Normalization of Some Holographic Dark Energy Models

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We propose the normalization of some holographic dark energy (HDE) models. Applying the normalization method, we derive the general equation of normalization of original HDE model and General HDE (GHDE) model; obtain that the coefficient w_{de} is inversely proportional to the square of the parameter c_L which is variable; get the normalized equations of original HDE model, GHDE model, agegraphic dark energy (ADE) model and New HDE (NHDE) model; obtain n = 2.894 which is in good agreement with $n = 2.886^{+0.169}_{-0.163}$ in ADE model and $c_{L\mathrm{n}}$= 3 which agrees well with 1.41 < c < 3.09 in NHDE model; and interpret the physical meaning of the ratio f_{de} and its average value by dimensional analysis. We suggest that the normalization of some HDE models is interesting and significant.

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1. Introduction

The holographic dark energy (HDE) model [1] [2] [3] [4] [5] is one of the competitive and promising models to explain the cosmic accelerated expansion [6]. In order to determine the numerical value of the parameter c [1] [2] [3] [4] in it, parameterization and data fitting are applied widely [2] [7] [9]. However the normalization method is used by Y. Bao to the original HDE model and General HDE (GHDE) model [8], the value of parameter c_L is in good agreement with c = 0.495 ± 0.039 obtained from Planck+WP+ BAO+HST+lensing [9]. This is an interesting and significant method, but the physical meaning of the ratio f_{de} and its average value isn't interpreted. In this paper, we apply the same method to some HDE models.

The paper is organized as follows. In Sec. 2, we derive the general equation of normalization of original HDE model [1] and GHDE model [3]; obtain the normalized equations of them. In Sec. 3, we get the normalized equations of the agegraphic dark energy (ADE) model [5] and New HDE (NHDE) model [4]; obtain n = 2.894 in ADE model and $c_{L\mathrm{n}}$= 3 in NHDE model. In Sec. 4, we interpret the physical meaning of f_{de} and its average value $\langle f_{de} \rangle$. We conclude in Sec. 5.

2. Normalization of Original HDE Model and GHDE Model

In this section, we review [8]; derive the general equation of normalization of original HDE model and GHDE model by the normalization method; determine the relation between the coefficient w_{de} and parameter c_L; and obtain the normalized equations of original HDE model and GHDE model.

2.1. General equation

First let us briefly review [8]. The equation of HDE model [1] [2] [3] can be rewritten as (we work with ħ = c = 1 units)

$$\rho_{de} = 3c_L^2 \mathrm{M}_{\mathrm{pl}}^2 L^{-2} \tag{1}$$

where ρ_{de} is the HDE density, $c_L \geq 0$ is a dimensionless model parameter, $\mathrm{M}_{\mathrm{Pl}} \equiv 1 / \sqrt{8\pi \mathrm{G}}$ is the reduced Planck mass and L is the cosmic cutoff. From Eq. (1), using $p = w\rho$, we obtain

$$L^2 p_{de} = w_{de} L^2 \rho_{de} = 3 w_{de} c_L^2 \mathrm{M}_{\mathrm{pl}}^2 \tag{2}$$

where p_{de} is the negative pressure of HDE, $w_{de} < 0$ is the coefficient of state. From (2), we can define

$$f_{de} = L^2 p_{de} / \mathrm{M}_{\mathrm{pl}}^2 = 3 w_{de} c_L^2 \tag{3}$$

where f_{de} is a ratio.

Applying the normalization method, we can define

$$\langle f_{de} \rangle = | \, (\sum_{i=1}^{n} f_{de\,i}) \, / \, n \, | \, , \; i = 1, 2, 3 \ldots n \qquad (4)$$

where $\langle f_{de} \rangle$ is the average ratio, $f_{de\,i}$ is the ratio of each halo. In order to calculate properly, $\langle f_{de} \rangle$ equals to unity [8]

$$\langle f_{de} \rangle = | \, (\sum_{i=1}^{n} f_{de\,i}) \, / \, n \, | = 1 \qquad (5)$$

Using (3) to (5), we obtain

$$\sum_{i=1}^{n} w_{dei} c_{Li}^2 = - n / 3 \qquad (6)$$

where $w_{de\,i}$ is the coefficient of state of each halo, c_{Li} is the dimensionless model parameter of each halo. This is the general equation of normalization of original HDE model and GHDE model.

