Normalization Some Holographic Dark Energy Models

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We propose the normalization of some holographic dark energy (HDE) models. Applying the normalization method, we define a dimensionless ratio f_{de}, interpret the physical meaning of f_{de} and its average value by the dimensional analysis, propose an assumption which the HDE average force equates to the negative reduced Planck force or negative Planck force statistically. We obtain the normalizable equations of original HDE model, get the parameter $c_L = 0.535$ which is very close to the upper limit of $c = 0.495 \pm 0.039$ obtained from Planck+WP+BAO+HST+lensing; derive the general equation of normalization of General HDE (GHDE) model; obtain that the coefficient w_{de} is inversely proportional to the square of c_L which is variable; get the normalizable equations of GHDE model, obtain $c_L(z) = 0.471$ when $\Omega_{de}(z) = 0.683$ which is in good agreement with c; give the normalizable equations of agegraphic dark energy (ADE) model and New HDE (NHDE) model; get $n = 2.894$ which agrees well with $n = 2.886^{+0.169}_{-0.163}$ in ADE model and $c_L = 3$ which is in agreement with $1.41 < c < 3.09$ in NHDE model. We suggest that the normalization of some HDE models is interesting and significant.

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1. Introduction

In order to explain the cosmic accelerated expansion [1], numerous theoretical models have been proposed [2]. Thereinto, M. Li's original holographic dark energy (HDE) model is the first in good agreement with the observational data [3]. It has a parameter c, so it is very important for the numerical value of c. Summarize relative literatures recently; there are two ways to calculate c approximately. One is parameterization, please refer to [4] [5] [6]. Another is data fitting, please see [7]. Moreover, M. Li *et al* proposed a new model of HDE (NHDE) with action principle, considered solving the causality problem and the circular logic problem concerning the future event horizon [8]

As a complete theory of HDE, c can be calculated from it, not only being determined through the observation.

The paper is organized as follows. In Sec. 2, we brief introduce the normalization method, define a dimensionless ratio; propose an assumption; obtain the normalizable equations of original HDE [3] model, and calculate the parameter c_L; derive the general equation of normalization of General HDE (GHDE) model [6], obtain its normalizable equations, and calculate $c_L(z)$. In Sec. 3, we get the normalizable equations of the agegraphic dark energy (ADE) model [10] and NHDE model [8]; calculate the numerical factor n in ADE model and c_L in NHDE model. We conclude in Sec. 4.

2. Normalization of Original HDE Model and GHDE Model

In this section, we brief introduce the normalization method, define a dimensionless ratio; interpret the physical meaning of it and its average value, propose an assumption; obtain the normalizable equations of original HDE model, and calculate c_L; derive the general equation of normalization of GHDE model; determine the relation between the coefficient w_{de} and c_L; obtain the normalizable equations of GHDE model, and calculate $c_L(z)$.

2.1. Normalization method, a dimensionless ratio and an assumption

We use the normalization method which makes non-dimensionalization and equals to negative one because the property of coefficient of state of dark energy [2].

The equation of HDE model [3] can be rewritten as (we work with ħ = c = 1 units)

$$\rho_{de} = 3c_L^2 M_{pl}^2 L^{-2} \quad (1)$$

where ρ_{de} is the HDE density, $c_L \geq 0$ is a dimensionless model parameter, $M_{Pl} \equiv 1/\sqrt{8\pi G}$ is the reduced Planck mass and L is the cosmic cutoff. From Eq. (1), using $p = w\rho$, we obtain

$$L^2 p_{de} = w_{de} L^2 \rho_{de} = 3 w_{de} c_L^2 M_{pl}^2 \quad (2)$$

where p_{de} is the negative pressure of HDE, $w_{de} < 0$ is the coefficient of state. From (2), we can define

$$f_{de} = L^2 p_{de} / M_{pl}^2 = 3 w_{de} c_L^2 \quad (3)$$

where f_{de} is a dimensionless ratio.

