

Ladies and gentlemen,

the following theory has been concluded, quantified and is in a position to explain freely and easily the phenomena connected to the High-temperature-superconductivity. I therefore ask you to forward my paper to your referee committee.

Sincerely,  
Hans Christian Haunschild

## **Theory of high-temperature-superconductivity in Cuprates**

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### **Abstract**

The theory presented here is able to explain all attributes of cuprate-based High-temperature-Superconductors in the normal- and in the SC-state as well. More than 300 experimentally measured values will be compared with the predictions of theory. In more than 90% there is quantitative, otherwise qualitative accordance between experiment and theory. For the theoretical calculations parameters gained by experiments only are used.

**Reluctance is senseless:** This theory will succeed.

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## 1. Introduction

In 1987 superconductors with critical  $T_C$ -values above 100 K were discovered. According to some estimations  $T_C$ -values exceeding 40 K cannot be explained by electron-phonon interaction, because for creating even higher  $T_C$ -values the interactions will be so heavy, that they will cause rearrangements of the lattice. Up to now there does not exist any universally acknowledged theory which explains these  $T_C$ -values. Certain is only, that in all superconductors known up to now the superconductivity current is transported by pairs of charge-carriers. Therefore the main task of any theory will be to explain the unusual thermal stability of these pairs in HTSC.

Up to now quite a few possible explanations have been debated. Naturally at first scientists tried to apply the model of electron-phonon-interaction to HTSC despite the difficulties

mentioned above<sup>1</sup>. For some time a model was debated which was based on the separation of electron-spin and -charge<sup>2et3</sup>. At present many scientists assume that magnetic interactions play the dominant part<sup>4</sup>. All these models however are not able to explain simultaneously the superconducting **and** the completely unusual normally conducting attributes of HTSC:

In most cases in the normal state the electrical resistance follows a linear temperature law down to deepest temperatures –in contradiction to the Landau-theory of Fermi-liquid (**chapters 6. to 8.**). The conduction-electrons obviously do not create a Fermi-liquid.

Accordingly an anomaly in the thermal conductivity appears too. Oddly enough with most of the HTSC –also in a normally conducting state- there will be an energy gap in the density of states (DOS), the so-called “pseudo-gap”.

In HTSC with a low density of holes a huge isotope effect is noticed: With increasing density of holes the effect decreases continuously for strong increasing again above optimal doping

Therefore an interaction must be discovered which

- a) leads to the formation of hole-pairs at high temperatures
- b) is equally responsible for the “non-Fermi-liquid-behaviour”
- c) also creates a gap in the density of state (DOS) of the mobile charge-carriers in the normal state
- d) permits a sufficiently plausible explanation as regards the isotope effects

All these phenomena (and some more) will be brought in connection and explained by this theory.

## **2. The Pseudogap**

## 2.1 Definition

In this work the conduction charge carriers will generally be called “holes”. The quasi-bound charge carriers in the lower Cu-3d-band of HTSC will always be called electrons (abbreviated as “El”). When speaking of “El” there will always be referred to these quasibounded electrons in the lower Cu-3d-band.

## 2.2. The Hubbard model and HTSC

This abstract is based on the idea that the Hubbard model of theoretical solid state physics<sup>5</sup> can also be applied to HTSC. According to this model the valence band of HTSC is split into three partial bands: two Cu-3d-bands and one oxygen-2p-band. If each lattice site (of ions) is occupied by exactly one electron, then (if  $T = 0K$ ) the oxygen band and the lower Cu-3d-band are totally unoccupied. As the bands do not overlap –but are separated from each other by energy gaps –then there will be an antiferromagnetic insulator.

These energy gaps are of a different origin than the energy gaps in semi-conductors: They originate from the mutual Coulomb repel of the electrons. This Coulomb repel prevents the delocalization of the El and links them –similar to atomic rests- to destined sites in the lattice.

This status is similar to a Wigner lattice in two dimensions. For memory: In **two** dimensions a Wigner lattice will exist at **high** densities of electrons only<sup>6</sup>:

$$\Gamma = \frac{e^2 \sqrt{N_S}}{4\sqrt{\pi} \epsilon_0 k_B T} = 3 \cdot 10^{-5} \frac{\sqrt{N_S}}{T} \quad (1)$$

Thereby is  $N_S$  the density of electrons per square metre. If  $\Gamma$  is large enough the mutual repel will overcome the attraction and it is assumed that there will be a diffraction pattern as a

periodic lattice<sup>6</sup>. In HTSC the occupied Cu-3d-band is created in a different way than in band insulators: in a band insulator no states can be occupied immediately above the upper band edge. In HTSC the occupied Cu-3d-band is the result of the mutual Coulomb repel of the EL. This repel does not exclude the stimulation of small oscillations of the EL. If and when EL are stimulated to such “lattice” oscillations, these EL will occupy states above the occupied Cu-3d-bands. Similarly to atomic residues these quasi-crystallized EL, which were studied here, can be excused out of their positions. The fact, that the EL are localized at a lattice site does not prohibit the excursion out of their normal positions.

