

Nine conjectures on the infinity of certain sequences of primes

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Abstract. In this paper I enunciate nine conjectures on primes, all of them on the infinity of certain sequences of primes.

Conjecture 1:

For any prime p there exist an infinity of positive integers n such that the number $n \cdot p - n + 1$ is prime.

Examples:

: For $p = 19$ we have the following primes: $2 \cdot 19 - 1 = 37$; $4 \cdot 19 - 3 = 73$; $6 \cdot 19 - 5 = 109$; $7 \cdot 19 - 6 = 127$; $9 \cdot 19 - 8 = 163$; $10 \cdot 19 - 9 = 181$ etc.

Conjecture 2:

For any prime p there exist an infinity of positive integers n such that the number $n \cdot p + n - 1$ is prime.

Examples:

: For $p = 11$ we have the following primes: $2 \cdot 11 + 1 = 23$; $4 \cdot 11 + 3 = 47$; $5 \cdot 11 + 4 = 59$; $6 \cdot 11 + 5 = 71$; $7 \cdot 11 + 6 = 83$; $9 \cdot 11 + 8 = 107$ etc.

Conjecture 3:

For any prime p there exist an infinity of positive integers n such that the number $n^2 \cdot p - n + 1$ is prime.

Examples:

: For $p = 7$ we have the following primes: $3^2 \cdot 7 - 2 = 61$; $4^2 \cdot 7 - 3 = 109$; $7^2 \cdot 7 - 6 = 337$; $10^2 \cdot 7 - 9 = 691$; $12^2 \cdot 7 - 11 = 997$ etc.

Conjecture 4:

For any prime p there exist an infinity of positive integers n such that the number $n^2 \cdot p + n - 1$ is prime.

Examples:

: For $p = 11$ we have the following primes: $3^{2*11} + 2 = 101$; $4^{2*11} + 3 = 179$; $6^{2*11} + 5 = 401$; $10^{2*11} + 9 = 1109$; $13^{2*11} + 12 = 1871$ etc.

Conjecture 5:

For any prime p there exist an infinity of positive integers n such that the number $n^p - p + n$ is prime.

Examples:

: For $p = 5$ we have the following primes: $1*5 + 2 = 7$; $2*5 + 3 = 13$; $3*5 + 4 = 19$; $5*5 + 6 = 31$; $6*5 + 7 = 37$; $7*5 + 8 = 43$ etc.

Conjecture 6:

For any prime p there exist an infinity of positive integers n such that the number $n^p - p - n$ is prime.

Examples:

: For $p = 5$ we have the following primes: $1*5 - 2 = 3$; $2*5 - 3 = 7$; $5*5 - 6 = 19$; $6*5 - 7 = 23$; $8*5 - 9 = 31$; $11*5 - 12 = 43$ etc.

Conjecture 7:

For any prime p there exist an infinity of positive integers n such that the number $(n - 1)^{2*p} + n$ is prime.

Examples:

: For $p = 7$ we have the following primes: $2^{2*7} + 3 = 31$; $3^{2*7} + 4 = 67$; $5^{2*7} + 4 = 179$; $6^{2*7} + 5 = 257$; $7^{2*7} + 6 = 349$ etc.

Conjecture 8:

For any prime p there exist an infinity of positive integers n such that the number $(n - 1)^{2*p} - n$ is prime.

Examples:

: For $p = 7$ we have the following primes: $3^{2*7} - 4 = 59$; $4^{2*7} - 5 = 107$; $8^{2*7} - 9 = 439$; $9^{2*7} - 10 = 557$; $15^{2*7} - 16 = 1559$ etc.

Conjecture 9:

For any two distinct primes greater than three p and q there exist an infinity of positive integers n such that the number $(p^2 - 1)n + q^2$ is prime, also an infinity of positive integers m such that the number $(q^2 - 1)n + p^2$ is prime.

Examples:

: For $(p, q) = (7, 11)$ we have the following primes of the form $48n + 121$: 313, 409, 457, 601, 937, 1033 etc. and the following primes of the form $120n + 49$: 409, 769, 1009, 1129, 1249, 1489 etc.

Note:

The idea of these sequences didn't come to me from "nowhere". Many from the types of primes presented in this paper are met in the study of Fermat pseudoprimes.