Electro-Gravity Via Geometric Chronon Field

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 there are small corrections in (35) and in (B.1),(B.3),(B.7). Addition of Muon/Electron mass ratio in (B.18) – page 35, W bosn to Tau in (B.22),(B.25) page 36, addition of appendix C. (27)-(37) pages 39-41, breakthrough in the Fine Structure Constant

Abstract. In De Sitter / Anti De Sitter space-time and in other geometries, reference sub-manifolds from which proper time is measured along integral curves, are described as events. We introduce here a foliation with the help of a scalar field. The scalar field need not be unique but from the gradient of the scalar field, an intrinsic Reeb vector of the foliations perpendicular to the gradient vector is calculated. The Reeb vector describes the acceleration of a physical particle that moves along the integral curves that are formed by the gradient of the scalar field. The Reeb vector appears as a component of an anti-symmetric matrix which is a part of a rank-2, 2-Form. The 2-form is extended into a non-degenerate 4-form and into rank-4 matrix of a 2-form, which when multiplied by a velocity of a particle, becomes the acceleration of the particle. The matrix has one U(1) degree of freedom and an additional SU(2) degrees of freedom in two vectors that span the plane perpendicular to the gradient of the scalar field and to the Reeb vector. In total, there are U(1) x SU(2) degrees of freedom. SU(3) degrees of freedom arise from three dimensional foliations but require an additional symmetry to exist in order to have a valid covariant meaning. The model aims at Causal Sets, that when not aligned along geodesic curves, force material clocks of different types, not to move geodetically, thus meaning forces and matter. This paper mostly deals with U(1) type clocks but also discusses SU(2) and SU(3) and in a more detailed way in appendix C.

Matter in the Einstein Grossmann equation is replaced by the action of the acceleration field, i.e. by a geometric action which is not anticipated by the metric alone. This idea leads to a new formalism that replaces the conventional stress-energy-momentum-tensor. The formalism will be mainly developed for classical physics but will also be discussed for quantized physics based on events instead of particles. The result is that a positive charge manifests small attracting gravity and a stronger but small repelling acceleration field that repels even uncharged particles that have a rest mass. Negative charge manifests a repelling anti-gravity but also a stronger acceleration field that attracts even uncharged particles that have rest mass.

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1. Introduction

The motivation of this theory is to show that matter can be put into correspondence with an acceleration field. There are two ways that measurement of proper time by a physical clock between events will be shortened: either by gravity in which the clock moves along geodesic curves but in curved space-time or by other interactions that prevent the clock from moving along a geodesic curve. These two approaches have to appear in the equation of gravity in order to describe Nature by a fully geometric model. The latter is not anticipated by the metric tensor alone and therefore it requires a new approach.

In this work, we will study the gradient of a real scalar field P which is $P_i \equiv \frac{dP}{dx^i}$. If a physical particle moves along the integral curves that are formed by P_i then its velocity is $\frac{V^i}{c} = \frac{P^i}{\sqrt{P^s P_s}} \Rightarrow V^i V_i = c^2$ where c is the speed of light. For convenience, throughout the paper we will use the notation $Z \equiv N^2 \equiv \|P_s\|^2 = P^s P_s$. The next step is to calculate the acceleration of such a particle, based on P_i. Taking the exterior derivative of $\frac{P_i}{\sqrt{Z}} dx^i$ as a 1-form, we will derive a 2-Form, while we continue with differential geometry conventions, comma as derivative and semi-colon as covariant derivative, $Z,_i \equiv Z_i \equiv \frac{dZ}{dx^i}$ and for a vector field V_k, $V_k,_i \equiv \frac{dV_k}{dx^i}$ where x^i are the contravariant coordinates. Also, the covariant derivatives are defined as in differential geometry, $V_k;_i \equiv V_k,_i - \Gamma^s{}_{ki} V_s$ and $V^k;_i \equiv V^k,_i + \Gamma^k{}_{si} V^s$ where $\Gamma^k{}_{si}$ are the affine connection, also known as second-type Christoffel symbols. We derive,

$$d\frac{P_i}{\sqrt{Z}} dx^i = (\frac{P_i}{\sqrt{Z}}),_j dx^i \wedge dx^j = ((\frac{P_i}{\sqrt{Z}}),_j - (\frac{P_j}{\sqrt{Z}},_i))dx^i \wedge dx^j \Rightarrow$$

$$(\frac{P_i}{\sqrt{Z}}),_j - (\frac{P_j}{\sqrt{Z}},_i) = (\frac{P_i,_j}{\sqrt{Z}} - \frac{P_i Z_j}{2Z^{3/2}}) - (\frac{P_j,_i}{\sqrt{Z}} - \frac{P_j Z_i}{2Z^{3/2}}) = \frac{P_j Z_i}{2Z^{3/2}} - \frac{P_i Z_j}{2Z^{3/2}} \tag{1}$$

From which

$$d(\frac{P_i}{\sqrt{Z}})dx^i = (\frac{P_j z_i}{2Z^{3/2}} - \frac{P_i z_j}{2Z^{3/2}})dx^i \wedge dx^j = A_{ij} dx^i \wedge dx^j = -A_{ji} dx^i \wedge dx^j$$

We now contract this anti-symmetric matrix with our original vector,

$$\frac{U_i}{2} \equiv A_{ij} \frac{P^j}{\sqrt{Z}} = \frac{P_j Z_i}{2Z^{3/2}} \frac{P^j}{\sqrt{Z}} - \frac{P_i Z_j}{2Z^{3/2}} \frac{P^j}{\sqrt{Z}} = \frac{Z_i}{2Z} - \frac{Z_j P^j}{2Z^2} P_i \tag{2}$$

It is immediately evident that the vector $\frac{U_i}{2}$ is perpendicular to $\frac{P_i}{\sqrt{Z}}$ both from

$$\frac{Z_i}{2Z} \frac{P^i}{\sqrt{Z}} - \frac{Z_j P^j}{2Z^2} P_i \frac{P^i}{\sqrt{Z}} = \frac{Z_i P^i}{2Z^{3/2}} - \frac{Z_j P^j}{2Z^{3/2}} = 0$$

and from a contraction of an anti-symmetric matrix A_{ij} by $\frac{P^i}{\sqrt{Z}}\frac{P^j}{\sqrt{Z}}=0$.

Physical meaning: A_{ij} transforms the vector $\frac{P_i}{\sqrt{Z}}$ to $\frac{U_i}{2}$ as a rotation and scaling transformation and is therefore, of rank 2. It can be extended to a non-degenerate matrix of rank 4, $\tilde{A}_{ij}$ which defines a field of acceleration, i.e. $\tilde{A}_{ij}\frac{V^j}{c}=\frac{a_i}{c^2}=g_{ij}\frac{1}{c^2}\frac{dV^j}{d\tau}$ where a_i is the covariant acceleration of the mass that interacts with the field, c is the speed of light, τ is proper time and g_{ij} is the metric tensor. The acceleration matrix $\tilde{A}_{ij}$ will be discussed later and will appear as the sum of two matrices $A_{ij}+B_{ij}$. If the A_{ij} and B_{ij} are real then $Det(A_{ij}+B_{ij})=\pm Det(A_{ij}+\tilde{B}_{ij})$ for other choices of $\tilde{B}_{ij}\neq B_{ij}$.

$$A_{ij}=\frac{U_i}{2}\frac{P_j}{\sqrt{Z}}-\frac{U_j}{2}\frac{P_i}{\sqrt{Z}}=(\frac{Z_i}{2Z}\frac{P_j}{\sqrt{Z}}-\frac{Z_kP^kP_i}{2Z^2}\frac{P_j}{\sqrt{Z}})-(\frac{Z_j}{2Z}\frac{P_i}{\sqrt{Z}}-\frac{Z_kP^kP_j}{2Z^2}\frac{P_i}{\sqrt{Z}})= \\ \frac{Z_i}{2Z}\frac{P_j}{\sqrt{Z}}-\frac{Z_j}{2Z}\frac{P_i}{\sqrt{Z}} \qquad (3)$$

We identify this representation with foliation theory,(Reeb, 1948, 1952 [1] and Godbillon-Vey, 1971, [2],[3]). We can write $\omega\equiv\frac{P_i}{\sqrt{Z}}$ and $\eta\equiv\frac{U_i}{2}$ and we reach the Reeb representation $d\omega=\eta\wedge\omega$ where $\eta\equiv\frac{U_i}{2}$ is known as the Reeb vector [1] of the foliation that is perpendicular to the 1-Form ω. The representation of the vector η leads to a far simpler term than the one represented by the Reinhart-Wood formula [2]. For this cohomology class, the following holds,

$$\omega\wedge d\omega=\frac{P_k}{\sqrt{Z}}(\frac{P_jz_i}{2Z^{3/2}}-\frac{P_iz_j}{2Z^{3/2}})dx^k\wedge dx^i\wedge dx^j=(\frac{P_jP_kz_i}{2Z^2}-\frac{P_iP_kz_j}{2Z^2})dx^k\wedge dx^i\wedge dx^j=0$$

The Godbillon-Vey 3-Form, of the foliation F that is perpendicular to the 1-Form ω is defined as $GV(F)=\eta\wedge d\eta$. Its De Rham Cohomology class is $[\eta\wedge d\eta]\in H^3(M,R)$ where here R denotes the real numbers. This cohomology class is an invariant of the foliation F and is a closed 3-Form. An interesting property of the Reeb vector is that its restriction to the foliation F integrates to 0 on each closed curve on F.

A generalization of (2) to the complex numbers is easily defined

$$\frac{U_i}{2}\equiv A_{ij}\frac{P^{*j}}{\sqrt{Z}}=\frac{Z_i}{2Z}-\frac{Z_jP^{*j}}{2Z^2}P_i \qquad (4)$$

And $Z=(P_iP^{*i}+P^*_iP^i)/2$ and because the metric tensor is symmetric and real, we can write $Z=P_iP^{*i}$. An interesting way to reach the Reeb vector $\frac{U_i}{2}$ is by the Lie derivative [4]

$$Lie(\frac{P*^m}{\sqrt{Z}},\frac{P_i}{\sqrt{Z}}) = \frac{P*^m}{\sqrt{Z}}(\frac{P_i}{\sqrt{Z}}),_m + (\frac{P*^m}{\sqrt{Z}}),_i \frac{P_m}{\sqrt{Z}} \tag{5}$$

In which the second term is positive because the differentiated $\frac{P_i}{\sqrt{Z}}$ vector has a low index.

The first term becomes,

$$\frac{P*^m}{\sqrt{Z}}(\frac{P_i}{\sqrt{Z}}),_m = \frac{P*^m P_i,_m}{Z} - \frac{P*^m}{\sqrt{Z}}\frac{P_i Z_m}{2Z^{3/2}} = \frac{P*^m P_i,_m}{Z} - \frac{Z_m P*^m P_i}{2Z^2} \tag{6}$$

The second term is,

$$(\frac{P*^m}{\sqrt{Z}}),_i \frac{P_m}{\sqrt{Z}} = \frac{P*^m,_i P_m}{Z} - \frac{P*^m P_m Z_i}{2Z^2} = \frac{P*^m,_i P_m}{Z} - \frac{Z_i}{2Z} \tag{7}$$

We add (6) and (7) to get (5) and notice that $\frac{P*^m P_i,_m}{Z} + \frac{P*^m,_i P_m}{Z} = \frac{P*^m P_m,_i}{Z} + \frac{P*^m,_i P_m}{Z}$ from which (5) becomes

$$Lie(\frac{P*^m}{\sqrt{Z}},\frac{P_i}{\sqrt{Z}}) = \frac{Z_i}{Z} - \frac{Z_i}{2Z} - \frac{Z_m P*^m P_i}{2Z^2} = \frac{Z_i}{2Z} - \frac{Z_m P*^m P_i}{2Z^2} = \frac{U_i}{2} \tag{8}$$

It is again the Reeb vector. It is important to say that the foliation F is covariant because its tangent vectors $T(F)$ consist of vectors which are perpendicular to $\omega \equiv \frac{P_i}{\sqrt{Z}}$ and orthogonality of vectors is invariant under local change of coordinates. The question that we should ask now, is how is the Reeb vector $\frac{U_i}{2}$ related to the curvature of the integral curves which are generated by ω? First of all, we have to notice that $Z = P_i P*^i$ is not constant and therefore $Z_k \equiv \frac{dZ}{dx^k} \neq 0$ unlike the case of a velocity of a particle $V_i V^i = c^2$. The squared curvature of the integral curves that are generated by P_i is expressible, according to differential geometry, by the measurement of how much the unit vector $\frac{P_i}{\sqrt{Z}}$ changes along an arc length parameterization t of the integral curves. Calculation of the second power of trajectory curvature of integral curves along a conserving field, can be left as an exercise to the reader but the author prefers to present its calculation. This calculation is valid for all integral curves that are generated by vector fields that are scalar gradients. In our case, the integral curves should not be geodesic if they pass through material fields.

Caution: The t parameterization may not be the time measured by any physical particle because the scalar field from which the vector field is derived may be the result of an intersection of multiple trajectories along which P is measured. However, a particle that follows the gradient curves will indeed measure t even if its trajectory is not geodesic. Let t be the arc length measured along the curves formed by the vector field P_μ. By differential geometry, we know that the second power of curvature along these curves is simply

$$Curv^2 = \frac{d}{dt}\frac{P_\lambda}{\sqrt{P_k P^k}}\frac{d}{dt}\frac{P_\mu}{\sqrt{P_k P^k}}g^{\lambda\mu} \tag{9}$$

such that $g^{\lambda\mu}$ is the metric tensor. (9) is an excellent candidate for an action operator. For convenience, we will write $Norm \equiv \sqrt{P^k P_k}$ and $\dot{P}_\lambda \equiv \frac{d}{dt}P_\lambda$. For the arc length parameter t. Here is the main trick, as was mentioned about $Z = Norm^2$, $Norm$ may not be constant because P_λ is not the 4-velocity of any particle, (to see an example of changing $Norm$, see "APPENDIX – The time field in the Schwarzschild solution"), An arc length parameterization along these curves is equivalent to proper time measured by a particle that moves along the curves, and in the real numbers case, P can be indeed time. Unlike velocity's squared norms, Z is not constant.

Let W_λ denote: $W_\lambda = \frac{d}{dt}(\frac{P_\lambda}{\sqrt{P_k P_k}}) = \frac{\dot{P}_\lambda}{Norm} - \frac{P_\lambda}{Norm^3}P_k\dot{P}_\nu g^{k\nu}$

Obviously

$$W_\lambda P_k g^{\lambda k} = \frac{\dot{P}_\lambda P_k g^{\lambda k}}{Norm} - \frac{P_\lambda P_s g^{\lambda s}}{Norm^3}P_k\dot{P}_\nu g^{k\nu} = \frac{\dot{P}_\lambda P_k g^{\lambda k}}{Norm} - \frac{P_k\dot{P}_\nu g^{k\nu}}{Norm} = 0$$

Thus

$$Curv^2 = W_\lambda W^\lambda = \frac{\dot{P}_\lambda \dot{P}_\nu g^{\lambda\nu}}{Norm^2} - \frac{P_\lambda \dot{P}_s g^{\lambda s}}{Norm^4}P_k\dot{P}_\nu g^{k\nu} = \frac{\dot{P}_\lambda \dot{P}^\lambda}{Norm^2} - (\frac{P_\lambda \dot{P}^\lambda}{Norm^2})^2$$

Following the curves formed by $P_\lambda = P,_\lambda = \frac{dP}{dx^\lambda}$, the term $\frac{dx^r}{dt} = \frac{P_\lambda}{Norm}$ is the derivative of the normalized curve or normalized "velocity", using the upper Christoffel symbols, $P_\lambda;_r \equiv \frac{d}{dx^r}P_\lambda - P_s\Gamma^s_{\lambda r}$.

$\frac{d}{dt}P_\lambda = (\frac{d}{dx^r}P_\lambda - P_s\Gamma^s_{\lambda r})\frac{dx^r}{dt} = (P_\lambda;_r)\frac{P^r}{Norm}$ such that x^r denotes the local coordinates. If P_λ is a conserving field, then $P_\lambda;_r = P_r;_\lambda$ and thus $P_\lambda,_r P^r = \frac{1}{2}Norm^2,_\lambda$ and

$$Curv^2 = \frac{\dot{P}_\lambda \dot{P}^\lambda}{Norm^2} - (\frac{P_\lambda \dot{P}^\lambda}{Norm^2})^2 = \frac{1}{4}(\frac{Norm^2,_\lambda Norm^2,_k g^{\lambda k}}{Norm^4} - (\frac{Norm^2,_s P_r g^{sr}}{Norm^3})^2)$$

In the real case, we have achieved the Reeb vector,

$$U_m = \frac{(\mathrm{P}^\lambda \mathrm{P}_\lambda),_\mathrm{m}}{\mathrm{P}^\mathrm{i}\mathrm{P}_\mathrm{i}} - \frac{(\mathrm{P}^\lambda \mathrm{P}_\lambda),_\mu \mathrm{P}^\mu}{(\mathrm{P}^\mathrm{i}\mathrm{P}_\mathrm{i})^2}P_m = \frac{Z_\mathrm{m}}{Z} - \frac{Z_\mu \mathrm{P}^\mu}{Z^2}P_m \tag{10}$$

And our candidate for a trajectory curvature action

$Action = \frac{1}{4}U_m U^m$ where in the complex case

$$Action = \frac{1}{8}(U*_m U^m + U_m U*^m) \tag{11}$$

Non-geodesic motion, as a result of interaction with a field, is not a geodesic motion in a gravitational field, i.e. it is not free fall. Moreover, material fields by this interpretation prohibit geodesic motion curves of particles moving at speeds less than the speed of light and by this, reduce the measurement of proper time. We return to the idea of acceleration by material fields.

