

Five conjectures on primes based on the observation of Poulet and Carmichael numbers

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Abstract. In this paper I enunciate five conjectures on primes, based on the study of Fermat pseudoprimes and on the author's believe in the importance of multiples of 30 in the study of primes.

Conjecture 1:

For any p, q distinct primes, $p > 30$, there exist n positive integer such that $p - 30^n$ and $q + 30^n$ are both primes.

Note:

This conjecture is based on the observation of 2-Poulet numbers (see my paper "A conjecture about 2-Poulet numbers and a question about primes").

Conjecture 2:

For any p, q, r distinct primes there exist n positive integer such that the numbers $30^n - p, 30^n - q$ and $30^n - r$ are all three primes.

Note:

This enunciation is obviously equivalent to the enunciation that there exist m such that $p + 30^m, q + 30^m$ and $r + 30^m$ are all three primes (take $x = 30^n - p, y = 30^n - q$ and $z = 30^n - r$. Then there exist k such that $30^k - 30^n + p, 30^k - 30^n + q$ and $30^k - 30^n + r$ are all three primes).

Note:

This conjecture implies of course that for any pair of twin primes (p, q) there exist a pair of primes $(30^n - p, 30^n - q)$ so that there are infinitely many pairs of twin primes.

Note:

This conjecture is based on the observation of 3-Carmichael numbers (see my paper "A conjecture about primes based on heuristic arguments involving Carmichael numbers").

Conjecture 3:

There exist an infinity of pairs of distinct primes (p, q) , where $p < q$, both of the same form from the following eight ones: $30*k + 1$, $30*k + 7$, $30*k + 11$, $30*k + 13$, $30*k + 17$, $30*k + 19$, $30*k + 23$ and $30*k + 29$ such that the number $p*q + (q - p)$ is prime.

Note:

This conjecture is based on the observation of Carmichael numbers.

Examples:

- : $31*151 + (151 - 31) = 4801$ prime;
- : $37*127 + (127 - 37) = 4789$ prime;
- : $41*101 + (101 - 41) = 4201$ prime;
- : $13*103 + (103 - 13) = 1429$ prime;
- : $17*47 + (47 - 17) = 829$ prime;
- : $19*109 + (109 - 19) = 2161$ prime;
- : $23*53 + (53 - 23) = 1249$ prime.

Conjecture 4:

There exist an infinity of pairs of distinct primes (p, q) , where $p < q$, both of the same form from the following eight ones: $30*k + 1$, $30*k + 7$, $30*k + 11$, $30*k + 13$, $30*k + 17$, $30*k + 19$, $30*k + 23$ and $30*k + 29$ such that the number $p*q - (q - p)$ is prime.

Note:

This conjecture is based on the observation of Carmichael numbers.

Examples:

- : $31*61 - (61 - 31) = 1861$ prime;
- : $7*37 - (37 - 7) = 229$ prime;
- : $11*41 - (41 - 11) = 421$ prime;
- : $13*73 - (73 - 13) = 919$ prime;
- : $17*47 - (47 - 17) = 769$ prime;
- : $19*139 - (139 - 19) = 2521$ prime;
- : $23*293 - (293 - 23) = 6469$ prime.

Conjecture 5:

For any p prime there exist an infinity of primes q , $q > p$, where p and q are both of the same form from the following eight ones: $30*k + 1$, $30*k + 7$, $30*k + 11$, $30*k + 13$, $30*k + 17$, $30*k + 19$, $30*k + 23$ and $30*k + 29$ such that the number $p*q - (q - p)$ is prime.