By the dimensional analysis, we know $[L^2 p_{de}] = [\mathrm{MLT}^{-2}]$, where M, L, and T are the dimension of mass, length and time respectively, so it is the force F_{de} which is produced by the HDE on the cosmic cutoff. We can call it the HDE force. Because f_{de} is dimensionless, $F_{Pl} = c^3 M_{Pl}^2 / \hbar = M_{Pl}^2$ is called the reduced Planck force, $F_P = 8\pi F_{Pl} = M_P^2$ is called the Planck force. The physical meaning of f_{de} is the ratio between the HDE force and reduced Planck force or Planck force. It is possible that $F_{de} \geq F_{Pl}$ (or F_P) or $F_{de} < F_{Pl}$ (or F_P). In order to calculate felicitously, we propose an assumption which the HDE average force $\langle F_{de} \rangle$ equates to the negative reduced Planck force $\langle F_{de} \rangle = -F_{Pl}$ or negative Planck force $\langle F_{de} \rangle = -F_P$ statistically. It is the basis of the normalization of some HDE models.

2.2. Normalizable equations of original HDE model

From the assumption, we obtain

$$\langle f_{de} \rangle = -1 \text{ or } \langle f_{de} \rangle = -1 \quad (4)$$

where $\langle f_{de} \rangle$ is the average ratio of f_{de}. For the original HDE model, c_L is constant. Substituting $w_{de} = -(1/3) - 2\sqrt{\Omega_{de}} / 3c_L$ [3] into (3), where $\Omega_{de} = \rho_{de} / \rho_c$ is the radio of dark energy density, $0 \leq \Omega_{de} \leq 1$, $\rho_c = 3M_{pl}^2 H^2$ is the critical density of the universe, and H is the Hubble constant, we have

$$\langle f_{de} \rangle = \left[\int_0^1 f_{de} \, d\Omega_{de} \right] / (1-0) = -1 \quad (5)$$

The above equation used the mean value formula of the continuous function. Solving Eq. (5) we gain

$$c_L^2 + (4/3) c_L - 1 = 0 \quad (6)$$

It is the normalizable equation of original HDE model. Solving Eq. (6), we obtain

$$c_L = (\sqrt{13} - 2)/3 = 0.535 \quad (7)$$

Distinctly c_L is very close to the upper limit of $c = 0.495 \pm 0.039$ obtained from Planck+WP+BAO+HST+lensing [7].

2.3. Normalizable equations of GHDE model

In general c_L is variable [4] [5] [6]. By the mean value theorem of the double integral, we have

$$\int dw_{de} \int 3 w_{de} c_L^2 dc_L = \langle f_{de} \rangle \int dw_{de} \int dc_L = -\int dw_{de} \int dc_L \quad (8)$$

Because the double integral on both sides is arbitrary, we obtain

$$w_{de} c_L^2 = -1/3 \quad (9)$$

So w_{de} is inversely proportional to the square of c_L, when $c_L = 1$, $w_{de} = -1/3$; $c_L = \sqrt{1/3} = 0.577$, $w_{de} = -1$. This is the general equation of normalization of the GHDE model.

Substituting $c_L = c_L(z)$ and $w_{de} = w_{de}(z) = -(1/3) - 2\sqrt{\Omega_{de}(z)} / 3c_L(z)$ [6] into (9), where z is the redshift, we have

$$c_L(z)^2 + 2 c_L(z) \sqrt{\Omega_{de}(z)} = 1 \quad (10)$$

That is the normalizable equation of z. Solving Eq. (10), we gain

$$c_L(z) = \sqrt{\Omega_{de}(z) + 1} - \sqrt{\Omega_{de}(z)} \quad (11)$$

When $\Omega_{de}(z) = 0.683$, we obtain

$$c_L(z) = 0.471 \quad (12)$$

It is in good agreement with $c = 0.495 \pm 0.039$ [7]. When $\Omega_{de}(z) = 0$, we have $c_L(z) = 1$; $\Omega_{de}(z) = 1$, $c_L(z) = \sqrt{2} - 1 = 0.414$, so $0.414 \leq c_L(z) \leq 1$. We get Fig. 1.

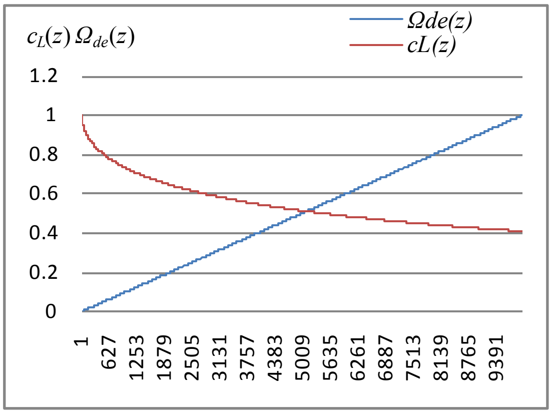

Fig. 1 $c_L(z)$ monotonously decreases when $\Omega_{de}(z)$ monotonously increases, $0.414 \leq c_L(z) \leq 1$.