We recall the work of Tzvi Scarr and of Yaakov Friedman [5] which used an anti-symmetric matrix to map a 4-velocity vector V^μ to a 4-acceleration vector a_ν. Since (2),

$$\frac{U_\nu}{2} = \frac{a_\nu}{c^2} = A_{\mu\nu}\frac{V^\mu}{c} \tag{12}$$

such that c is the speed of light, where V^μ is the 4-velocity of a material frame and $A_{\mu\nu}$ is the Scarr-Friedman matrix [5]. The known relation $a_\nu V^\nu = 0$ is obvious.

The real valued action above (11), will lead to a very different energy momentum tensor than that of a simple real valued scalar Klein Gordon energy momentum tensor, instead of $\frac{\hbar^2}{m}(2P_\mu P_\nu - P^\lambda P_\lambda g_{\mu\nu}) - mc^2P^2 g_{\mu\nu}$ we will see $\frac{c^4}{8\pi K}\frac{1}{4}(U_\mu U_\nu - \frac{1}{2}g_{\mu\nu}U_\lambda U^\lambda - 2U^k;_k \frac{P_\mu P_\nu}{P_s P^s})$ where $U_\mu = \frac{Z_\mu}{Z} - \frac{Z_k P^k}{Z_2}P_\mu$ and where $Z_\mu = dZ/dx^\mu$ and $Z = P_s P^s$ and $P_\mu = dP/dx^\mu$. The term $2U^k;_k \frac{P_\mu P_\nu}{P_s P^s}$ was not expected by the author. In almost flat geometry, considering $|U_0| << Max\{|U_1|,|U_2|,|U_3|\}$ leads to the interpretation of electric charge up to multiplication by a constant. Another perhaps more illuminating way is to look at the action $\tilde{F}_{\mu\nu} = \frac{1}{4}(A_{\mu\nu} + B_{\mu\nu})$ where $B_{\mu\nu}$ is the anti-symmetric matrix that was mentioned just before (3) and $\frac{U_\mu U^\mu}{4}\sqrt{-g} = \frac{\tilde{F}_{\mu\nu}\tilde{F}^{\mu\nu}}{4}\sqrt{-g}$ such that $\sqrt{-g}$ is the volume coefficient term that appears in General Relativity. Unlike in the classical covariant electro-magnetic action, there is no mixed 4-current J^μ and vector potential A_μ component $A_\mu J^\mu\sqrt{-g}$ since such a term is redundant due to the unexpected $\frac{1}{4}(-2U^k;_k \frac{P_\mu P_\nu}{P_s P^s})$ term that will appear in the Euler Lagrange equations of the Lagrangian

$$\frac{U_\mu U^\mu}{4}\sqrt{-g} = \frac{\tilde{F}_{\mu\nu}\tilde{F}^{\mu\nu}}{4}\sqrt{-g}\ .$$

To summarize:

$$\frac{c^4}{8\pi K}\frac{\tilde{F}_{\mu\nu}\tilde{F}^{\mu\nu}}{4}\sqrt{-g} \neq -(\frac{F_{\mu\nu}F^{\mu\nu}}{4\mu_0} + A_\mu J^\mu)\sqrt{-g}$$

where μ_0 is the permeability constant of vacuum, $F_{\mu\nu}$ is the electro-magnetic tensor.

2. SU(2) X U(1) symmetries – partially symplectic space-time

There is, however, a problem with $A_{\mu\nu}$. There is a degree of freedom in the matrix $A_{\mu\nu}$ which is defined by two vectors, $\frac{P_i}{\sqrt{Z}}$ and by $\frac{U_i}{2}$. That means two additional vectors can be defined in order to express acceleration in the plane which is perpendicular to the local plane spanned by $\frac{P_i}{\sqrt{Z}}$ and $\frac{U_i}{2}$.

We continue with the TzviScarr and Yaakov Friedman acceleration representation matrix [5] and for simplicity, we restrict our discussion to the real case. $A_{\mu\nu}$ is singular and we can easily define a matrix that rotates vectors in a plane perpendicular to both U_μ and to P_ν in order to extend $A_{\mu\nu}$ to a regular matrix by adding to $A_{\mu\nu}$ a second singular matrix, denoted by $B_{\mu\nu}$. That is the matrix

$$B^{\alpha\beta} \equiv \frac{1}{\sqrt{2}} \varepsilon^{\mu\nu\alpha\beta} A_{\mu\nu} \tag{13}$$

where $\varepsilon^{\mu\nu\alpha\beta}$ is the Levi-Civita tensor (not symbol as the Levi-Civita symbol is a tensor density and not a tensor). It is easily verified that

$$(A^{\alpha\beta} + B^{\alpha\beta})(A_{\alpha\beta} + B_{\alpha\beta}) = A^{\alpha\beta} A_{\alpha\beta} + B^{\alpha\beta} B_{\alpha\beta}$$

and also

$$B^{\alpha\beta} B_{\alpha\beta} = \frac{1}{2} \varepsilon^{\mu\nu\alpha\beta} A_{\mu\nu} \varepsilon_{ij\alpha\beta} A^{ij} = \frac{1}{2} \varepsilon^{\mu\nu}{}_{ij} A_{\mu\nu} A^{ij} = \frac{1}{2} (\delta_i^\mu \delta_j^\nu - \delta_j^\mu \delta_i^\nu) A^{ij} A_{\mu\nu} =$$

$$\frac{1}{2} (A^{\mu\nu} A_{\mu\nu} - A^{\nu\mu} A_{\mu\nu}) = A^{\mu\nu} A_{\mu\nu}$$

Therefore $(A^{\alpha\beta} + B^{\alpha\beta})(A_{\alpha\beta} + B_{\alpha\beta}) = A^{\alpha\beta} A_{\alpha\beta} + B^{\alpha\beta} B_{\alpha\beta} = 2A^{\mu\nu} A_{\mu\nu}$ where δ_i^j is the Kronecker delta.In the real numbers case, there are two ways to extend $A_{\alpha\beta}$ to a regular matrix and to keep the norm of the acceleration vector after the extended matrix is multiplied by vectors perpendicular to both $\omega = \frac{P_\mu}{\sqrt{Z}}$ and to U_μ. These matrices are,

$$A_{\alpha\beta} + B_{\alpha\beta} \text{ and } A_{\alpha\beta} - B_{\alpha\beta} \tag{14}$$

and it is easy to see that $(A_{\alpha\beta} + B_{\alpha\beta})(A^{\alpha\beta} - B^{\alpha\beta}) = 0$ (14) is the matrix we have been looking for and it also results in an immediate degree of freedom in the representation of the acceleration matrix by two additional vectors to ω and U_μ but **not** in the matrix itself. (14) is quite similar to Dirac matrices but unlike them, it describes two acceleration planes and not a bi-spinor [6]. In particular, $\tilde{\gamma}_\alpha{}^\mu$ rotations in $SU(4)$, see [7], that do not affect $A_{\alpha\beta}$ may be applied to $B_{\alpha\beta}$. These rotations are in $SU(2)xU(1)$, $B_{\alpha\beta} = \pm\tilde{\gamma}_\alpha{}^\mu B_{\mu\nu} \tilde{\gamma}_\beta{}^\nu$ and $A_{\alpha\beta} = \tilde{\gamma}_\alpha{}^\mu A_{\mu\nu} \tilde{\gamma}_\beta{}^\nu$. There is no $SU(2)$

degree of freedom in $B_{\alpha\beta}$ itself but only in its representation vectors, i.e., the normalized gradient of a scalar and its Reeb vector. As was suggested in (14), the singular $A_{\alpha\beta}$ acceleration matrix is replaced with $A_{\alpha\beta} \to A_{\alpha\beta} + \tilde{\gamma}_\alpha{}^\mu B_{\mu\nu}\tilde{\gamma}_\beta{}^\nu$

In the complex case, consider $e^{i\theta}B_{\alpha\beta}$ instead of $B_{\alpha\beta}$. $e^{i\theta}$ is a $U(1)$ member at angle θ.

2.1. Partially Symplectic space-time

For space-time to be symplectic, it is enough to show that for $A_{ij} = \frac{Z_i}{2Z}\frac{P_j}{\sqrt{Z}} - \frac{Z_j}{2Z}\frac{P_i}{\sqrt{Z}}$ there exists a $B_{ij} = \frac{W_i}{2W}\frac{Q_j}{\sqrt{W}} - \frac{W_j}{2W}\frac{Q_i}{\sqrt{W}}$ such that Q is a scalar function $W = Q_k Q^k$ such that $Q_k = dQ/dx^k$ is the K[th] derivative of Q, and $W_j = dW/dx^j$ with local coordinates x^s and the following holds: $P_iQ^i = P_iW^i = Z_iQ^i = Z_iW^i = 0$ or in other words, the two forms A_{ij} and B_{ij} define transformations in perpendicular planes of the tangent space of space-time.

The result is that the following form is not zero $\frac{Z_i}{2Z}\frac{P_j}{\sqrt{Z}}\frac{W_s}{2W}\frac{Q_t}{\sqrt{W}}dx^i \wedge dx^j \wedge dx^s \wedge dx^t \neq 0$.

We define $\beta = (\frac{Z_i}{2Z}\frac{P_j}{\sqrt{Z}} + \frac{W_i}{2W}\frac{Q_j}{\sqrt{W}})dx^i \wedge dx^j$. It is immediate that β is the exterior derivative of the 1-Form $\omega = (\frac{P_i}{\sqrt{Z}} + \frac{Q_i}{\sqrt{W}})dx^i$ and that β is a closed form and obviously $\beta \wedge \beta = 2\frac{Z_i}{2Z}\frac{P_j}{\sqrt{Z}}\frac{W_s}{2W}\frac{Q_t}{\sqrt{W}}dx^i \wedge dx^j \wedge dx^s \wedge dx^t$ which is a 4[th] order non-degenerate form. The demand for a manifold to be symplectic, is that there will be a 1-Form ω such that $\beta = d\omega$ and such that $\beta^n = \beta \wedge \beta \wedge ... \beta$ will be of order $2n$ which is the dimension of the manifold and that β will be a closed form as in our case. By a theorem of Darboux [8], there exists a local basis (x^0, x^1, y^0, y^1) in which $\beta = x^0 \wedge y^0 + x^1 \wedge y^1$ or in a more illuminating way,

$$\begin{pmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{pmatrix}.$$

In a $2n$ dimensional Riemannian or Pseudo-Riemannian manifold, if there is a series of, n vectors, $\omega(1) = \frac{P(1)_\mu}{\sqrt{P(1)_\lambda P(1)^\lambda}}dx^\mu, \omega(2) = \frac{P(2)_\mu}{\sqrt{P(2)_\lambda P(2)^\lambda}}dx^\mu, ..., \omega(2) = \frac{P(n)_\mu}{\sqrt{P(n)_\lambda P(n)^\lambda}}dx^\mu$ which have non-degenerate exterior derivatives and therefore non-degenerate Reeb vectors $U(1)_\mu, ..., U(n)_\mu$ then show that the $n \times n$ Gram determinant $Det(U(i)_\mu U(j)^\mu)$ is invariant for all such choices. In the real case, it can be shown that any other choice is reducible to $U(1)_{\mu},\ldots,U(n)_{\mu}$ by a composition of an $SO(n)$ transformation followed by a reflection which generalizes to $SU(n)$ and a reflection in the complex case. Consider an action operator as the third root of Gram's determinant of 3 Reeb vectors.

3. SU(3) symmetry

We may want to express the acceleration matrix $A_{\alpha\beta}$ by three scalar fields that are defined in the foliation F that is perpendicular to $\frac{P_i}{\sqrt{Z}}$. This is because P_i is a geometric object that defines foliations of space-time and can be conversely defined by the foliations. Another motivation is to show that $SU(3)$ that is seen in Quantum Chromo-Dynamics, may originate from geometry. By a theorem of Frobenius, necessary conditions for 3 vectors $h(j=1,2,3)$ to span the foliation F is that the vectors $h(s)$ are Holonomic if their Lie brackets depend on them $[h(i),h(k)]=\sum_{j=1}^{3}c_j h(j)$ for some coefficients c_j. The Lie brackets of each two vectors must depend on the vectors that span $T(F)$. We may write our 3 scalars a,b,c (here c is not the speed of light and not the previous coefficients but a scalar field) and their gradients that span the foliation's tangent space $T(F)$ as follows, $h_k(1)=\frac{d}{dx^k}a$, $h_k(2)=\frac{d}{dx^k}b$, $h_k(3)=\frac{d}{dx^k}c$.

We now express $A_{\alpha\beta}$ by $h_k(1)$, $h_k(2)$, $h_k(3)$ in a covariant formalism but we need some constraint on P_μ.

Condition: $P_{\mu};^{\lambda} P^{*\mu} = P^*_{\mu};^{\lambda} P^{\mu}$

This condition is not trivial and in general, $P_{\mu};^{\lambda} P^{*\mu} \neq P^*_{\mu};^{\lambda} P^{\mu}$.

Consider the following matrix:

$$D_{ij} = g_{ij} - a^*_i a_j - b^*_i b_j - c^*_i c_j \tag{15}$$

and $a_i = \frac{\alpha_i}{\sqrt{(\alpha^{\lambda}\alpha^*_{\lambda} + \alpha^{*\lambda}\alpha_{\lambda})/2}}$ for some scalar function α whose gradient α_i is in the foliation perpendicular to P_μ etc. and in the same manner replace β_i by a normalized unit vector b_i and γ_i by a normalized $c_i = \frac{\gamma_i}{\sqrt{\gamma_\mu \gamma^{*\mu} + \gamma^*_\mu \gamma^\mu}}$ vector. Also, we demand orthogonality, $a_i b^{*i} = a_i c^{*i} = b_i c^{*i} = 0$. Obviously $D_{ij}a^{*j} = D_{ij}b^{*j} = D_{ij}c^{*j} = 0$ and $D_{ij}\frac{P^{*j}}{\sqrt{Z}} = \frac{P^{*i}}{\sqrt{Z}}$ which then shows that $D_{ij} = \frac{P^*_i P_j}{Z}$, $Z = (P_{\lambda}P^{*\lambda} + P^*_{\lambda}P^{\lambda})/2$. That is very interesting, because as was already said, we can represent $P^*_i P_j$ by orthogonal fields that span the tangent space of the perpendicular foliation to P_j, namely T(F). Consider the following:

$$D_{ij};_k - D_{ik};_j = \left(\frac{P*_i;_k P_j + P*_i P_j;_k}{Z} - \frac{P*_i P_j Z_k}{Z^2}\right) - \left(\frac{P*_i;_j P_k + P*_i P_k;_j}{Z} - \frac{P*_i P_k Z_j}{Z^2}\right) =$$
$$\left(\frac{P*_i P_k Z_j}{Z^2} - \frac{P*_i P_j Z_k}{Z^2}\right) + \left(\frac{P*_i;_k P_j}{Z} - \frac{P*_i;_j P_k}{Z}\right) \tag{16}$$

Now comes a little trick:

$$(D_{ij};_k - D_{ik};_j)\frac{P^i P*^j}{Z} =$$
$$\left(\frac{P*_i P_k Z_j}{Z^2} - \frac{P*_i P_j Z_k}{Z^2}\right)\frac{P^i P*^j}{Z} + \left(\frac{P*_i;_k P_j}{Z} - \frac{P*_i;_j P_k}{Z}\right)\frac{P^i P*^j}{Z} =$$
$$\left(\frac{Z_j P*^j P_k}{Z^2} - \frac{Z_j}{Z}\right) + \left(\frac{P*_i;_k P^i}{Z} - \frac{P^i P*^j P*_i;_j P_k}{Z^2}\right)$$

By (4), it is obvious that the first two terms constitute minus twice the Reeb vector, $\frac{Z_j P*^j P_k}{Z^2} - \frac{Z_j}{Z} = -U_k = -2(\frac{U_k}{2})$. For the last two terms, we need a special condition $P_\mu;^\lambda P*^\mu = P*_\mu;^\lambda P^\mu$ although usually $P_\mu;^\lambda P*^\mu \neq P*_\mu;^\lambda P^\mu$. Then by this condition, $\frac{P*_i;_k P^i}{Z} - \frac{P^i P*^j P*_i;_j P_k}{Z^2} = \frac{U_k}{2}$ and therefore

$$(D_{ij};_k - D_{ik};_j)D*^{ij} = (D_{ij};_k - D_{ik};_j)\frac{P^i P*^j}{Z} = -\frac{U_k}{2} \tag{17}$$

Consider our assumption, $D_{ij} = g_{ij} - a*_i a_j - b*_i b_j - c*_i c_j$ and we have obtained an expression of the Reeb vector by the orthonormal vectors that represent the foliation. The additives of (15) are tensors. This leads us to an open question as follows: Is the condition $P_\mu;^\lambda P*^\mu = P*_\mu;^\lambda P^\mu$, the minimal condition which is needed for a representation of the Reeb vector by a_j, b_j, c_j as the sum of tensor terms ? In other words, is the condition $P_\mu;^\lambda P*^\mu = P*_\mu;^\lambda P^\mu$ a necessity for the tensor representation of the acceleration matrix by the foliation scalars, a, b, c ?