2.2. Normalized equations of original HDE model

For the original HDE model, $c_{Li} = c_L$ is constant, we have

$$\sum_{i=1}^{n} w_{dei} = - n / 3c_L^2 \qquad (7)$$

Substituting $w_{de\,i} = -(1/3) - 2\sqrt{\Omega_{de\,i}} / 3c_L$ [1] into (7), where $\Omega_{de\,i}$ is the fractional dark energy density of each halo, we obtain

$$c_L^2 + [(2\, c_L / n) \sum_{i=1}^{n} \sqrt{\Omega_{dei}} \,] = 1 \qquad (8)$$

It is the normalized equation in [8].

2.3. Relation between w_{den} and c_{Ln}

In general c_{Li} is variable [3], we need to solve the Eq. (6) by the similar subtraction in [8]. When n = 1, we have

$$w_{de\,1} c_{L1}^2 = - 1 / 3 \qquad (9)$$

Then Eq. (6) can be rewritten as

$$w_{de\,n} c_{Ln}^2 + \sum_{i=1}^{n-1} w_{dei} c_{Li}^2 = - n / 3 \qquad (10)$$

When i =1, 2, 3… n – 1, we have

$$\sum_{i=1}^{n-1} w_{dei} c_{Li}^2 = - (n - 1) / 3, \; n \geq 2 \qquad (11)$$

Taking (11) to (10), we obtain

$$w_{de\,n} c_{Ln}^2 = -1 / 3 \qquad (12)$$

So w_{den} is inversely proportional to the square of c_{Ln}, when $c_{Ln} = 1$, $w_{den} = -1/3$; $c_{Ln} = \sqrt{1/3} = 0.577$, $w_{den} = -1$.

2.4. Normalized equations of GHDE model

For the GHDE model, substituting $c_{Ln} = c_L(z)$ and $w_{de\,n} = w_{de}(z) = -(1/3) - 2\sqrt{\Omega_{de\,n}(z)} / 3c_L(z)$ [3] into (12), where z is the redshift, we have

$$c_L(z)^2 + 2\, c_L(z) \sqrt{\Omega_{de}(z)} = 1 \qquad (13)$$

That is the normalized equation of z in [8].

3. Normalized Equations of ADE Model and NHDE Model

In this section, we redefine the ratio f_{de} in ADE model and NHDE model respectively; obtain the normalized equations of them.

3.1. Normalized equations of ADE model

For the ADE model, because of $\rho_{de} = 3n^2 M_{pl}^2 t^{-2}$, we can redefine

$$f_{de} = w_{de} \rho_{de} t^2 / 8\pi M_{pl}^2 = 3 w_{de} n^2 / 8\pi \qquad (14)$$

Substituting $w_{de} = -1 + 2\sqrt{\Omega_{de}} / 3na$ [5] into it, where n is a numerical factor, t is the time, a is the scale factor, $\Omega_{de} = \rho_{de} / \rho_c$ is the fractional dark energy density, $\rho_c = 3M_{pl}^2 H^2$ is the critical density of the universe, and H is the Hubble constant, we obtain

$$\langle f_{de} \rangle = | \, (1/n) \sum_{i=1}^{n} (3 w_{de}\, n^2 / 8\pi) \, | = 1 \qquad (15)$$

Solving (15) we have

$$3n^2 - 2n\sqrt{\Omega_{de}} / a - 8\pi = 0 \qquad (16)$$

This is the normalized equation of ADE model. Solving it we obtain

$$n = (\sqrt{\Omega_{de}/a^2 + 24\pi} + \sqrt{\Omega_{de}} / a) / 3 \qquad (17)$$

When $a \to \infty$, $n \to \sqrt{24\pi} / 3 = 2.894$, it is in good agreement with $n = 2.886^{+0.169}_{-0.163}$ [10].