3. Normalizable Equations of ADE Model and NHDE Model

In this section, we redefine the dimensionless ratio in ADE model and NHDE model respectively; obtain the normalizable equations of them, and calculate n and c_L.

3.1. Normalizable equations of ADE model

For the ADE model, because of $\rho_{de}= 3n^2 M_{pl}^2 t^{-2}$, we can redefine

$$f_{de}= w_{de}\rho_{de} t^2 / 8\pi M_{pl}^2 = 3w_{de} n^2 / 8\pi \qquad (13)$$

Substituting $w_{de} = -1+2\sqrt{\Omega_{de}} / 3na$ [10] into it, where n is a numerical factor, t is the time, and a is the scale factor, we obtain

$$\int da \int 3(-1+2\sqrt{\Omega_{de}} / 3na) n^2 dn / 8\pi = \langle f_{de}\rangle \int da \int dn = -\int da \int dn \qquad (14)$$

From Eq. (14) we gain

$$3n^2 - 2n\sqrt{\Omega_{de}} / a - 8\pi = 0 \qquad (15)$$

This is the normalizable equation of ADE model. Solving it we get

$$n = (\sqrt{\Omega_{de}/a^2 + 24\pi} + \sqrt{\Omega_{de}} / a) / 3 \qquad (16)$$

When $a \to \infty$, $n \to \sqrt{24\pi} / 3 = 2.894$, it agrees well with $n = 2.886^{+0.169}_{-0.163}$ [11].

3.2. Normalizable equations of NHDE model

For the NHDE model, because $\rho_{de}= [(\lambda-\lambda(0)) / 2a^4 - c_L / a^2L^2] / 24\pi G$ [8], we redefine

$$f_{de}= \rho_{de} a^2L^2 / M_{pl}^2 = \lambda L^2 / 6a^2 - c_L / 3, \ (\lambda(0) = 0) \qquad (17)$$

where L is a decreasing function and $\dot{\lambda} = -4\alpha c_L / L^3$, we have

$$\int d\lambda \int dY \int (\lambda Y^2 / 6 - c_L / 3) dc_L = \langle f_{de}\rangle \int d\lambda \int dY \int dc_L = -\int d\lambda \int dY \int dc_L \qquad (18)$$

where $Y = L / a$, From Eq. (18) we gain

$$c_L = 3+\lambda Y^2/2 \qquad (19)$$

To solve c_L we need to know the numerical value of λ and Y. If $\lambda < 0$, $c_L < 3$; $\lambda \geq 0$, $c_L \geq 3$. When $a \to \infty$, $Y \to 0$ [8], $c_L \to 3$, that is in agreement with $1.41 < c < 3.09$ [8].

4. Conclusion

In this paper, we have defined a dimensionless ratio f_{de}; interpreted the physical meaning of it and its average value, proposed an assumption which the HDE average force equates to the negative reduced Planck force or negative Planck force statistically; obtained the normalizable equations of original HDE model [3], got the parameter $c_L = 0.535$ which is very close the upper limit of $c = 0.495 \pm 0.039$ obtained from Planck+WP+BAO+HST+lensing [7]; derived the general equation of normalization of the GHDE model [6], obtained the coefficient w_{de} being inversely proportional to the square of c_L which is variable; gave the normalizable equations of GHDE model, and got $c_L(z) = 0.471$ when $\Omega_{de}(z) = 0.683$ [9], which are in good agreement with c [7]; redefined f_{de} in ADE model [10] and NHDE model [8] respectively, obtained the normalizable equations of them, gave the expression of n in ADE model and one of c_L in NHDE model, obtained $n = 2.894$ which agrees well with $n = 2.886^{+0.169}_{-0.163}$ [11] in ADE model and $c_L = 3$ which is in agreement with $1.41 < c < 3.09$ [8] in NHDE model.

Only the models which their HDE density is inversely proportional to the square of the cosmic cutoff or time can be normalized. So we investigate original HDE model, GHDE model, ADE model and NHDE model simply. Our method can give the better results with others. The problem is that we can't explain why f_{de} is the ratio between the HDE force and reduced Planck force in original HDE model, GHDE model and

NHDE model, but between HDE force and Planck force in ADE model; and can't explain $\langle f_{de} \rangle$ also. We will research them in after work. At last, we suggest that the normalization of some HDE models is interesting and significant.

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