4. Invariance of the Reeb vector under different functions of P

Here we wish to explore another degree of freedom in the action operator of the "acceleration field" which results from the Reeb vector, as shown by a representative vector field $\frac{dP}{dx^i}$ which is tangent to a non-geodesic integral curve. We wish to show that P can be replaced with a smooth function $f(P)$ and that U_m is invariant under such a transformation.

We revisit our acceleration field and write $U_m = \frac{N^2{,}_m}{N^2} - \frac{N^2{,}_\mu P^{*\mu}}{N^4} P_m$ *s.t.* $Z = N^2 \equiv (P^{*i} P_i + P^{*i} P_i)/2$ (also found as Z in this paper) we can omit the comma for the sake of brevity the same way we write P_i instead of $\mathrm{P}_{,\mathrm{i}}$ for $\frac{dP}{dx^i}$ and write $U_m = \frac{N^2{}_m}{N^2} - \frac{N^2{}_\mu P^{*\mu}}{N^4} P_m$. We will prove the invariance of U_m where P is real, however, a similar proof is also valid where P is complex and where P is replaced with a smooth function of P.

Suppose that we replace P by $f(P)$ such that f is positive and increasing or decreasing, then $f(P)_i \equiv \frac{df(P)}{dx^i} = \frac{df(P)}{dP}\frac{dP}{dx^i} = f_p(P)P_i$. Let $N^2 \equiv \mathrm{P}^\lambda \mathrm{P}_\lambda$ then $\hat{N}^2 \equiv f(P)_\lambda f(P)^\lambda = N^2 f_p(P)^2$ and $\frac{\hat{N}^2{}_k}{\hat{N}^2} = \frac{N^2{}_k}{N^2} + \frac{2 f_{pp}(P)}{f_p(P)} P_k$ but also

$$\hat{U}_k = \frac{\hat{N}^2{}_k}{\hat{N}^2} - \frac{\hat{N}^2{}_s f_p(P)P^s f_p(P)P_k}{\hat{N}^2} =$$

$$\frac{N^2{}_k}{N^2} + \frac{2 f_{pp}(P)}{f_p(P)} P_k - \left(\frac{N^2{}_s}{N^2} + \frac{2 f_{pp}(P)}{f_p(P)} P_s\right) \frac{f_p(P)P^s f_p(P)P_k}{N^2 f_p(P)^2} =$$

$$\frac{N^2{}_k}{N^2} - \frac{N^2{}_\mu P^\mu}{N^4} P_k = U_k \tag{18}$$

Which proves the invariance of the Reeb vector $\frac{U_k}{2}$ under different parameterizations of the scalar field P.

5. Energy density by an acceleration field – Reeb vector at the classical non-covariant limit

We now show, how much energy density does this term $\frac{1}{4}U_k U^k$ represent.

For a clock that moves along the integral curves, formed by $\frac{P_\mu}{\sqrt{Z}}$, we have from (2) and (11)

$$\frac{a_\mu}{c^2} = \frac{U_\mu}{2} \tag{19}$$

In special relativity, the squared curvature of a trajectory of a particle is expressible by its 4-acceleration, divided by the squared speed of light, $\frac{dV_\mu / c}{c d\tau} = \frac{a_\mu}{c^2}$ where the proper time τ differentiates the velocity V_μ and where τ is an arc-length parameterization.

The classical limit of a gravitational field is not covariant and that even worse, the classical field is intrinsic to the body of mass M that generates gravity; however, it is valid tool for the assessment of a physical model. We consider small mass at rest in a Newtonian (obviously not covariant) gravitational field. By the principle of equivalence, this mass is accelerated, otherwise it would freely fall. So, if a force field can keep small mass from falling, the field's classical limit of energy, is the same as the energy of the classical non-covariant gravitational field. This results hints at the energy of an acceleration field that opposes weak gravity. Summation of the squared norm of non-covariant 3-acceleration, a^2 of clocks that are kept from falling in the weak gravity generated by the mass M is

$$\iiint\limits_{V=Volume} a^2 dV = \int\limits_{r0}^{\infty}\left(\frac{(KM)^2}{r^2}\right)^2 4\pi\, r^2 dr = K\frac{4\pi KM^2}{r_0} \tag{20}$$

Where K is Newton's gravity constant. Now we calculate the non-relativistic and non-covariant negative potential energy $-E_g$,

$$\int\limits_0^M \left(\frac{Km}{r_0}\right) dm = \frac{KM^2}{2r_0} = -E_g \tag{21}$$

So from (20) and (21)

$$\frac{1}{8\pi K}\iiint\limits_{V=Volume} a^2 dV = -E_g \tag{22}$$

(22) implies the following relation between energy and the non-gravitational acceleration field that prohibits geodesic motion, where $\rho\, c^2$ is the energy density and ρ is the mass density.

$$\frac{a^2}{c^4} = \frac{8\pi K\rho}{c^2} \Rightarrow Energy-Density = \frac{1}{2}\frac{a^2}{4\pi K} = \frac{a^2}{8\pi K} \tag{23}$$

where c is the speed of light. (23) dictates in four dimensions,

$$\frac{U_\mu U^\mu}{4} = \frac{a_\mu a^\mu}{c^4} = -\frac{8\pi K}{c^4} * \mathrm{Energy - Density} \tag{24}$$

Note that unlike (24), (23) is not a covariant expression. What does it mean in the non-covariant classical limit of the electro-static field E. Since an electric field is also a form of an energy density, $Energy-Density = \frac{\varepsilon_0}{2}E^2$ where ε_0 is the permittivity of vacuum and from (24) we can infer the following non-covariant classical limit, $8\pi K\frac{\varepsilon_0}{2}E^2 = a^2$ where E^2 and a^2 are square norms of the 3-vectors $\vec{E}$ and $\vec{a}$. We can infer,

$$\sqrt{4\pi K\varepsilon_0}\,E = a \tag{25}$$

The acceleration in (25) is dauntingly small and very difficult to measure. It requires an immense field of 1 million volts over 1 millimetre to expose an acceleration of uncharged clocks, which is about 8.61cm/sec^2, less than 0.01 g, providing that there are no other fields that cancel out this acceleration. In fact, we will see below that charge also generates gravity and that for the choice $8\pi K$ in (23), the acceleration will be about 4.305 cm/sec^2. By the principle of parsimony, the fact that this acceleration field stores energy, i.e. $Energy-Density = \frac{a^2}{8\pi K}$ means that this acceleration is aligned with the electric charge, electro-static field curves because this can explain the electric charge attraction and repulsion by simply, increasing or decreasing the energy stored in such a weak acceleration field. As we shall see, if instead of $8\pi\ K$ we develop this theory such that $4\pi\ K$ divides the square norm of acceleration, no acceleration of neutral particles will be measured within a homogeneous electrostatic field. This is because, we will develop the Euler Lagrange equations of the Ricci scalar plus (11) and see that charge also generates gravity and not only inertial mass does.

We now use a covariant terminology of 4-acceleration a_μ and $a^2 \equiv a_\mu a^\mu$.

As a more general theory, we can write $Energy\_Density = -\frac{a^2}{\sigma K} = -\frac{a_\mu a^\mu}{\sigma K}$, such that $\sigma = 8\pi$.

Another important remark is that in the classical non-covariant limit, the divergence of the electric field can be written as,

$$\frac{U_\mu;^\mu}{2} = Div(\frac{U_\mu}{2}) = Div(a_\mu / c^2) \approx$$

$$\frac{\sqrt{4\pi K\varepsilon_0}}{c^2} Div(E) = \frac{\sqrt{4\pi K\varepsilon_0}}{c^2}\frac{\rho}{\varepsilon_0} = \sqrt{\frac{4\pi K}{\varepsilon_0}}\frac{\rho}{c^2} \tag{26}$$

Such that ρ denotes charge density and not the previously defined energy density.

By experiments done by Hector Serrano, for NASA, the author believes that the acceleration field of even uncharged clocks in an electric field is towards the electron and out of the proton. A relation between charge and gravity can be developed, leading to unprecedented repercussions on the feasibility of Alcubierre Warp Drive, reference is given where it is discussed later.

There is a remark of Serrano [9], about a moving capacitor in vacuum, in a reply to Peter Liddicoat: *"Actually by the generally accepted definition of what constitutes high vacuum 10^-6 Torr is about in the middle. This pressure is about equal to low Earth orbit. More importantly at this pressure the 'Mean Free Path' of the molecules in the chamber is far too great to support Corona/Ion wind effects. We've tested from atmosphere to 10^-7 Torr with no change in performance either. However, I'm glad the results have you thinking. It looks simple, but trust me it's not"..*

Hector Serrano has mentioned in a patent [10], that a capacitor manifests weak thrust also in vacuum. Another indirect evidence is the Flyby Anomaly [11] which is possibly caused by ionosphere charge. For further evidence, see Timir Datta et. al. work as an elegant way to focus field lines by metal cone and plane and to observe an effect [12]. The author believes the acceleration of charge-less particles in an electric field is from positive to negative. In section 9 it is shown that there is an electro-gravitational effect opposite in direction to the acceleration of an uncharged particle in an electro-static field. There is at least informal evidence that the elecro-gravitational effect shows thrust of the entire dipole towards the positive direction [9] and the author does not imply asymmetrical capacitors of 1 - 0.1 Pico-Farad with 45000 Volts. It is shown that such capacitors - according to the calculations in section 9, assuming a roughly approximated acceleration proportional to the gravitational field – will not manifest any measurable effect of at least 1 micro Newton thrust. Most likely is that any measurable thrust, using such small capacitors, will be solely based on ionic wind.

6. Experimental problems – electron mobility

The down side of the non-geodesic acceleration is that it is about 10 orders of magnitude smaller than the accepted and known electric field interaction. For example, negative charge suspended above the

Earth will cause charge to move in the ground. This charge will have a much stronger effect than the interaction with the acceleration field as is, and will cause a shielding effect i.e. the fields will cancel out within the Earth. Even the almost ideal insulator, i.e. diamond crystals, have impurities such as Nitrogen Vacancies [13] that allow charge carriers to move in the lattice i.e. high electron mobility. In the purest diamonds, the NV impurities are about 10^{18} nodes per cm^{-3} comparing to $1.77x10^{23}$ carbon atoms per cm^{-3}. The donor electrons lie deep in the band gap of 5.47eV, at about 1.7 eV.

7. Vaknin's theory

We quote here one of the four models of Vaknin [14] as follows: This work contains a possible realization of space-time as an ideal geometric object that becomes physically accessible only where a wave function which is called "chronon" collapses. The physical model is therefore of events and not of particles. This paper offers the idea that matter occurs where the Reeb vector is not zero. Showing consistency of this model with Quantum Mechanics is a very difficult task although it is possible to show that the energy of an electric field is stored in an acceleration field by replacement of the electro-magnetic tensor with the anti-symmetric acceleration field.

Vaknin's description of the realization of event is as follows:*"Time as a wave function with observer-mediated collapse. Entanglement of all Chronons at the exact "moment" of the Big Bang. A relativistic QFT with Chronons as Field Quanta (excited states.) The integration is achieved via quantum superpositions".*

The main difference between Vaknin's approach and the author's approach is that Vaknin's approach is algebraic where the author's approach is geometric. Thus, the outcome is two different theories that discuss a similar idea. We now show the simplest implementation of Vaknin's model as a quantization idea of time by collapsible events, as an additional constraint to the action

$$\int_{\Omega^4} \frac{c^4}{\sigma K} Curv^2 \sqrt{-g}\, d\Omega^4 \text{ where } \sigma = 8\pi$$

Where $\sqrt{-g}$ is the root of the negative metric tensor determinant for the volume element, such that $P = \lim_{n\to\infty} \psi(1) + \psi(2) + \ldots + \psi(n)$ and such that: $\int_{\Omega^4} \psi(k)\psi^*(k)\sqrt{-g}\, d\Omega^4 = 1$ And $0 < j < k < \infty \Rightarrow \int_{\Omega^4} \psi(j)\psi^*(k)\sqrt{-g}\, d\Omega^4 = 0$.

The term c^4 is the speed of light to the power of 4 and K is Newton's gravity constant. $Curv^2$ is a generalization of the square norm of the Reeb vector to the complex numbers field,

$$Curv^2 = \frac{(U_j U^{*j} + U^*_j U^j)}{8} \text{ where } \frac{U_j}{2} = \frac{Z_j}{2Z} - \frac{Z_k P^{*k}}{2Z^2} P_j = \left(\frac{Z_j P_s}{2Z^{3/2}} - \frac{Z_s P_j}{2Z^{3/2}}\right)\frac{P^{s*}}{\sqrt{Z}}$$

and $A_{sj} = \frac{Z_j P_s}{2Z^{3/2}} - \frac{Z_s P_j}{2Z^{3/2}}$ can be extended to a regular matrix $A_{sj} + B_{sj}$ where $B^{\alpha\beta} \equiv r\frac{1}{\sqrt{2}}\varepsilon^{\mu\nu\alpha\beta}A_{\mu\nu}$ and $\varepsilon^{\mu\nu\alpha\beta}$ is the Levi Civita tensor (not symbol) and $rr^* = 1$ due to a degree of freedom.

7.1. *Physical meaning:* The field $A_{js}+B_{js}$ will rotate and scale a scalar wave function φ of a particle, $\varphi_j=\frac{d\varphi}{dx^j}$ where $(A_{js}+B_{js})\varphi^{*s}=\frac{1}{c}\frac{d\varphi_j}{d\tau}$ and where τ measures proper time.

We generalize the acceleration field energy density from $-(\frac{a_j a^j}{8\pi K})=-\frac{U_j U^j}{4}\frac{c^4}{8\pi K}$ to $-(\frac{a_j a^{*j}+a^*_j\, a^j}{16\pi K})=-\frac{U_j U^{*j}+U^*_j\, U^j}{8}\frac{c^4}{8\pi K}$ and $\frac{a_j}{c^2}=\frac{U_j}{2}$.

As we saw in (18), P does not have to be the proper time measured along curves. Instead, it can be a function of such proper time. (18) motivates the decomposition of P into wave functions because P does not have to be a monotonically increasing function of the proper time measured along integral curves formed by P_k. The problem is that P is not any wave function of a particle. The simplest physical interpretation of the ψ wave function is that it describes events in space-time and not particles. Therefore, P becomes a sum of wave functions and $P=\lim_{n\to\infty}\psi(1)+\psi(2)+...+\psi(n)$ is a decomposition of the function P as a sum of wave functions.

As quantum states, these event wave functions $\psi(1)$, $\psi(2)$,... must be normalized to probability 1 on the space-time manifold and they should be independent of each other as was written in two integral constraints. The best motivation for the constraints $\int_{\Omega^4}\psi(k)\psi^*(k)\sqrt{-g}\,d\Omega^4=1$ and for $\int_{\Omega^4}\psi(k)\psi^*(j)\sqrt{-g}\,d\Omega^4=0 \text{ s.t. } k\neq j$ is given by Vaknin [14] and Storkin [15] where they emphasize that physical events are discrete. Storkin considers decreasing probability functions of the number of events in a given -volume, e.g. Poisson distribution within 2+1 dimensions Minkowsky space-time. Causal sets are partially ordered graphs of events along paths in space-time. The approach of this paper is more robust than that of Storkin because causal sets are the result of the order of events along the integral curves that are naturally formed by P_k along with the mentioned constraints that induce a countable set of wave functions.

7.2. *Auto-rotation:* In this section, we will study the field of a particle, whose rest mass energy is presumably stored in an acceleration field. The equation $(A_{js}+B_{js})\varphi^{*s}=\frac{1}{c}\frac{d\varphi_j}{d\tau}$ can be better understood, by recalling (8) and by replacing $\frac{P_k}{\sqrt{Z}}$ in (8) with φ_k, as in the following: $A_{js}\varphi^{*s}=Lie(\frac{P^{*\lambda}}{\sqrt{Z}},\varphi_j)=\frac{P^{*\lambda}}{\sqrt{Z}}\varphi_{j},_{\lambda}+\left(\frac{P^{*s}}{\sqrt{Z}}\right),_j\varphi_s$ in which the right hand side describes the acceleration of φ_s along the vector $\frac{P^{*\lambda}}{\sqrt{Z}}$. On the other hand, the left hand side, $A_{js}\varphi^{*s}$ describes the acceleration of φ^{*s} along $\frac{P_s}{\sqrt{Z}}$ which does not include the portion of the acceleration $B_{js}\varphi^{*s}$, and which results in the equation above. By (8), If we replace φ_s with $\frac{P_s}{\sqrt{Z}}$, we get $\frac{U_j}{2}$ on both sides of the equation. What is the meaning of φ_k ? Dirac's equation consists of spinors [6], and of matrix blocks as basis elements of the Lie Algebra of a rotation group. In Dirac's equation, φ_k is not the gradient of a scalar function, the indices correspond to orthogonal unit vectors and $\varphi_k\varphi^{*k}$ or $(\varphi_k\varphi^{*k} + \varphi^{*}{}_k\,\varphi^k)/2$ can be reduced to a probability density. φ_k is not a vector in the usual sense of General Relativity because it transforms between different coordinate systems with the help of spin connections and not with the help of affine connections. The philosophy of the Dirac equation, stems from the motivation to represent spatial spin axes as three orthogonal quantum states and to predict their probabilities. Any measurement of a spin is either +spin or -spin, no matter from which angle the physical measurement is performed. This property of the spin is very different than that of the classical mechanics spin. The philosophy of this paper is very different than that of Dirac, because it is based on a fully geometric interpretation of matter as acceleration fields. We begin with a quest for φ_k that will be an ordinary vector, unlike in Dirac's equation. We know that if $\varphi_k\varphi^{*k}$ is a probability density, then by (24), if all the energy of the particle is in its acceleration field, then we must have $\frac{8\pi K}{c^4}\varphi_k\varphi^{*k}\, mc^2 = -\frac{U_k U^k}{4}$ which results in two equations, $A_{js}\varphi^{*s} = \frac{P^{*\lambda}}{\sqrt{Z}}\varphi_{j},_{\lambda} + \left(\frac{P^{*s}}{\sqrt{Z}}\right),_j \varphi_s$ and $\frac{8\pi K}{c^4}\varphi_k\varphi^{*k}\, mc^2 = -\frac{U_k U^k}{4}$ where mc^2 is the energy of the particle and $\varphi_k\varphi^{*k}\, mc^2$ is the energy density of the particle. There is a new open problem and a new ongoing research which is intended to answer whether φ_k, as presented here, offers a useful way to describe matter on the quantum level.

