

In summary, this paper introduced the General Universal Uncertainty Principle (UUP) which is an extension to the legendary Heisenberg Uncertainty Principle (HUP). The principle, in its simplified version (simplified UUP), has already shown that the Berkenstein-Hawking equation for the black hole temperature is an approximation to a more general law introduced in the above mentioned paper [2].

Thus part of the potential of this formulation has already been unveiled. However, it is to early to envisage the full implications of the UUP principle, but one thing is for sure: a number of physical laws will need to be re-written.

REFERENCES

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