3.2. Normalized equations of NHDE model

For the NHDE model, because $p_{de} = [\lambda - \lambda(0)] / 48\pi a^4 - c_L / 24\pi a^2 L^2$ [4], we redefine

$$f_{de} = p_{de} a^2 L^2 / M_{pl}^2 = \lambda L^2 / 6a^2 - c_L / 3, \; (\lambda(0) = 0) \qquad (18)$$

where L is a decreasing function and $\dot{\lambda} = -4\alpha c_L / L^3$, we have

$$\langle f_{de} \rangle = | \, (1/n) \sum_{i=1}^{n} [\lambda_i Y^2/6 - c_{Li}/3] \, | = 1 \qquad (19)$$

where $Y = L / a$, from (19), we obtain

$$\sum_{i=1}^{n} [\lambda_i Y^2/2 - c_{Li}] = - 3n \qquad (20)$$

It is the normalized equation of NHDE model. Solving Eq. (20) we have

$$c_{Ln} = 3 + \lambda_n Y^2/2 \qquad (21)$$

To solve c_{Ln} we need to know the numerical value of λ_n and Y. If $\lambda_n < 0$, $c_{Ln} < 3$; $\lambda_n \geq 0$, $c_{Ln} \geq 3$. When $a \to \infty$, $Y \to 0$, $c_{Ln} \to 3$, that agrees well with $1.41 < c < 3.09$ [7].

4. Physical Meaning of f_{de} and Its Average Value

In this section, we interpret the physical meaning of the ratio f_{de} and its average value by dimensional analysis.

Clearly f_{de} is the key. From (3), (14) and (18), the definability is different. So what is the physical meaning of f_{de} and $\langle f_{de} \rangle$? By the dimensional analysis, we know $[L^2 p_{de}]$ = $[w_{de} \rho_{de} t^2]$ = $[p_{de} a^2 L^2]$ = $[\mathrm{MLT}^{-2}]$, where M, L, and T are the dimension of mass, dimension of length and dimension of time respectively, so it is the force F_{de} which is produced by the HDE on the cosmic cutoff. We can call it the HDE force. Because f_{de} is dimensionless, $\mathrm{F_{Pl}} = \mathrm{c}^3 \mathrm{M_{Pl}^2} / \hbar = \mathrm{M_{Pl}^2}$ is called the reduced Planck force. The physical meaning of f_{de} is the ratio between the HDE force and reduced Planck force. In ADE model, $\mathrm{F_P} = 8\pi \mathrm{c}^3 \mathrm{M_{Pl}^2} / \hbar = 8\pi \mathrm{M_{Pl}^2} = \mathrm{M_P^2}$ is called the Planck force, where $\mathrm{M_P} \equiv 1/\sqrt{\mathrm{G}}$ is the Planck mass, so f_{de} is the ratio between HDE force and Planck force. The physical meaning of $\langle f_{de} \rangle$ is that the HDE average force $\langle F_{de} \rangle$ is assumed to equate to the negative reduced Planck force or negative Planck one statistically. This is the basis for its normalization.

5. Conclusion

In this paper, we have derived the general equation of normalization of original HDE model and GHDE model; obtained the coefficient w_{de} being inversely proportional to the square of the parameter c_L which is variable; got the normalized equations of original HDE model and GHDE model; redefined the ratio f_{de} in ADE model and NHDE model respectively; obtained the normalized equations of them; gave the expression of n in ADE model and one of c_{Ln} in NHDE model; obtain n = 2.894 which is in good agreement with $n = 2.886^{+0.169}_{-0.163}$ [10] in ADE model and c_{Ln}= 3 which agrees well with $1.41 < c < 3.09$ [7] in NHDE model; and interpreted the physical meaning of f_{de} and its average value $\langle f_{de} \rangle$.

Non-all HDE models can be normalized; only the models which their HDE density is inversely proportional to the square of the cosmic cutoff or time can be. So we investigate original HDE model, GHDE model, ADE model and NHDE model only. Our method can give the better results with data-fitting. The problem is that we can't explain why f_{de} is the ratio between the HDE force and reduced Planck force in original HDE model, GHDE model and NHDE model, but between HDE force and Planck force in ADE model; and can't explain $\langle f_{de} \rangle$ also. We will research them in after work. At last, we suggest that the normalization of some HDE models is interesting and significant.

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