8. General Relativity for the deterministic limit

By General Relativity, we have to add the Hilbert-Einstein action [16][17][18] to the negative sign of the square curvature of the gradient of the scalar field in order to replace the energy-momentum tensor in the Einstein's field equations. Negative means that the curvature operator is mostly negative. As before, we assume $\sigma = 8\pi$ (from the previously discussed term, $-a_\mu a^\mu / 8\pi K$ as an energy density).

$$Z = N^2 = \mathrm{P}_\mu \mathrm{P}^\mu \text{ and } U_\lambda = \frac{Z_\lambda}{Z} - \frac{Z_k P^k P_\lambda}{Z^2} \text{ and } \mathrm{L} = \frac{1}{4}\mathrm{U}^k\mathrm{U}_k$$

$$R = Ricci \text{ curvature.}$$

$$Min\,Action = Min\int_\Omega \left(R - \frac{8\pi}{\sigma}L\right)\sqrt{-g}\,d\Omega =$$

$$Min\int_\Omega \left(R - \frac{1}{4}\mathrm{U}^k\mathrm{U}_k\right).\sqrt{-g}\,d\Omega \text{ s.t. } \sigma = 8\pi \tag{27}$$

$\sqrt{-g}$ is a scalar density of the volume element, R is the Ricci curvature [16] and $\sqrt{-g}$ is the determinant of the metric tensor used for the 4-volume element as in tensor densities [17].

The variation of the Ricci scalar is well known. It uses the Platini identity and Stokes theorem to calculate the variation of the Ricci curvature and reaches the Einstein tensor [18], as follows, $\delta R = R_{\mu\nu}\delta g^{\mu\nu}$ and $\delta\sqrt{-g} = -\frac{1}{2}g_{\mu\nu}\delta g^{\mu\nu}\sqrt{-g}$ by which we infer $\delta(R\sqrt{-g}) = (R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu})\delta g^{\mu\nu}$ which will be later added to the variation of $\left(\frac{1}{2}R - \frac{8\pi}{\sigma}L\right).\sqrt{-g}$ by $\delta g^{\mu\nu}$.

The following Euler Lagrange equations have to hold,

$$\left(\frac{\partial}{\partial g^{\mu\nu}} - \frac{d}{dx^m}\frac{\partial}{\partial(g^{\mu\nu},_m)} + \frac{d^2}{dx^m dx^s}\frac{\partial}{\partial(g^{\mu\nu},_m,_s)}\right)\left((\frac{1}{2}R - \frac{1}{4}\mathrm{U}^k\mathrm{U}_k)\sqrt{-g}\right) = 0$$

and

$$\left(\frac{\partial}{\partial p} - \frac{d}{dx^m}\frac{\partial}{\partial(P_m)} + \frac{d^2}{dx^m dx^s}\frac{\partial}{\partial(P,_m,_s)}\right)\left((\frac{1}{2}R - \frac{1}{4}\mathrm{U}^k\mathrm{U}_k)\sqrt{-g}\right) = 0$$

$\mathrm{U}^k\mathrm{U}_k = \frac{Z_\mu Z^\mu}{Z^2} - \frac{(Z_s P^s)^2}{Z^3}$ which we obtain from the minimum Euler Lagrange equation because $U_\lambda P^\lambda = \frac{Z_\lambda P^\lambda}{Z} - \frac{Z_k P^k P_\lambda P^\lambda}{Z^2} = 0$. In order to calculate the minimum action Euler-Lagrange equations, we will separately treat the Lagrangians, $L = \frac{Z_\mu Z^\mu}{Z^2}$ and $L = \frac{(Z_s P^s)^2}{Z^3}$ to derive the Euler Lagrange equations of the Lagrangian $L = \frac{Z_\mu Z^\mu}{Z^2} - \frac{(Z_s P^s)^2}{Z^3} = U_\mu U^\mu$. The Euler Lagrange operator of the Ricci scalar $\left(\frac{\partial}{\partial g^{\mu\nu}} - \frac{d}{dx^m}\frac{\partial}{\partial(g^{\mu\nu},_m)} + \frac{d^2}{dx^m dx^s}\frac{\partial}{\partial(g^{\mu\nu},_m,_s)}\right.$.

The reader may skip the following equations up to equation (33). Equations (33), (34) and (36) are however crucial.

$$L=\frac{(P^{\lambda}Z_{\lambda})^{2}}{Z^{3}}\ \ s.t.\ Z=P_{\mu}P^{\mu}$$

$$\frac{\partial(L\sqrt{-g})}{\partial g^{\mu\nu}}-\frac{d}{dx^{m}}\frac{\partial(L\sqrt{-g})}{\partial(g^{\mu\nu},_{m})}=$$

$$\left(\begin{array}{l}
-2(\frac{(P^{\lambda}P_{\lambda}),_{s}P^{s}}{Z^{3}}P_{\mu}P_{\nu}P^{m});_{m}+\\
2\frac{(P^{\lambda}P_{\lambda}),_{s}P^{s}}{Z^{3}}(\Gamma_{\mu}{}^{i}{}_{m}P_{i}P_{\nu}P^{m}+\Gamma_{\upsilon}{}^{i}{}_{m}P_{\mu}P_{i}P^{m})+\\
+2\frac{(P^{\lambda}P_{\lambda}),_{s}P^{s}}{Z^{3}}(P_{\mu}P_{\nu});_{m}P^{m}\\
-2\frac{(P^{\lambda}P_{\lambda}),_{s}P^{s}}{Z^{3}}(\Gamma_{\mu}{}^{i}{}_{m}P_{i}P_{\nu}P^{m}+\Gamma_{\upsilon}{}^{i}{}_{m}P_{\mu}P_{i}P^{m})+\\
+2(\frac{(P^{\lambda}P_{\lambda}),_{s}P^{s}}{Z^{3}})Z_{\mu}P_{\nu}-3(\frac{((P^{\lambda}P_{\lambda}),_{s}P^{s})^{2}}{Z^{4}})P_{\mu}P_{\nu}\\
-\frac{1}{2}\frac{(P^{\lambda}Z_{\lambda})^{2}}{Z^{3}}g_{\mu\nu}
\end{array}\right)\sqrt{-g}=$$

$$\left(\begin{array}{l}
-2(\frac{(P^{\lambda}P_{\lambda}),_{m}P^{m}}{Z^{3}}P^{k});_{k}P_{\mu}P_{\nu}\\
-2\frac{(P^{\lambda}Z_{\lambda})^{2}}{Z^{3}}\frac{P_{\mu}P_{\nu}}{Z}+2(\frac{(P^{\lambda}P_{\lambda}),_{s}P^{s}}{Z^{3}})Z_{\mu}P_{\nu}+\\
-\frac{1}{2}\frac{(P^{\lambda}Z_{\lambda})^{2}}{Z^{3}}g_{\mu\nu}-\frac{(P^{\lambda}Z_{\lambda})^{2}}{Z^{3}}\frac{P_{\mu}P_{\nu}}{Z}
\end{array}\right)\sqrt{-g}\qquad(28)$$

$$L = \frac{Z^{\lambda} Z_{\lambda}}{Z^2} \;\; s.t. \; Z = P_{\mu} P^{\mu}$$

$$\frac{\partial (L\sqrt{-g})}{\partial g^{\mu\upsilon}} - \frac{d}{dx^m} \frac{\partial (L\sqrt{-g})}{\partial g^{\mu\upsilon},_m} =$$

$$\left(\begin{array}{l} -2(\frac{Z^m P_{\mu} P_{\upsilon}}{Z^2});_m \\ +2\frac{(\Gamma_{\mu}{}^{i}{}_{m} P_i P_{\nu} Z^m + \Gamma_{\upsilon}{}^{i}{}_{m} P_{\mu} P_i Z^m)}{Z^2})+ \\ +2\frac{(P_{\mu} P_{\upsilon});_m Z^m)}{Z^2} \\ -2\frac{(\Gamma_{\mu}{}^{i}{}_{m} P_i P_{\nu} Z^m + \Gamma_{\upsilon}{}^{i}{}_{m} P_{\mu} P_i Z^m)}{Z^2})+ \\ +\frac{Z_{\mu} Z_{\nu}}{Z^2} - 2\frac{Z_s Z^s}{Z^3} P_{\mu} P_{\nu} - \frac{1}{2}\frac{Z_m Z^m}{(P^i P_i)^2} g_{\mu\nu} \end{array} \right) \sqrt{-g} =$$

$$(-2(\frac{Z^m}{Z^2});_m P_{\mu} P_{\nu}$$

$$-2\frac{Z^{\lambda} Z_{\lambda}}{Z^2}\frac{P_{\mu} P_{\nu}}{Z} - \frac{1}{2}\frac{Z_k Z^k}{Z^2} g_{\mu\nu} + \frac{Z_{\mu} Z_{\nu}}{Z^2})\sqrt{-g}$$

$$L = \frac{Z^{\lambda} Z_{\lambda}}{Z^2} \;\; s.t. \; Z = P_{\mu} P^{\mu}$$

$$\frac{\partial (L\sqrt{-g})}{\partial g^{\mu\upsilon}} - \frac{d}{dx^m} \frac{\partial (L\sqrt{-g})}{\partial g^{\mu\upsilon},_m} =$$

$$\left(\begin{array}{l} -2(\frac{Z^m P_{\mu} P_{\upsilon}}{Z^2});_m + 2\frac{(\Gamma_{\mu}{}^{i}{}_{m} P_i P_{\nu} Z^m + \Gamma_{\upsilon}{}^{i}{}_{m} P_{\mu} P_i Z^m)}{Z^2})+ \\ +2\frac{(P_{\mu} P_{\upsilon});_m Z^m)}{Z^2} - 2\frac{(\Gamma_{\mu}{}^{i}{}_{m} P_i P_{\nu} Z^m + \Gamma_{\upsilon}{}^{i}{}_{m} P_{\mu} P_i Z^m)}{Z^2})+ \\ +\frac{Z_{\mu} Z_{\nu}}{Z^2} - 2\frac{Z_s Z^s}{Z^3} P_{\mu} P_{\nu} - \frac{1}{2}\frac{Z_m Z^m}{(P^i P_i)^2} g_{\mu\nu} \end{array} \right) \sqrt{-g} =$$

$$(-2(\frac{Z^m}{Z^2});_m P_{\mu} P_{\nu} - 2\frac{Z^{\lambda} Z_{\lambda}}{Z^2}\frac{P_{\mu} P_{\nu}}{Z} - \frac{1}{2}\frac{Z_k Z^k}{Z^2} g_{\mu\nu} + \frac{Z_{\mu} Z_{\nu}}{Z^2})\sqrt{-g}$$

(29)

We subtract (28) from (29)

$$Z = P_\mu P^\mu \ and \ U_\lambda = \frac{Z_\lambda}{Z} - \frac{Z_k P^k P_\lambda}{Z^2} and \ L = U^\kappa U_k = \frac{Z_\lambda Z^\lambda}{Z^2} - \frac{(Z_k P^k)^2}{Z^3}$$

$$\left(\frac{\partial(L\sqrt{-g})}{\partial g^{\mu\upsilon}} - \frac{d}{dx^m}\frac{\partial(L\sqrt{-g})}{\partial g^{\mu\upsilon},_m}\right)U^\kappa U_k =$$

$$\begin{pmatrix} +2\left(\frac{(P^\lambda P_\lambda),_m P^m}{Z^3}P^k\right);_k P_\mu P_\nu + \\ 2\frac{(P^\lambda Z_\lambda)^2}{Z^3}\frac{P_\mu P_\nu}{Z} - 2\left(\frac{(P^\lambda P_\lambda),_s P^s}{Z^3}\right)Z_\mu P_\nu + \\ +\frac{1}{2}\frac{(P^\lambda Z_\lambda)^2}{Z^3}g_{\mu\nu} + \frac{(P^\lambda Z_\lambda)^2}{Z^3}\frac{P_\mu P_\nu}{Z} + \\ \left(-2\left(\frac{Z^m}{Z^2}\right);_m P_\mu P_\nu - 2\frac{Z^\lambda Z_\lambda}{Z^2}\frac{P_\mu P_\nu}{Z} - \frac{1}{2}\frac{Z_k Z^k}{Z^2}g_{\mu\nu} + \frac{Z_\mu Z_\nu}{Z^2}\right) \end{pmatrix}\sqrt{-g} =$$

$$\begin{pmatrix} +2\left(\left(\frac{(P^\lambda P_\lambda),_m P^m}{Z^3}P^k\right);_k - 2\left(\frac{Z^m}{Z^2}\right);_m\right)P_\mu P_\nu + \\ +2\frac{(P^\lambda Z_\lambda)^2}{Z^3}\frac{P_\mu P_\nu}{Z} - 2\frac{Z^\lambda Z_\lambda}{Z^2}\frac{P_\mu P_\nu}{Z} + \\ +\frac{1}{2}\frac{(P^\lambda Z_\lambda)^2}{Z^3}g_{\mu\nu} - \frac{1}{2}\frac{Z_k Z^k}{Z^2}g_{\mu\nu} + \\ +\frac{Z_\mu Z_\nu}{Z^2} - 2\left(\frac{(P^\lambda P_\lambda),_s P^s}{Z^3}\right)Z_\mu P_\nu + \frac{(P^\lambda Z_\lambda)^2}{Z^3}\frac{P_\mu P_\nu}{Z} \end{pmatrix}\sqrt{-g} =$$

$$\begin{pmatrix} +2\left(\left(\frac{(P^\lambda P_\lambda),_m P^m}{Z^3}P^k\right);_k - 2\left(\frac{Z^m}{Z^2}\right);_m\right)P_\mu P_\nu + \\ +2\frac{(P^\lambda Z_\lambda)^2}{Z^3}\frac{P_\mu P_\nu}{Z} - 2\frac{Z^\lambda Z_\lambda}{Z^2}\frac{P_\mu P_\nu}{Z} + \\ +U_\mu U_\nu - \frac{1}{2}U_k U^k g_{\mu\nu} \end{pmatrix}\sqrt{-g} =$$

$$\left(U_\mu U_\nu - \frac{1}{2}U_k U^k g_{\mu\nu} - 2U^k;_k \frac{P_\mu P_\nu}{Z}\right)\sqrt{-g} \tag{30}$$

$$L=\frac{(Z^s P_s)^2}{Z^3} \quad s.t.\ Z=P^\lambda P_\lambda \ and\ Z_m=(P^\lambda P_\lambda)_{,m}$$

$$\frac{\partial(L\sqrt{-g})}{\partial P_\mu}-\frac{d}{dx^\nu}\frac{\partial(L\sqrt{-g})}{\partial P_{\mu,\nu}}=$$

$$\left(\begin{array}{l} -4\left(\frac{(Z_s P^s)}{Z^3}P^\mu P^\nu\right)_{;\nu}+4\frac{(Z_s P^s)}{Z^3}\Gamma_i{}^\mu{}_\nu P^i P^\nu+ \\ +4\frac{(Z_s P^s)}{Z^3}P^\mu{}_{;\nu}P^\nu-4\frac{(Z_s P^s)}{Z^3}\Gamma_i{}^\mu{}_k P^i P^k+ \\ +2\frac{Z_m P^m Z^\mu}{Z^3}-6\frac{(Z_m P^m)^2}{Z^4}P^\mu \end{array}\right)\sqrt{-g}=$$

$$\left(-4\left(\frac{(Z_s P^s)P^\nu}{Z^3}\right)_{;\nu}P^\mu+2\frac{Z_m P^m Z^\mu}{Z^3}-6\frac{(Z_m P^m)^2}{Z^4}P^\mu\right)\sqrt{-g} \tag{31}$$

$$L=\frac{Z^s Z_s}{Z^2} \quad s.t.\ Z=P^\lambda P_\lambda \ and\ Z_m=(P^\lambda P_\lambda)_{,m}$$

$$\frac{\partial(L\sqrt{-g})}{\partial P_\mu}-\frac{d}{dx^\nu}\frac{\partial(L\sqrt{-g})}{\partial P_{\mu,\nu}}=$$

$$\left(\begin{array}{l} -4\left(\frac{P^\mu Z^\nu}{Z^2}\right)_{;\nu}+\frac{4}{Z^2}\Gamma_i{}^\mu{}_k P^i Z^k+ \\ +\frac{4}{Z^2}P^\mu{}_{;\nu}Z^\nu-\frac{4}{Z^2}\Gamma_i{}^\mu{}_k P^i Z^k+ \\ -4\frac{Z_m Z^m}{Z^3}P^\mu\sqrt{-g} \end{array}\right)\sqrt{-g}=$$

$$\left(-4\left(\frac{Z^\nu}{Z^2}\right)_{;\nu}-4\frac{Z_m Z^m}{Z^3}\right)P^\mu\sqrt{-g} \tag{32}$$

We subtracted the Euler Lagrange operators of $\frac{(Z^s P_s)^2}{Z^3}\sqrt{-g}$ in (28) from the Euler Lagrange operators of $\frac{Z^\lambda Z_\lambda}{Z^2}\sqrt{-g}$ in (29) and got (30) and we will subtract (31) from (32) to get two tensor equations of gravity, these will be (33), and (36).

Assuming $\sigma = 8\pi$, where the metric variation equations (27), (28), (29) and (30) yield

$$
\begin{aligned}
&\mathrm{Z} = N^2 = \mathrm{P}_\mu \mathrm{P}^\mu \ , \ U_\lambda = \frac{Z_\lambda}{Z} - \frac{Z_k P^k P_\lambda}{Z^2} \ , L = \frac{1}{4} U_i U^i \ \text{ and } \mathrm{Z} = \mathrm{P}^k \mathrm{P}_k \\
&\frac{8\pi}{\sigma}\frac{1}{4}\left(\begin{array}{l}
+2\left(\left(\frac{(\mathrm{P}^\lambda \mathrm{P}_\lambda),_{\mathrm{m}} P^m}{Z^3} P^k\right);_k -2\left(\frac{Z^m}{Z^2}\right);_m\right) P_\mu P_\nu + \\
+2\frac{(P^\lambda Z_\lambda)^2}{Z^3}\frac{P_\mu P_\nu}{Z} - 2\frac{Z^\lambda Z_\lambda}{Z^2}\frac{P_\mu P_\nu}{Z} + \\
+U_\mu U_\nu - \frac{1}{2} U_{\mathrm{k}} U^k g_{\mu\nu}
\end{array}\right) = \\
&\frac{8\pi}{\sigma}\frac{1}{4}\left(U_\mu U_\nu - \frac{1}{2} U_{\mathrm{k}} U^k g_{\mu\nu} - 2U^k;_k \frac{P_\mu P_\nu}{Z}\right) = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} \\
&\text{s.t. } \ R = R_{\mu\upsilon} g^{\mu\upsilon} \\
&\text{s.t. } \ R_{kj} = (\Gamma_{jk}{}^P),_p - (\Gamma_{pk}{}^P),_j + \Gamma_{p\mu}{}^P \Gamma_{jk}{}^\mu - \Gamma_{pj}{}^\mu \Gamma_{k\mu}{}^p
\end{aligned} \tag{33}
$$

$R_{\mu\nu}$ is the Ricci tensor and $R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}$ is the Einstein tensor [18].In general, by (27) and $\sigma = 8\pi$ (33) can be written as

$$
\frac{1}{4}\left(U_\mu U_\nu - \frac{1}{2} U_{\mathrm{k}} U^k g_{\mu\nu} - 2U^k;_k \frac{P_\mu P_\nu}{Z}\right) = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} \tag{34}
$$

If we consider Vaknin's model [14] of realization of space-time by collapse events $\psi(i = 1,2,3,\ldots, n \to \infty)$ then we have to add the lambda * constraint to the action operator and the resulting Euler Lagrange equations for a vanishing metric variation are:

$$
\begin{gathered}
P = \lim_{n\to\infty} \psi(1) + \psi(2) + \cdots + \psi(n) \\
\int_{\Omega_4} \psi(\mathrm{k})\psi * (\mathrm{k})\sqrt{-g}\, d\Omega_4 = 1 \\
0 < j < k \Rightarrow \int_{\Omega_4} \psi(\mathrm{j})\psi * (\mathrm{k})\sqrt{-g}\, d\Omega_4 = 0 \\
\frac{1}{8}\left(U_\mu U *_\gamma + U *_\mu U_\gamma - \frac{1}{2}(U *^k U_k + U^k U *_k) g_{\mu\gamma} - 2(U^k;_k + U *^k;_k) \frac{(P_\mu P *_\gamma + P *_\mu P_\gamma)}{2Z}\right) = \\
R_{\mu\gamma} - \frac{1}{2} R g_{\mu\gamma}
\end{gathered} \tag{35}
$$

for some cosmological constant λ . Also, note our choice $\sigma = 8\pi$.

We can also see that the ordinary local conservation laws are modified if $U^k;_k \neq 0$ unless the local average around charge $(-2\overline{U}^k;_k \frac{\overline{P}_\mu \overline{P}_\nu}{\overline{Z}});^\nu = 0$ which is expected due to symmetry around the charge.

Charge-less field: The term $-2U^k;_k \frac{P_\mu P_\nu}{Z}$ in (33) can be generalized to:

$-2((U^k;_k + U*^k ;_k)/2)\frac{(P_\mu P*_\nu + P*_\mu P_\nu)/2}{Z}$ and can be zero under the following condition:

$$4(A_{\mu\nu};^\mu \frac{P*^\nu}{\sqrt{Z}} + A*_{\mu\nu};^\mu \frac{P^\nu}{\sqrt{Z}}) = U_\mu U*^\mu + U*_\mu U^\mu \Rightarrow U^k;_k + U*^k ;_k = 0$$

The complimentary matrix $B^{\mu\nu} = \frac{1}{\sqrt{2}} A_{\alpha\beta}\varepsilon^{\alpha\beta\mu\nu}$ can be transformed to a real matrix due to the SU(2) x U(1) degrees of freedom and also be imaginary. From (31), (32) we have,

$$\frac{d}{dx^\mu}(\frac{\partial}{\partial P_\mu} - \frac{d}{dx^\nu}\frac{\partial}{\partial P_\mu,_\nu})(U_k U^k \sqrt{-g}) = W^\mu;_\mu \sqrt{-g} = 0$$

We recall, $W^\mu = (\frac{\partial}{\partial P_\mu} - \frac{d}{dx^\nu}\frac{\partial}{\partial P_\mu,_\nu})(U_k U^k \sqrt{-g})$

$$W^\mu =$$

$$(-4(\frac{Z^\nu}{Z^2});_\nu - 4\frac{Z_m Z^m}{Z^3})P^\mu + 4(\frac{(Z_s P^s)P^\nu}{Z^3});_\nu P^\mu - 2\frac{Z_m P^m Z^\mu}{Z^3} + 6\frac{(Z_m P^m)^2}{Z^4} P^\mu =$$

$$-4(\frac{Z^\nu}{Z^2});_\nu P^\mu - 4\frac{Z_m Z^m}{Z^3} P^\mu +$$

$$+4(\frac{(Z_s P^s)P^\nu}{Z^3});_\nu P^\mu + 4\frac{(Z_m P^m)^2}{Z^4} P^\mu$$

$$-2\frac{Z_m P^m}{Z^2}(\frac{Z^\mu}{Z} - \frac{Z_m P^m P^\mu}{Z^2}) =$$

$$-4((\frac{U^k}{Z});_k + \frac{U^k U_k}{Z})P^\mu - 2\frac{Z_m P^m}{Z^2} U^\mu = 0$$

$$W^\mu;_\mu = \left(-4U^\nu;_\nu \frac{P^\mu}{Z} - 2\frac{(Z_m P^m)}{Z^2} U^\mu\right);_\mu = 0 \tag{36}$$

9. electro-gravity –unexpected gravity induced by electric charge

We return to (34)

$$\frac{1}{4}(U_\mu U_\nu - \frac{1}{2}U_k U^k g_{\mu\nu} - 2U^k;_k \frac{P_\mu P_\nu}{Z}) = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}$$

and see a startling property of the term $\frac{1}{4}(-2U^k;_k \frac{P_\mu P_\nu}{Z})$. In comparison, $\frac{1}{4}(U_\mu U_\nu - \frac{1}{2}U_k U^k g_{\mu\nu})$ looks like an energy momentum tensor of a perfect fluid and in contrast,

$\frac{P_\mu P_\nu}{Z}$ consists of the unit vector $\frac{P_\mu}{\sqrt{Z}} = \frac{P_\mu}{\sqrt{P_k P^k}}$ which points to a perpendicular direction to U_μ.

The "source" of the Reeb vector U_μ can be defined by a non-zero $U^k;_k$ and its velocity need not be parallel to $\frac{P_\mu}{\sqrt{Z}}$. This property means that this term does not behave like as expected from an ordinary Energy – Momentum tensor. From (26) and from Einstein – Grossman's equation in vacuo,

We have that $-\frac{U^k;_k}{2} = \frac{\sqrt{\frac{4\pi K}{\varepsilon_0}\frac{\rho(\text{Charge})}{c^2}}}{\frac{8\pi K}{c^4}} = \rho(\text{Charge - Gravitational - Mass})c^2$ from which we infer $\frac{\rho(\text{Charge - Gravitational - Mass})}{\sqrt{16\pi K\varepsilon_0}} = \rho(Gravitational - Mass)$ or in terms of charge Q and mass M instead of charge density and mass density,

$$M_{\text{Charge-Gravitational-Mass}} = \frac{\pm Q}{\sqrt{16\pi K\varepsilon_0}} \tag{37}$$

This means that charge can cause gravity or anti-gravity and its sign is opposite to the acceleration field around the charge. A more general form is

$M_{\text{Charge-Gravitational-Mass}} = \frac{\pm Q}{\sqrt{2\sigma K\varepsilon_0}}$ where $\sigma = 8\pi$, K is Newton's gravity constant and ε_0 is the permittivity of vacuum.

We will calculate $\frac{\pm Q}{\sqrt{16\pi K\varepsilon_0}}$ for ± 20 Coulombs.

$$\frac{\pm 1Coulomb}{\sqrt{16\pi\varepsilon_0 K}} =\sim \pm 5.8023x10^9 Kg.$$

Multiplied by 20 we have

$$\frac{\pm 20Coulomb}{\sqrt{16\pi\varepsilon_0 K}} =\sim \pm 1.1605x10^{11} Kg.$$

This renders Alcubierre Warp Drive [19] a feasible technology by charge separation. Capacitors of several pico-Farads will not yield any measurable thrust [20] because they do not separate enough charge. However, separation of virtual charge that appears during transition states of electrons as they interact with photons, and by a short lived vacuum charge, is not ruled out because they can explain the effect known as EMDrive [21] by Warp Drive [19] caused by (37).

10. Total acceleration around electric charge

In the classical non-relativistic limit, acceleration a around a charge Q at radius r will be the result of (37) and by the acceleration field that prohibits geodesic motion, see (25),

$$-\delta a \approx \frac{-KQ}{r^2\sqrt{2\sigma K\varepsilon_0}} + \sqrt{\frac{\sigma K\varepsilon_0}{2}}\frac{Q}{4\pi\varepsilon_0 r^2} = \sqrt{\frac{\sigma K}{2\varepsilon_0}}\frac{Q}{r^2}\left(\frac{1}{\sigma} - \frac{1}{4\pi}\right) = g_{Electro\_gravity} - a \tag{38}$$

If $\sigma = 8\pi$ then $-\delta a \approx \sqrt{\frac{\sigma K}{2\varepsilon_0}}\frac{Q}{r^2}\left(\frac{1}{8\pi} - \frac{1}{4\pi}\right) = g_{Electro\_gravity} - a$

$$-\delta a \approx \sqrt{\frac{4\pi K}{\varepsilon_0}}\frac{Q}{r^2}\left(\frac{1}{8\pi} - \frac{1}{4\pi}\right) = -\sqrt{\frac{K}{16\pi\varepsilon_0}}\frac{Q}{r^2}$$

which results in a sum of accelerations $\delta a \approx \sqrt{\frac{K}{16\pi\varepsilon_0}}\frac{Q}{r^2}$

11. Proof of conservation

Theorem: Conservation law of the real Reeb vector.
From the vanishing of the divergence of Einstein tensor and (33) in the paper, we have to prove the following:

$$\frac{1}{4}\left(U_\mu U_\nu - \frac{1}{2}U_k U^k g_{\mu\nu} - 2U^k{;_k}\frac{P_\mu P_\nu}{Z}\right){;^\mu} = G_{\mu\nu}{;^\mu} = (R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}){;^\mu} = 0 \tag{39}$$

Proof:
From the zero variation by the scalar time field (36)

$$W^\mu{;_\mu} = \left(-4U^\nu{;_\nu}\frac{P^\mu}{Z} - 2\frac{(Z_m P^m)}{Z^2}U^\mu\right){;_\mu} = 0 \tag{40}$$

$$-\left(2U^\nu{;_\nu}\frac{P^\mu}{Z}\right){;_\mu} = \left(\frac{(Z_m P^m)}{Z^2}U^\mu\right){;_\mu} \tag{41}$$

$$\left(-2U^k{;_k}\frac{P^\mu P^\nu}{Z}\right){;_\mu} = \left(\frac{(Z_m P^m)}{Z^2}U^\mu\right){;_\mu}P^\nu - \left(2U^k{;_k}\frac{P^\mu}{Z}\right)P^\nu{;_\mu} =$$
$$\left(\frac{(Z_m P^m)}{Z^2}U^\mu\right){;_\mu}P^\nu - U^k{;_k}\frac{Z^\nu}{Z} \tag{42}$$

Now let $t \equiv Z_m P^m$

$$\left(\frac{t}{Z^2}U^\mu\right){;_\mu}P^\nu - U^k{;_k}\frac{Z^\nu}{Z} = \left(\frac{t}{Z^2}\right){;_\mu}U^\mu P^\nu + \frac{t}{Z^2}U^\mu{;_\mu}P^\nu - U^k{;_k}\frac{Z^\nu}{Z} =$$
$$-U^\mu{;_\mu}U^\nu + \left(\frac{t}{Z^2}\right){;_\mu}U^\mu P^\nu$$

so,

$$\left(-2U^k{;_k}\frac{P^\mu P^\nu}{Z}\right){;_\mu} = -U^\mu{;_\mu}U^\nu + \left(\frac{t}{Z^2}\right){;_\mu}U^\mu P^\nu \tag{43}$$

$$\left(U^\mu U^\nu - \frac{1}{2}U_k U^k g^{\mu\nu} - 2U^k;_k \frac{P^\mu P^\nu}{Z}\right);^\mu =$$

$$U^\mu;_\mu U^\nu + U^\mu U^\nu;_\mu - \frac{1}{2}(U_k;_\mu U_s + U_k U_s;_\mu)g^{ks}g^{\mu\nu} -$$

$$U^\mu;_\mu U^\nu + (\frac{t}{Z^2});_\mu U^\mu P^\nu =$$

$$U^\mu U^\nu;_\mu - \frac{1}{2}(U^s U_s);^\nu + (\frac{t}{Z^2});_\mu U^\mu P^\nu = 0 \tag{44}$$

Notice that

$$U^\mu U^\nu;_\mu - \frac{1}{2}U^s U_s;^\nu =$$

$$U^\mu\left((\frac{Z_k}{Z});_\mu - (\frac{t}{Z^2});_\mu P_k - (\frac{t}{Z^2})P_k;_\mu\right)g^{k\nu} -$$

$$U^s\left((\frac{Z_s}{Z});_k - (\frac{t}{Z^2});_k P_s - (\frac{t}{Z^2})P_s;_k\right)g^{k\nu} =$$

$$-U^\mu(\frac{t}{Z^2});_\mu P^\nu \tag{45}$$

Since $-(\frac{t}{Z^2});_k P_s U^s = 0$

$$U^\mu U^\nu;_\mu - \frac{1}{2}(U^s U_s);^\nu + (\frac{t}{Z^2});_\mu U^\mu P^\nu = -U^\mu(\frac{t}{Z^2});_\mu P^\nu + (\frac{t}{Z^2});_\mu U^\mu P^\nu = 0 \tag{46}$$

and we are done.

Conclusion

An upper limit on measurable time from each event backwards to the "big bang" singularity as a limit or from a manifold of events as in de Sitter or anti - de Sitter, may exist only as a limit and is not a practical physical observable because it can only be theoretically measured. Since more than one curve on which such time can be virtually measured intersects the same event - as is the case in material fields which prohibit inertial motion, i.e. prohibit free fall - such a time can't be realized as a coordinate. Nevertheless, using such time as a scalar field, enables to describe matter as acceleration fields by using the gradient of the scalar field and it allows new physics to emerge by a replacement of the stress-energy-momentum tensor. One arrives at electro-gravity as a neat explanation of the Dark Matter effect and the advent of Sciama's Inertial Induction, which becomes realizable by separation of high electric charge. This paper totally rules out any measurable Biefeld Brown effect in vacuum on Pico-Farad or less, Ionocrafts due to insufficient amount of electric charge [20]. The electro-gravitational effect is due to field divergence and not directly due to intensity or gradient of the square norm. Inertial motion prohibition by material fields, e.g. intense electrostatic field, can be measured as a very small mass dependent force on neutral particles that have rest mass and thus can measure proper time. The non-gravitational acceleration should be from the positive to the negative charge. The electro-gravitational effect which is opposite in direction and half in intensity, requires large amounts of separated charge carriers and acts on the entire negative to positive dipole.

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Also, in the Book of Principals by the philosopher Rabbi Joseph Albo, essay 18 appears to be the first known historical account of what Measurable Time – In Hebrew "Zman Meshoar" and Immeasurable Time "Zman Bilti Meshoar" are. His Idea of the immeasurable time as a limit [22], is the very reason for 11 years of research and for this paper.

AppendixA– The time field in the Schwarzschild solution
Motivation: To make the reader familiar with the idea of maximal proper time from a sub-manifold and to calculate the background scalar time field of the Schwarzschild solution from that sub-manifold. We choose as a sub-manifold, a small 3 dimensional 3-spehere around the "Big Bang" singularity and therefore this example is limited to a "Big Bang" manifold. So, we want to connect each event in a Schwarzschild solution to a primordial sub-manifold a fraction of second after the presumed "Big Bang", with the longest possible curve under the assumption that no closed time-like curves occur.
In this limited case, the scalar field is uninteresting as it does not represent interactions with any charged particle or with other force fields and therefore, the Reeb vector is zero.

We would like to calculate $\frac{U_s U^s}{4} = \frac{1}{4}\left(\frac{(P_\lambda P^\lambda),_m (P_\lambda P^\lambda),_k g^{mk}}{(P_i P^i)^2} - \frac{((P_\lambda P^\lambda),_m P^m)^2}{(P_i P^i)^3}\right)$ in Schwarzschild coordinates for a freely falling particle. This theory predicts that where there is no matter, the result must be zero. The speed U of a falling particle from very far away, as measured by an observer in the gravitational field is

$$V^2 = \frac{U^2}{c^2} = \frac{R}{r} = \frac{2GM}{rc^2} \tag{A.1}$$

Where R is the Schwarzschild radius. If speed V is normalized in relation to the speed of light then $V = \frac{U}{c}$. For a far observer, the deltas are denoted by dt', dr' and,

$$\dot{r}^2 = \left(\frac{dr}{dt}\right)^2 = V^2\left(1 - \frac{R}{r}\right) \tag{A.2}$$

because $dr = dr' / \sqrt{1 - R/r}$ and $dt = dt'\sqrt{1 - R/r}$.

$$P=\int_0^t\left((1-\frac{R}{r})-\frac{\dot{r}^2}{(1-\frac{R}{r})}\right)^{\frac{1}{2}}dt=\int_0^t\left((1-\frac{R}{r})-\frac{\frac{R}{r}(1-\frac{R}{r})^2}{(1-\frac{R}{r})}\right)^{\frac{1}{2}}dt=$$

$$\int_0^t\left((1-\frac{R}{r})^2\right)^{\frac{1}{2}}dt=\int_0^t(1-\frac{R}{r})dt$$

which results in,

$$P_t=\frac{dP}{dt}=(1-\frac{R}{r}) \tag{A.3}$$

Here t is not a tensor index and it denotes derivative by t!
On the other hand

$$P=\int_0^r\left((1-\frac{R}{r})\frac{1}{\dot{r}^2}-\frac{1}{(1-\frac{R}{r})}\right)^{\frac{1}{2}}dr=\int_0^r\left(\frac{(1-\frac{R}{r})\frac{r}{R}}{(1-\frac{R}{r})^2}-\frac{1}{(1-\frac{R}{r})}\right)^{\frac{1}{2}}dr=\int_0^r\left(\frac{\frac{r-R}{R}}{\frac{r-R}{r}}\right)^{\frac{1}{2}}dr=$$

$$\int_0^r\sqrt{\frac{r}{R}}dr$$

Which results in

$$P_r=\frac{dP}{dr}=\sqrt{\frac{r}{R}} \tag{A.4}$$

Here, r is not a tensor index and it denotes derivative by r !
For the square norms of gradients, we use the inverse of the metric tensor,

So, we have $(1-\frac{R}{r})\rightarrow(1-\frac{R}{r})^{-1}$ and $(1-\frac{R}{r})^{-1}\rightarrow(1-\frac{R}{r})$

So, we can write

$$N^2=P_\lambda P^\lambda=(1-\frac{R}{r})P_r^{\,2}-(1-\frac{R}{r})^{-1}P_t^{\,2}=(1-\frac{R}{r})(\frac{r}{R}-1)=\frac{r}{R}+\frac{R}{r}-2$$

$$N^2=\frac{r}{R}+\frac{R}{r}-2 \tag{A.5}$$

$N^2{}_\lambda=\dfrac{dN^2}{dx^\lambda}$ And we can calculate

$$\frac{N^2{}_\lambda N^{2\lambda}}{(N^2)^2}=\frac{(1-\frac{R}{r})^2(\frac{1}{R}-\frac{R}{r^2})^2}{(\frac{r}{R}+\frac{R}{r}-2)^2} \qquad (A.6)$$

We continue to calculate

$$N^2{}_t P_t=(1-\frac{R}{r})^2(\frac{1}{R}-\frac{R}{r^2})\sqrt{\frac{R}{r}} \text{ and } \frac{N^2{}_t P_t}{(1-\frac{R}{r})}=(1-\frac{R}{r})(\frac{1}{R}-\frac{R}{r^2})\sqrt{\frac{R}{r}} \qquad (A.7)$$

Note that here t is not a tensor index and it denotes derivative by t !

$$(1-\frac{R}{r})N^2{}_r P_r=(1-\frac{R}{r})(\frac{1}{R}-\frac{R}{r^2})\sqrt{\frac{r}{R}} \qquad (A.8)$$

Please note, here r is not a tensor index and it denotes derivative by r !

$$N^2{}_\lambda P^\lambda=(1-\frac{R}{r})(\frac{1}{R}-\frac{R}{r^2})(\sqrt{\frac{r}{R}}-\sqrt{\frac{R}{r}}) \text{ and}$$

$$(N^2{}_\lambda P^\lambda)^2=(1-\frac{R}{r})^2(\frac{1}{R}-\frac{R}{r^2})^2(\frac{r}{R}+\frac{R}{r}-2) \qquad (A.9)$$

So

$$\frac{(N^2{}_\lambda P^\lambda)^2}{(N^2)^3}=\frac{(1-\frac{R}{r})^2(\frac{1}{R}-\frac{R}{r^2})^2}{(\frac{r}{R}+\frac{R}{r}-2)^2} \qquad (A.10)$$

And finally, from (A.6) and (A.10) we have,

$$\frac{(P^\lambda P_\lambda),_m (P^s P_s),_k g^{mk}}{(P_i P^i)^2}-\frac{(P^\lambda P_\lambda),_m P^m}{(P_i P^i)^3}=$$

$$\frac{N^2{}_\lambda N^{2\lambda}}{(N^2)^2}-\frac{N^2{}_\lambda P^\lambda}{(N^2)^3}=\frac{(1-\frac{R}{r})^2(\frac{1}{R}-\frac{R}{r^2})^2}{(\frac{r}{R}+\frac{R}{r}-2)^2}-\frac{(1-\frac{R}{r})^2(\frac{1}{R}-\frac{R}{r^2})^2}{(\frac{r}{R}+\frac{R}{r}-2)^2}=0 \qquad (A.11)$$

which shows that indeed the gradient of time measured, by a falling particle until it hits an event in the gravitational field, has zero curvature as expected.

Appendix B – Planck Area Gravity – Based on a lecture by professor Seth Lloyd of the M.I.T combined with the Geometric Chronon model and its correlation with sub-atomic particles

Suppose we have an atomic length L , The speed of light is c so the maximal acceleration will be

$\frac{\frac{c}{L}}{c} = \frac{c^2}{L}$. By the real case (33), $\frac{1}{4}(U_\mu U_\nu - \frac{1}{2}U_k U^k g_{\mu\nu} - 2U^k;_k \frac{P_\mu P_\nu}{Z}) = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}$

Then

$$\frac{1}{4}(U_\mu U_\nu - \frac{1}{2}U_k U^k g_{\mu\nu} - 2U^k;_k \frac{P_\mu P_\nu}{Z})\frac{P^\mu P^\nu}{Z} = (R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu})\frac{P^\mu P^\nu}{Z}$$

which becomes

$$-\frac{1}{8}U_k U^k - \frac{1}{2}U^k;_k = (R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu})\frac{P^\mu P^\nu}{Z} \tag{B.1}$$

If the right-hand side if multiplied by $\frac{1}{2}$ and then by $\frac{\pi}{12}L^4$ then it yields the missing or added area to the sphere perpendicular to the unit vector $\frac{P^\mu}{\sqrt{Z}}$ [23], [24]. The term $\frac{1}{2}$ is required because $\frac{1}{2}(R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu})\frac{P^\mu P^\nu}{Z}$ is the sum of sectional curvatures of the infinitesimal 3-volume ball which is perpendicular to the vector P^μ. When this sum of sectional curvatures is multiplied by $\frac{\pi}{12}L^4$, it yields the area that is subtracted or added due to gravity or anti-gravity.

$$\frac{\pi}{24}(R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu})\frac{P^\mu P^\nu}{Z}L^4 = -Area \tag{B.2}$$

Then by (B.1)

$$\frac{\pi}{24}(-\frac{1}{8}U_k U^k - \frac{1}{2}U^k;_k)L^4 = -Area \tag{B.3}$$

We replace $-\frac{1}{2}U_k$ by 4-acceleration divided by c^2 and we have $-\frac{1}{2}U_k = \frac{c^2}{Lc^2} = \frac{a_k}{c^2} = \frac{1}{L}$ so (B.3) becomes

$$\frac{\pi}{24}(\frac{1}{2}\frac{1}{L^2} \pm \frac{1}{c^2}\frac{c^2}{L}\frac{1}{L})L^4 = \frac{\pi}{24}L^2(\frac{1}{2} \pm 1) = -Area \tag{B.4}$$

Because $-\frac{1}{8}U_k U^k g_{\mu\nu} = \frac{1}{2}\frac{1}{L}\frac{1}{L} = \frac{1}{2}\frac{1}{L^2}$ and also $-\frac{1}{2}U^k;_k = \pm\frac{1}{c^2}\frac{c^2}{L^2}$

Which is either an addition to the area or subtraction from the area due to the divergence term $\frac{1}{4}(-2U^k;_k) = -\frac{1}{2}U^k;_k$ so, $-\frac{\pi}{24}L^2\frac{3}{2} = Area$ or $+\frac{\pi}{24}L^2\frac{1}{2} = Area$

Divide these areas by the area of the two-dimensional sphere $4\pi L^2$ and we have ratios,

$$\frac{-\frac{\pi}{24}L^2\frac{3}{2}}{4\pi L^2} = \frac{-\frac{1}{24}\frac{3}{2}}{4} = -\frac{1}{64} \quad \text{or} \quad \frac{+\frac{1}{24}\frac{1}{2}}{4} = +\frac{1}{192} \tag{B.5}$$

If we consider that we have an "arrow" which is the Reeb vector U_μ alone with an infinitesimally small support and it is within a ball of radius r and we attribute the divergence of the field to this "arrow" only, then we should not consider the increase or decrease of area around the ball due to the Ricci curvature as influencing U_μ due to Gauss law as we should if we consider the source of the field as a charge – like phenomenon. Then we can say that the field appears along a distance L and has induced a geodesic motion prohibition as an acceleration, $\frac{c^2}{L}$ and $\frac{1}{4}U_\lambda U^\lambda = \frac{1}{L^2}$ and $\frac{1}{2}U_k;^k = \frac{1}{L^2}$.
We can play with the added portion area as a portion of energy around a particle. Let us consider, for instance, the $\frac{1}{192}$ portion of the Muon energy, 105.6583745(24) MeV
This value is about 0.550304033979167 MeV and the electron mass is about 0.5109989461 MeV.
It is a nice thought experiment but we need much more than that. We can't solve (34) or (35) yet but we can at least have a better idea of the field behavior in the Planck scale.

We didn't take into account that the geodesic motion prohibition field i.e. acceleration field changes its density on the sphere in accordance with increased or decreased area ratio, $\alpha = \frac{Area}{4\pi L^2} = 1 + \frac{added\ area}{4\pi L^2}$. We consider the Gauss law around an electric charge. So here we present a second approach to area addition and subtraction around an electrically charged particle. (B.4) can be rewritten as a more enlightening term,

$$1+\frac{\frac{\pi}{24}\left(-\frac{1}{2}\frac{1}{\alpha^2 L^2}\pm\frac{1}{c^2}\frac{c^2}{L\alpha}\frac{1}{L}\right)L^4}{4\pi L^2} = 1+\frac{1}{96}\left(-\frac{1}{2}\alpha^{-2}\pm\alpha^{-1}\right) = \frac{4\pi L^2 + Area}{4\pi L^2} = \alpha \quad \text{(B.6)}$$

Such that α is either bigger than 1 or smaller than 1 and denotes the increase or decrease in area. Note that the term α measures how much the square acceleration field changes as the area grows or dwindles.
The resulting equation is a cubic equation: $1+\frac{1}{96}\left(-\frac{1}{2}\alpha^{-2}\pm\alpha^{-1}\right)=\alpha$ that can be easily solved numerically.

$$1+\frac{1}{96}\left(-\frac{1}{2}a^{-2}\pm a^{-1}\right) = a \Rightarrow 192a^3 = 192a^2 \pm 2a - 1 \Rightarrow a^3 = \frac{192a^2 \pm 2a - 1}{192} \quad \text{(B.7)}$$

The area is increased or decreased by α and the portion of the area that changes is

$$\alpha \sim= 1.00520819\,3610747100 \Rightarrow \left(\frac{1}{1-\alpha}\right)^{-1} \sim= +(192.005150\,87160028...)^{-1}$$

around a negative charge or

$$\alpha \sim= 0.99207267\,636432284 \Rightarrow \left(\frac{1}{\alpha-1}\right)^{-1} \sim= -(62.6395393\,39674555...)^{-1}$$

around a positive charge. The problem is that there is no stable charged particle without spin and therefore our discussion could mean a temporary decomposition of electrically neutral Bosons into two energy states, one temporarily behaving like a negative charge and one like a positive one. The reasoning behind such a claim is that if matter is expressible by a weak acceleration field and the weak acceleration field energy is the energy of an electric field, then elementary neutral particles, even with zero magnetic momentum and with zero electric dipole, should have an internal electric field.
The question is how to infer such a structure. The idea is that area changes are relative to energy ratios even if they are changes due to charge electro-gravity and not due to inertial mass. It is a manifestation of a holographic principle [23], [24]. Our modest test will be to divide the Higgs energy by 2 and then either by 192.005150 ...,or by 62.6395393 .... That is by Beta = **384.010301743200560** or by Alpha =**125.279078679349110**.

For example: 125 GeV / 125.279078679349110 ~= **0.9977 GeV** which should be a Baryonic energy state. Another energy is 125 GeV / 384.01030174320056 ~= **325.5 MeV**
This energy is the model dependent vacuum constituent Quark energy according to Zhao Zhang et. al. [25].

According to this paper, no neutral particle can avoid having an internal structure, otherwise, the particle would not be able to manifest an acceleration field as energy. This leads to the possible model of BS Meson, Z Boson and Higgs Boson as either oscillating + and – charge such that both the magnetic and electric dipoles are zero, or as spinning + and - charge such that both magnetic and electric dipoles vanish. The problem is the Z and the Higgs bosons which are considered elementary particles. The Z boson mass is $91.1876 \pm 0.0021\, GeV/C^2$. If we split this mass into two charges, then 1/ 192.00515087160028 of area around the negative charge will be added, which is considered as proportional to mass [23]. But that portion is of half of the mass that splits to two charges, so we seek1/384.00258393161619 of the mass of the Z boson as having a physical meaning.

$$91.1876\,GeV/(384.0103) = 237.4613\ \mathrm{MeV} \quad \text{(B.8)}$$

which is the energy difference between the Phi (previously Eta) and Omega Mesons !

By the Checkered Board Model and EMS [26] the mass of the up Quark deviates from the Standard Model's $\sim 2.3\,\mathrm{MeV/C^2}$ and is $237.31\,\mathrm{MeV/C^2}$ according to that very same model, the down Quark is $42.39\mathrm{Mev/C^2}$ unlike the S.M. $\sim 4.8\,\mathrm{MeV/C^2}$. The Z boson can contribute to mass fluctuations through half of its mass by area fluctuations around a positive charge too but that yields $727.87572\ \mathrm{MeV}$ and there is no known 727.876 MeV resonance in the particles world.

Here is a summary of the electro-gravity energy in the Planck scale around a positive and a negative charge that split an elementary boson and by this, these energies are beyond the Standard Model.

Table 1. Presumed Beta and Alpha energy ratios due to a splitting of an Elementary charge-less boson into positive and negative charge – a supposed process which is beyond the Standard Model. A second assumption is that the magnetic moments and the electric dipoles of such bosons are zero, therefore split charge should be fluctuating and so is the area around positive and negative charge.

	Energy delta (GeV)	**Energy sum (GeV)**	Area expansion ratio around e- r2=1.005208194 Beta=2/(r2-1) **1/384.0103087 of the energy**	Area contraction ration around e+ r1=0.984035643 Alpha=2/(1-r1) **1/125.27908 of the energy**
Higgs Boson energy portion	125 GeV *(1/Alpha-1/Beta)	125 GeV *(1/Alpha+1/Beta)	125 GeV/Beta	125 GeV/Alpha
Higgs Boson electro-gravitational energy	0.67226024 Delta of the last two cells in this row	1.323284428 About **1.323GeV** Sum of the next two cells	Gravitational energy delta around e- 0.325512095 GeV About **325.5 MeV**	Gravitational energy delta around e+ 0.997772333 GeV About **1.0 GeV**
Z Boson	91.1876Ge	91.1876GeV	91.1876	91.1876

Energy portion	V *(1/Alpha-1/Beta)	*(1/Alpha+1/Beta)	GeV/Beta	GeV/Alpha
Z Boson electro-gravitationa -l energies	0.49041438 Delta of the last two cells in this row	0.965337049 About **0.965GeV** Sum of the next two cells	Gravitational energy delta around e- 0.237461334 GeV About **237.46 MeV**	Gravitational energy delta around e+ 0.727875715 GeV About **728 MeV**

The mass ratio between Muon and the Electron

(B.7) took into account the change of a unit field through a unit area as that area expands due to non – zero Einstein Tensor. The field however should be concentrated along an "equator" of a ball and zero at its poles due to spin around a negative charge. In this calculation, we only take into account the uneven distribution of the field but not its motion due to spin. The integration of the acceleration field should be of a field,

$$\frac{c^2}{r}\cos(\emptyset) \tag{B.9}$$

Where $\emptyset$ is the angle from the "equator". A ball integration on two hemispheres is then,

$$\begin{gathered} 2\int_{\emptyset=0}^{\emptyset=\frac{\pi}{2}} \frac{c^2}{r}\cos(\emptyset)\cdot 2\pi \mathrm{r}\cdot\cos(\emptyset)\cdot \mathrm{r}\cdot d\emptyset = \\ 4\pi c^2 \mathrm{r}\int_{\emptyset=0}^{\emptyset=\frac{\pi}{2}} \frac{1+\cos^2(\emptyset)}{2} d\emptyset = \\ 4\pi c^2 \mathrm{r}\cdot \left.\frac{\emptyset+\frac{1}{2}\sin(2\emptyset)}{2}\right]_{\emptyset=0}^{\emptyset=\frac{\pi}{2}} = 4\pi c^2 \mathrm{r}\cdot\frac{\pi}{4} \end{gathered} \tag{B.10}$$

If the field is uniform then the integration would be

$$2\int_{\emptyset=0}^{\emptyset=\frac{\pi}{2}} \frac{c^2}{r} 2\pi \mathrm{r}\cdot\cos(\emptyset)\cdot \mathrm{r}\cdot d\emptyset = 4\pi \mathrm{r} c^2 \tag{B.11}$$

And the ratio between (B.11) and (B.10) is

$$\frac{4}{\pi} \tag{B.12}$$

which means that the acceleration field has to grow by $\frac{4}{\pi}$ in order to sum up as in (B.11). We can imagine U_λ as a vector that points towards or outwards - of an integral curve in space-time but that the Minkowsky norm of the field is always the same, only the probability that this vector points towards a certain direction in space-time changes. This idea leads to the compensating scaling value in (B.12).

So equation (B.7) becomes a different equation,

$$1+\frac{1}{96}\left(-\frac{1}{2}\left(\frac{4}{\pi}\right)^2 a^{-2}+\frac{4}{\pi}a^{-1}\right)=a \quad \text{(B.13)}$$

But $a=1+\frac{Added\ or\ subtracted\ area}{4\pi r^2}$

The ratio $\frac{4\pi r^2}{Added\ or\ subtracted\ area}=\frac{1}{a-1}\cong \mathbf{206.751340}$, which is very close to the ratio between the mass of the Muon to the mass of the Electron, **206.768277**. The difference is expected because we did not take into account a spinning field. We followed the M.I.T professor Seth Lloyd offer that addition or subtraction of quantum area, means addition or subtraction of energy and reached 206.751340. For the solution of

$$1+\frac{1}{96}\left(-\frac{1}{2}\left(\frac{4}{\pi}\right)^2 a^{-2}-\frac{4}{\pi}a^{-1}\right)=a \quad \text{(B.14)}$$

We get about 44.63955018. These ratios ~1/45 and ~1/207 could mean a decay path for charged leptons where the numerical stability of 1/45 is worse than that of 1/207.

A more exact root for (B.13) yields,

$$\frac{4\pi r^2}{Added\ or\ subtracted\ area}=\frac{1}{a-1}\cong 206.75133988502202 \quad \text{(B.15)}$$

The difference in accuracy in this alue by 64 and 128 bits is just the last 2 - 3 digits.
If we divide the Muon energy by this value we get very close to the energy of the electron and the delta in Mega electron volts is:
105.658745 MeV / 206.75133988502202 – 0.5109989461 (MeV) = 0.00004187500790 MeV
Which is **41.87500790 eV**. That energy is small but beyond the energy of any Neutrino mass. It is an unknown energy. Should it be a particle, this particle is beyond the Standard Model and its existence should manifest itself through a g-2 Muon anomaly.
The ratio between the electron's energy and this energy is
0.5109989461 / 0.00004187500790 which is approximately**, 12202.95760492718728** almost 12203.
We can get this value if we return to the (B.7) roots and see that their multiplications a bit lower than 12203, but (B.7) assumes a gravitational field caused by an acceleration field around a negative and around a positive charge with no spin. The roots of (B.7) yield 1+ and 1- area ratios. These area ratios multiply to 12027.11454948692699 and not to 12027 < 12203. The exact number is obtained if we look at the following polynomials:

$$1+\frac{1}{96}\left(-\frac{1}{2}(1-\frac{1}{96})^2 a^{-2}\pm\left(1-\frac{1}{96}\right)a^{-1}\right)=a \quad \text{(B.16)}$$

Which is 1 + or 1- the portion of area added around a negative charge or subtracted around a positive charge such that the acceleration field is smaller by a factor of $(1-\frac{1}{96})$.
The idea to use a damping of $(1-\frac{1}{96})$ is because of the factor $\frac{1}{96}$ in (B.7). This implies that charge fluctuations could be of the order $\frac{1}{96}$ of the charge of the electron e.
The two polynomials in (B.16) with the $\pm$ sign have each 3 roots each and the big roots are a=1.00520707510980 for (+) and b=0.98426221868924 for (-).
And

$$\left(\frac{1}{a-1}\right)\left(\frac{1}{1-b}\right)=12202.88874066467724 \quad \text{(B.17)}$$

Which with numerical accuracy is even closer to 12202.95760492718728.

Other choices except for 96 in $(1-\frac{1}{96})$ are further away from 12202.95760492718728 even after 4 digits after the floating point. The third root is from (B.13) $1+\frac{1}{96}\left(-\frac{1}{2}\left(\frac{4}{\pi}\right)^2 c^{-2}+\frac{4}{\pi}c^{-1}\right)=c$
So the following system is

$$1+\frac{1}{96}\left(-\frac{1}{2}(1-\frac{1}{96})^2 a^{-2}+\left(1-\frac{1}{96}\right)a^{-1}\right)=a$$
$$1+\frac{1}{96}\left(-\frac{1}{2}(1-\frac{1}{96})^2 b^{-2}-\left(1-\frac{1}{96}\right)b^{-1}\right)=b$$
$$1+\frac{1}{96}\left(-\frac{1}{2}\left(\frac{4}{\pi}\right)^2 c^{-2}+\frac{4}{\pi}c^{-1}\right)=c$$

$$MuonMass\,\frac{(c-1)}{(1+(a-1)(1-b))}=ElctronMass \qquad \text{(B.18)}$$

~ 0.51099894586371 MeV instead of 0.5109989461 MeV.

The mass ratio between W Boson and the Tau particle

The calculation in (B.18) turned out to be very accurate. In fact, if we update the Muon energy from 105..6583745 MeV to 105.65837455 MeV then (B.18) yields the energy 0.5109989461 MeV.
We did not use, however, the other root of (B.14) $1+\frac{1}{96}\left(-\frac{1}{2}\left(\frac{4}{\pi}\right)^2 c^{-2}-\frac{4}{\pi}\mathrm{c}^{-1}\right)=c$ for which $44.63955018=\frac{1}{(1-\mathrm{c})}$
This ratio is very close to the mass ratio between the W boson and the Tau particle.

$\frac{80385\ MeV}{44.639550144.6395501759681596986501771036198}$ =1800.75739300965255 MeV

The energy of the Tau particle is 1776.82 MeV so the delta is 23.93739300965255 MeV. Dividing this energy by the Tau energy yields 0.01347204163035791470154545761529
$=\frac{1}{74.227799129316732925524252770539}$

To understand where such a value can come from, we will return to (B.1) but this time with $\mathrm{U}_\lambda\mathrm{U}^\lambda=0$ when U_λ is in the real numbers format.
(B.7) transforms into a simpler equation $a^2=\frac{192a\pm2}{192}$ instead of $a^3=\frac{192a^2\pm2a-1}{192}$ because $\mathrm{U}_\lambda\mathrm{U}^\lambda=0$ which means $1+\frac{1}{96}\left(-\frac{1}{2}a^{-2}\pm a^{-1}\right)=a \Rightarrow 1+\frac{1}{96}(\pm a^{-1})=a$.
In the very same manner (B.13) and (B.14) turn into $1+\frac{1}{96}\left(\mp\frac{4}{\pi}a^{-1}\right)=a$ which yields the following second order polynomial root equations, $a^2-a-\frac{1}{96}\left(\mp\frac{4}{\pi}\right)=0$. The roots are easy to calculate especially that we know that one has to be slightly above 1 and one root is below 1.

$$a=\frac{1+\sqrt{1-4*\frac{4}{\pi}\frac{1}{96}}}{2}=\frac{1+\sqrt{1-\frac{1}{6\pi}}}{2} \qquad \text{(B.19)}$$

$$b=\frac{1+\sqrt{1+4*\frac{4}{\pi}\frac{1}{96}}}{2}=\frac{1+\sqrt{1+\frac{1}{6\pi}}}{2} \qquad \text{(B.20)}$$

$$a = 0.98655635652581$$

and $\frac{1}{(1-a)} = \sim 74.384596848307264667856634332603$

very close to the ratio 74.227799129316732925524252770539.

From the roots of the polynomials

$$a^2 - a + \frac{1}{96}\left(\frac{4}{\pi}\right) = 0 \tag{B.21}$$

$$1 + \frac{1}{96}\left(-\frac{1}{2}\left(\frac{4}{\pi}\right)^2 c^{-2} - \frac{4}{\pi}c^{-1}\right) = c$$

We get a pretty good match to the ratio between the mass of the W boson and the mass of the Tau particle.

$$80385\ MeV\,\frac{1-c}{1+(1-a)} = \sim \mathbf{1776.86978906557101}\ MeV \tag{B.22}$$

The exact value of **1776.82** MeV is obtained if 80.385 GeV is replaced by **80.3829 GeV**, which is **80.385-0.0021 GeV**.

There are several questions that (B.21) raise. The first is why does (B.21) describe area reduction as expected around a positive charge ? The author expected calculations of area addition around a negative charge. The other question is, why the Reeb vector that is used by the area ratio in the first equation in (B.21) is a null vector, $U_\lambda U^\lambda = 0$? How is a null vector related to an energy potion of the W boson ?

We could choose a different polynomial than in (B.21). We could choose one for a negative charge and one for a positive and use the following field scaling factor.

$$(1 - \frac{1}{96})\left(\frac{4}{\pi}\right) = 0 \tag{B.23}$$

We used $(1 - \frac{1}{96})$ and $\left(\frac{4}{\pi}\right)$ before. The combination of the two by multiplication, reflects ideas from (B.18).

So we have two polynomials

$$1 + \frac{1}{96}\left(-\frac{1}{2}\left(\frac{4}{\pi}\right)^2 c^{-2} - \frac{4}{\pi}c^{-1}\right) = \mathrm{c} \tag{B.24}$$

$$1 + \frac{1}{96}\left(-\frac{1}{2}\left(\frac{4}{\pi}(1 - \frac{1}{96})\right)^2 \mathrm{a}^{-2} + \frac{4}{\pi}(1 - \frac{1}{96})a^{-1}\right) = a$$

and

$$1 + \frac{1}{96}\left(-\frac{1}{2}\left(\frac{4}{\pi}(1 - \frac{1}{96})\right)^2 \mathrm{b}^{-2} - \frac{4}{\pi}(1 - \frac{1}{96})b^{-1}\right) = b$$

The average of area ratios is a bit surprising if we take out the sign, which means that like in (B.18) we used the term $(a-1) * (1-b)$ for different values of $a,\ b$.

The result is then

$$\frac{\frac{1}{a-1}+\frac{1}{1-b}}{2}=\frac{2}{a-b}=74.24076204988859$$

That is even closer to the value 74.227799129316732925524252770539 we have been looking for. The value is also close to $(\frac{1}{a-1}*\frac{1}{1-b})^{1/2}=\sim\frac{2}{a-b}$ in this specific case.

$$80385\ MeV\frac{1-c}{1+(a-b)/2}=\sim\mathbf{1776.82412407487959}\ \boldsymbol{MeV} \quad \text{(B.25)}$$

Even closer to **1776.82 MeV**

So which seems a correct way to guess the rules that lead to mass of the Tau as a residual electro-gravitational energy of the W+ boson, (B.25) or (B.22) ? The author prefers (B.25) because a null Reeb vector doesn't seem plausible when discussing particles with rest masses.

We can calculate (B.25) based on null Reeb vectors and use (B.19) and (B.20)

In that case we have

$$80385\ MeV\frac{1-c}{1+(a-b)/2}=\sim\mathbf{1777.17852813806985}\ \boldsymbol{MeV} \quad \text{(B.26)}$$

which is too big. Only if we consider a lower W+ boson mass **80369 MeV** we then get a reasonable answer, **1776.82479477425545 MeV**

More accurate W boson results are required in order to find the best candidate polynomial roots.

The Python code that was used for the mass ratio calculations out of area ratios

```
import numpy as NP

class ELECTROGRAVITY_CLASS:

    def function_cubic_viete(self, a, b, c, d): # If all roots are real.

        # Viete's algorithm when all roots are real.

        b2 = NP.longdouble(b * b)
        b3 = NP.longdouble(b2 * b)
        a2 = NP.longdouble(a * a)
        a3 = a2 * a

        p = (3 * a * c - b2) / (3 * a2)

        q = (2 * b3 - 9 * a * b * c + 27 * a2 * d) / (27 * a3)

        offset = b / (3 * a)

        t1 = 2 * NP.sqrt(-p / 3) * NP.cos(NP.arccos(NP.sqrt(-3 / p) * (3 * q) / (2 * p)) / 3)
        t2 = 2 * NP.sqrt(-p / 3) * NP.cos(NP.arccos(NP.sqrt(-3 / p) * (3 * q) / (2 * p)) / 3 - NP.pi / 3)
        t3 = 2 * NP.sqrt(-p / 3) * NP.cos(NP.arccos(NP.sqrt(-3 / p) * (3 * q) / (2 * p)) / 3 - 2 * NP.pi / 3)

        x1 = t1 - offset
        x2 = t2 - offset
        x3 = t3 - offset

        return (x1, x2, x3)
```

```
MAIN_electrogravity_class = ELECTROGRAVITY_CLASS()

f = 1 - 1/96

x1, x2, x3 = \
MAIN_electrogravity_class.function_cubic_viete(1, -1, -f / 96, (f * f) / 192)

x4, x5, x6 = \
MAIN_electrogravity_class.function_cubic_viete(1, -1, f / 96, (f * f) / 192)

f = 4 / NP.pi

x7, x8, x9 = \
MAIN_electrogravity_class.function_cubic_viete(1, -1, -f / 96, (f * f) / 192)

print("Anti-gravity: X1,X2,X3 = (%.14lf, %.14lf, %.14lf)" %(x1, x2, x3))
print("Gravity: X4,X5,X6 = (%.14lf, %.14lf, %.14lf)" %(x4, x5, x6))
print("Anti-gravity: X7,X8,X9 = (%.14lf, %.14lf, %.14lf)" %(x7, x8, x9))

print("Muon mass in MeV/C^2 105.6583745")
print("Predicted electron mass im MeV/C^2 %.14lf" % ((105.6583745 * (x7 - 1)) / (1 + (x1-1)*(1-
x4))))
0
x4, x5, x6 = \
MAIN_electrogravity_class.function_cubic_viete(1, -1, f / 96, f*f / 192)

x8 = (1 + NP.sqrt(1 - 1/(NP.pi * 6)))/2

print("Gravity: X4,X5,X6 = (%.14lf, %.14lf, %.14lf)" %(x4, x5, x6))
print("Gravity: X8 = %.14lf" % x8)

print("Predicted Tau particle out of the W Boson 80385 MeV/C^2 = %.14lf " % (80385*(1-x4)/(1+1-
x8)))

f = (1 - 1/96) * (4 / NP.pi)

x7, x8, x9 = \
MAIN_electrogravity_class.function_cubic_viete(1, -1, -f / 96, (f * f) / 192)

x10, x11, x12 = \
MAIN_electrogravity_class.function_cubic_viete(1, -1, f / 96, (f * f) / 192)

print("Anti-gravity: X7,X8,X9 = (%.14lf, %.14lf, %.14lf)" %(x7, x8, x9))
print("Gravity: X10,X11,X12 = (%.14lf, %.14lf, %.14lf)" %(x10, x11, x12))
print("Average 1/(1 - (1/(X7-1) + 1/(1-x10))/2) = %.14lf" %(1/((x7 - x10)/2)))

print("Better prediction: Tau out of the W Boson 80385 MeV/C^2 = %.14lf " \
        % (80385*(1-x4)/(1+0.5*(x7-x10))))

x7 = (1 + NP.sqrt(1 + 1/(NP.pi * 6)))/2
```

```
x10 = (1 + NP.sqrt(1 - 1/(NP.pi * 6)))/2

print("Another prediction: Tau out of the W Boson 80369 MeV/C^2 = %.14lf " \
        % (80369 *(1-x4)/(1+0.5*(x7-x10))))

input("Press Enter to exit> ")
```

The Fine Structure constant

The fine structure constant can be viewed as related to an energy portion of an emitted photon to the energy of an emitting charge. This is the subject of this section.

We will continue with the idea of an acceleration field $\frac{C^2}{L}$ where here C is the speed of light and in the real numbers case we relate square acceleration to the Reeb vector square norm $\frac{U^{\mu}U_{\mu}}{4}=\frac{C^4}{L^2}$.

The fine structure constant is related to an interaction with a charged particle without spin transfer. If we think of a charge in such an interaction as a random point on a sphere then it is easy to see that the average distance between two random points on a sphere S2 is $\frac{4}{3}L$ such that L is the radius of the sphere. So if we ignore the addition of area around a negative charge or the subtraction of area around a positive charge, we have to multiply $\frac{C^2}{L}$ by $\left(\frac{4}{3}\right)^{-1}=\frac{3}{4}$ and we have $\frac{3C^2}{4L}$ and (B.7) transforms into:

$$1+\frac{1}{96}\left(-\frac{1}{2}\left(\frac{3}{4}\right)^2 a^{-2}\pm\left(\frac{3}{4}\right)a^{-1}\right)=a\Rightarrow 192a^3=192a^2\pm 2a\left(\frac{3}{4}\right)-\left(\frac{3}{4}\right)^2 \qquad \text{(B.27)}$$

or

$$1+\frac{1}{96}\left(-\frac{1}{2}\left(\frac{3}{4}\right)^2 a^{-2}+\left(\frac{3}{4}\right)a^{-1}\right)=a$$

$$1+\frac{1}{96}\left(-\frac{1}{2}\left(\frac{3}{4}\right)^2 b^{-2}-\left(\frac{3}{4}\right)b^{-1}\right)=b$$

and we find an emergent inaccurate approximation to the inverse of the fine structure constant:

$$\left(\frac{1}{(1-\mathrm{a})}\frac{1}{(\mathrm{b}-1)}\right)^{1/2}=\sim 137.2504256 \qquad \text{(B.28)}$$

Close to 137.035999173 which is the inverse of the fine structure constant but not enough.

We need to take into account the effect of dilation and contraction of areas around the electric charge in order to get a better value.

In flat geometry, the distance between two points on the sphere that have an angle θ with the center is

$$2\sin\left(\frac{\theta}{2}\right) \qquad \text{(B.29)}$$

Integrating on the sphere and dividing by the area, we get the average distance,

$$\frac{1}{4\pi}\int_0^{\pi} 2\sin\left(\frac{\theta}{2}\right) 2\pi\sin(\theta)\, d\theta = \frac{4}{3} \qquad \text{(B.30)}$$

Nice but why to see an interacting charge as a random point on a sphere ? That idea has a lot to do with a popular idea, the Holographic Principle.
The distance $2\sin\left(\frac{\theta}{2}\right)$ changes if the area around the center of a unit circle is not 4π .
It is easy to show that if the area changes by a factor of R then the (B.29) turns into

$$2\cos\left(\frac{\theta}{2}\right)\int_{\alpha=0}^{\alpha=\frac{\theta}{2}} \frac{\sqrt{\sin^2(\alpha)+R\cdot\cos^2(\alpha)}}{\cos^2(\alpha)}\, d\alpha \qquad \text{(B.31)}$$

The term $2\cos\left(\frac{\theta}{2}\right)\int_{\alpha=0}^{\alpha=\frac{\theta}{2}} \frac{\sqrt{\sin^2(\alpha)+R\cdot\cos^2(\alpha)}}{\cos^2(\alpha)}\, d\alpha$ is reduced to $2\sin\left(\frac{\theta}{2}\right)$ if $R = 1$.

(B.31) can be written in a slightly different way which is more convenient for numerical integration:

$$2\cos\left(\frac{\theta}{2}\right)\int_{\alpha=0}^{\alpha=\frac{\theta}{2}} \frac{\sqrt{1+(R-1)\cdot\cos^2(\alpha)}}{\cos^2(\alpha)}\, d\alpha \qquad \text{(B.32)}$$

So the average distance between two points on the sphere becomes,

$$D(R) = \frac{1}{4\pi}\int_{\theta=0}^{\theta=\pi} 2\pi\sin(\theta)\, 2\cos\left(\frac{\theta}{2}\right)\left(\int_{\alpha=0}^{\alpha=\frac{\theta}{2}} \frac{\sqrt{1+(R-1)\cdot\cos^2(\alpha)}}{\cos^2(\alpha)}\, d\alpha\right) d\theta \qquad \text{(B.33)}$$

The (B.27) polynomial equations can be written as:

$$1+\frac{1}{96}\left(-\frac{1}{2}\left(\frac{1}{D(a)}\right)^2 a^{-2} + \left(\frac{1}{D(a)}\right) a^{-1}\right) = a \qquad \text{(B.34)}$$

and

$$1+\frac{1}{96}\left(-\frac{1}{2}\left(\frac{1}{D(b)}\right)^2 b^{-2} - \left(\frac{1}{D(b)}\right) b^{-1}\right) = b$$

In a more numerically plausible form

$$\left(\frac{1}{192}\left(192a^2 + 2\left(\frac{1}{D(a)}\right)a - \left(\frac{1}{D(a)}\right)^2\right)\right)^{1/3} = a \qquad \text{(B.35)}$$

and

$$\left(\frac{1}{192}\left(192b^2 - 2\left(\frac{1}{D(b)}\right)b - \left(\frac{1}{D(b)}\right)^2\right)\right)^{1/3} = b \qquad \text{(B.36)}$$

(B.35) and (B.36) offer a way to calculate the area increase around a negative charge that randomly appears on a sphere and the decrease around a positive charge.

Integrals (B.32) and (B.33) seem very difficult to solve which calls for numerical integration. The problem with numerical integration is that errors accumulate but nevertheless, the solution to (B.35) and to (B.36) was achieved numerically and for integral (B.33) with 20000 refinement steps for $d\alpha$ and 20000 refinement steps for $d\theta$ and 100 iterations for the polynomials we get:

$$\left(\frac{1}{(1-a)}\frac{1}{(b-1)}\right)^{1/2} = \sim 136.963731695 \qquad \text{(B.37)}$$

This value is close to the real value of the inverse of the fine structure constant 137.035999173 and it is quite possible that with an analytic solution to the integral (B.33) we would have a better result.

1000 refinement steps in (B.33) for $d\alpha$ and 1000 refinement steps for $d\theta$ reached a worse result of 136.834964866 .
Here is the output for 100 iterations and with 1000 steps for $d\alpha, d\theta$, 10000 steps and 20000 steps, Using x1 for a, x2 for b, 1/f1 for 1/D(a) and 1/f2 for 1/(D(b)),

1000 refinement steps:
136.834964866, x1=1.004872358, x2=0.9890385852, 1/f1=0.749652088, 1/f2=0.7536327342

10000 refinement steps:
136.956912583, x1=1.004870307, x2=0.9890534882, 1/f1=0.748872721, 1/f2=0.7528407842

20000 refinement steps:
136.963731695, x1=1.004870192, x2=0.9890543202, 1/f1=0.748829194, 1/f2=0.7527965552

The software involves basic understanding in numerical analysis and was written in the C language and not with the default Python that is used in the academic world simply because Python is too slow for 20000 refinement steps that resulted in 400,000,000 integrand summations.

```
// FineStructure.cpp : Defines the entry point for the console application.
// Using multi-byte strings and not Unicode.

#include "stdafx.h"
#include <math.h>
#include <conio.h>

// #define FINE_STUCTURE_STEP 0.0001
#define FINE_STRUCTURE_LOOP 20000 // Must be 1 / step.
#define FINE_STUCTURE_STEP ((long double)1.0/FINE_STRUCTURE_LOOP)
#define FINE_STRUCTURE_PI 3.14159265358979323846264338327950
#define FINE_STRUCTURE_ITERATIONS 100

// Calculate 0.5 of the distance.
long double FUNCTION_distance(long double d_angle,long double d_area_ratio)
{
  long double d_x,d_cos2,d_height,d_step,d_sum,d_gamma;
  int i;

  d_step = d_angle * FINE_STUCTURE_STEP;
  d_sum = 0;
  d_gamma = d_area_ratio - 1;
```

```
  d_height = cos(d_angle) * d_step;

  for (i = 0, d_x = 0; i < FINE_STRUCTURE_LOOP; i++, d_x += d_step)
  {
    d_cos2 = cos(d_x);
    d_cos2 *= d_cos2;

    // d_cos2 is never 0. d_x < Pi/2.
    // d_cos1 is multiplied here for numerical stability.
    // It could be multiplied after the loop if not for numeical reasons.
    d_sum += sqrt(1 + d_gamma * d_cos2)*d_height / d_cos2;
  }

  return d_sum;
}

long double FUNCTION_average_distance(long double ad_area_ratio)
{
  long double ad_x,ad_step,ad_sum,ad_distance;
  int i;

  ad_sum = 0;
  ad_step = (long double)FINE_STRUCTURE_PI * FINE_STUCTURE_STEP;

  for (i = 0, ad_x = 0; i < FINE_STRUCTURE_LOOP; i++,ad_x+=ad_step)
  {
    ad_distance = FUNCTION_distance(ad_x * 0.5, ad_area_ratio);

    // 2 * 2Pi / 4Pi = 1.
    // Also area that grows by b and is divided an area that grows by b is 1.
    ad_sum += sin(ad_x) * ad_distance;
 }

  return ad_sum * ad_step;
}

void FUNCTION_roots(void)
{
  long double r_root1=1,r_root2=1;
  long double r_third,r_result;
  int i;

  r_third = (long double)1.0/3;

  for (i = 0; i < FINE_STRUCTURE_ITERATIONS; i++)
  {
    long double r_f1,r_f2;

    r_f1 = FUNCTION_average_distance(r_root1);

    r_root1 = (192 * r_root1 * r_root1 +
               2 * r_root1 / r_f1 -
               1.0/(r_f1 * r_f1))/192;

    r_root1 = pow(r_root1,r_third);

    r_f2 = FUNCTION_average_distance(r_root2);
```

```
    r_root2 = (192 * r_root2 * r_root2 -
               2 * r_root2 / r_f2 -
               1.0 / (r_f2 * r_f2)) / 192;

    r_root2 = pow(r_root2, r_third);

    r_result = sqrt(1 / ((r_root1 - 1)*(1 - r_root2)));

    printf("%.9lf, x1=%.9lf, x2=%.9lf2, 1/f1=%.9lf, 1/f2=%.9lf2\n",
           r_result,r_root1,r_root2,1/r_f1,1/r_f2);
  }
}


int main()
{
  while (_kbhit()) _getch(); // Clear keyboard input.

  FUNCTION_roots();

  while (_kbhit()) _getch(); // Clear keyboard input.

  puts("Press Enter to exit the console.");

  getchar();

  return 0;
}
```

C – Towards a unified force theory – 1,2,3 types of accelerated clocks

The following action can be extended to U(1) x SU(2) and to SU(3) symmetries by considering more than one Reeb vector.

$$Z = N^2 = (P_\mu P^{*\mu} + P^*_\mu P^\mu)/2 \text{ and } U_\lambda = \frac{Z_\lambda}{Z} - \frac{Z_k P^{*k} P_\lambda}{Z^2} \text{ and}$$
$$L = \frac{1}{4}\left(\frac{U^k U^*_k + U^{*k} U_k}{2}\right) \tag{C.1}$$

Since the matrix of a Symplectic form can be described as two rotation and scaling hyper-planes, there is a possibility to locally add another scalar $P(2)$ and the Reeb vector of its gradient $P(2)_\mu$

$$Z(2) = N^2(2) = (P(2)_\mu P(2)^{*\mu} + P(2)^*_\mu P(2)^\mu)/2$$

And $$Z(2)_\lambda = \frac{dZ(2)}{dx^\lambda}$$

$$U(2)_\lambda = \frac{Z(2)_\lambda}{Z(2)} - \frac{Z(2)_k P(2)^{*k}\ P(2)_\lambda}{Z(2)^2}$$

And such that

$$U(2)_\lambda U^{*\lambda} = P(2)_\lambda P^{*\lambda} = P(2)_\lambda U^{*\lambda} = P_\lambda U(2)^{*\lambda} = 0 \qquad \text{(C.2)}$$

and obviously $P_\lambda U^{*\lambda} = 0$ and $P(2)_\lambda U(2)^{*\lambda} = 0$ from the definition of a Reeb vector.
The action is then dictated by the root of the Gram determinant and is added to the previous action,

$$\mathrm{L} = \frac{1}{4}\left(\frac{\mathrm{U}^{\mathrm{k}}\mathrm{U}^*_{\mathrm{k}} + \mathrm{U}^{*\mathrm{k}}\,\mathrm{U}_{\mathrm{k}}}{2}\right) + \begin{vmatrix} \dfrac{U_\mu U^{*\mu} + U^*_\mu U^\mu}{8} & \dfrac{U_\mu U(2)^{*\mu} + U^*_\mu U(2)^\mu}{8} \\[2ex] \dfrac{U(2)_\mu U^{*\mu} + U(2)^*_\mu U^\mu}{8} & \dfrac{U(2)_\mu U(2)^{*\mu} + U(2)^*_\mu U(2)^\mu}{8} \end{vmatrix}^{\frac{1}{2}} \qquad \text{(C.3)}$$

The physical meaning of $U(2)_\mu$ is another acceleration field in another plane. We will consider (C.2) as the “Electro-Weak Geometric Chronon Action”.

In the three dimensional space, Minkowsky perpendicular to P_λ we can view three holonomic vectors fields that span the foliation tangent space as required by the Froben ius theorem . These can be locally described by three gradients,
$P(3)_\lambda, P(4)_\lambda, P(5)_\lambda$ and accordingly we can discuss their Reeb vectors, $S_\lambda, W_\lambda, T_\lambda$.
This time we can’t require the orthogonality condition which is described in (C.2) because there are no three Minkowsky – perpendicular hyper planes in space-time.
Now we need the third root of the determinant of the Gram matrix of these new three Reeb vectors and the action becomes,

$$\mathrm{L}=\frac{1}{4}\left(\frac{\mathrm{U}^{\mathrm{k}}\mathrm{U}*_{\mathrm{k}}+\mathrm{U}*^{\mathrm{k}}\,\mathrm{U}_{\mathrm{k}}}{2}\right)+$$

$$\begin{vmatrix} \frac{U_\mu U*^\mu+U*_\mu U^\mu}{8} & \frac{U_\mu U(2)*^\mu+U*_\mu U(2)^\mu}{8} \\ \frac{U(2)_\mu U*^\mu+U(2)*_\mu U^\mu}{8} & \frac{U(2)_\mu U(2)*^\mu+U(2)*_\mu U(2)^\mu}{8} \end{vmatrix}^{\frac{1}{2}}+$$

$$\begin{vmatrix} \frac{S_\mu S*^\mu+S*_\mu S^\mu}{8} & \frac{S_\mu W*^\mu+S*_\mu W^\mu}{8} & \frac{S_\mu T*^\mu+S*_\mu T^\mu}{8} \\ \frac{W_\mu S*^\mu+W*_\mu S^\mu}{8} & \frac{W_\mu W*^\mu+W*_\mu W^\mu}{8} & \frac{W_\mu T*^\mu+W*_\mu T^\mu}{8} \\ \frac{T_\mu S*^\mu+T*_\mu S^\mu}{8} & \frac{T_\mu W*^\mu+T*_\mu W^\mu}{8} & \frac{T_\mu T*^\mu+T*_\mu T^\mu}{8} \end{vmatrix}^{\frac{1}{3}}$$

(C.4)

And the Einstein – Grossmann minimum action equation can be written as

$$\begin{aligned} &R = Ricci\ \text{curvature.} \\ &Min\ Action = Min\int_\Omega\left(R-\frac{8\pi}{\sigma}L\right)\sqrt{-g}\,d\Omega = \\ &Min\int_\Omega\left(R-\frac{1}{4}\mathrm{U}^{\mathrm{k}}\mathrm{U}_{\mathrm{k}}\right).\sqrt{-g}\,d\Omega \qquad \text{s.t. } \sigma = 8\pi \end{aligned}$$

(C.5)

The latter third root of a determinant of Gram’s matrix can be viewed as the “Strong Geometric Chronon Action”.
The probabilities of events must sum to 1 so for example

$$\begin{aligned} &S=\sum_{i=1}^{n\to\infty}\psi(i,S) \\ &\int_\Omega\psi(k,S)\psi*(k,S)\sqrt{-g}\,d\Omega = 1,\ k=1,2,3,4,5,\ldots. \\ &\int_\Omega\psi(i,S)\psi*(k,S)\sqrt{-g}\,d\Omega = 0,\ i\neq k \end{aligned}$$

(C.6)

The sign of time

This idea is quite similar to Dr. Sam Vaknin’s idea of Quarks of time, see [14].

Consider the Levi – Civita tensor (and not the Levi – Civita symbol which is a tensor density and not a tensor) we can establish a vector which is parallel to the flow P^{β} but can have either a positive or negative sign when contracted with P_{μ}, we have
$V^{\beta} = \varepsilon^{\mu\nu\alpha\beta} P(3)_{\mu} P(4)_{\nu} P(5)_{\alpha}$

$$V^{\beta} = \varepsilon^{\mu\nu\alpha\beta} P(3)_{\mu} P(4)_{\nu} P(5)_{\alpha} \tag{C.7}$$

V^{β} defines the sign of the flow and therefore, the sign of $\frac{P_{\beta} V^{*\beta} + P^{*}_{\beta} V^{\beta}}{2}$ and obviously,
$P_{\beta} P(3)^{*\beta} = P_{\beta} P(4)^{*\beta} = P_{\beta} P(5)^{*\beta} = 0$.

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