When the system is doped with oxygen, the oxygen band will loose electrons and the system will be doped with holes. With increasing doping the ferromagnetism disappears, the Fermi-energy moves into the region of the oxygen band, the system becomes metallic. In this system now a band appears which contains freely moving holes. A second band will appear, the lower Cu-3d-band containing electrons which repel each other because of the Coulomb interaction.

The upper Cu-3d-band remains unoccupied. For the understanding of the following please keep in mind that the abbreviation EL used in connection with HTSC always refers to the Cu-3- $d_{x^2-y^2}$ -electrons, quasi-bound to Cu- or O-ions in the lower Cu-3d-band. The mobile charge-carriers in the oxygen band will be referred to as “holes”.

If energy is added, **both** types of charge carriers –holes and EL- can be scattered into unoccupied levels of energy.

### 2.3 Application of the Hubbard-model to HTSC

The existing theories of solid state physics consider a metal as a skeleton of ions surrounded by a gas of charge carriers. By interaction between these charge carriers and the ions the latter will be stimulated to virtual lattice oscillations. In contrast to the ions the mobile charge carriers in a metal cannot cause such oscillations, because there is not any restoring force, having an effect on a gas. (Plasma-oscillations of the charge carriers are no lattice oscillations.) In this essay it is assumed that the typical theories of metals do not work at the HTSC, but the EL in the Cu-3d-band, charging the highest energy can be shifted relatively to the ions. Because of the special electron-configuration in the HTSC, this quasibounded EL can make fast oscillations relatively to the much inert atomic rests. Because of these oscillations of quasibounded EL a polarization will be generated which is relaying an attraction between two holes. Thereby these oscillations of EL play the same role for HTSC as the oscillations of the atomic lattice play for conventional superconductivity.

In this connection should be remembered that according to the simplest theory of conventional superconductivity the critical temperature is inversely proportional to the root of the mass of oscillating particles<sup>7</sup>. Therefore the small mass of the electrons (EL) can manage to generate very high  $T_C$ 's. Thus the high  $T_C$ 's can be explained qualitatively at least.

At this point it will be remarked, that the virtual "lattice-oscillations" of Cu-3d-EL are Bosons, not obeying the Pauli-Principle. Perhaps this is a bit puzzling, because the carriers of this "lattice-oscillations are electrons, strictly obeying the Pauli-Principle. But lattice-oscillations do not possess a rest mass. A particle without rest mass possesses a whole-number spin and thereby obeys the Bose-Einstein-statistic. This is valid for lattice-oscillations in frozen hydrogen or in hydrides, too. And this is valid at every temperature, although the protons are fermions. In context with impact processes between protons and electrons, p. e. in Pd-H-systems, the protons appear as classical particles, because the protons are fixed at defined lattice-sites and because of this can be identified. From that the building in of statistical distributed protons in metals only leads to a largely T-independent RRR.

In contrary for impact processes of single movable holes with the Cu-3d-system in the HTSC the Fermi-Dirac-statistic is valid, because the holes and the Cu-3d-EL cannot be identified and possesses rest-masses with half-whole-number spin. Towards it results the dominating contribution to the electrical normal-state-resistance in the HTSC.

## 2.4 Quantitative derivating of the pseudogap and the formation of hole-pairing:

Quantitative the dielectrical function will be calculated by the following formulation:

$$m \cdot \frac{d^2x}{dt^2} - CX = eE$$

By their moving through the crystal the holes are “hauling” at the localized Cu-3d-EL with

the force:  $m_{EL} \cdot \frac{d^2x}{dt^2}$ . By the repulsion of the Cu-3d-EL against another, there results

a restoring force CX to this EL. The whole force, which has an effect at the EL's, is  $= eE$ .

The resolution of this fluxion-equation is  $X(t) = \frac{eE}{m\omega^2} = \frac{eE}{m}t^2$ .

Proof:  $\frac{d^2x}{dt^2} = 2eE/m$  and  $CX(t) = eE$ . Therefore this solution is correct mathematically.

Physically it is not strongly correct, really it is valid:  $C = \frac{m\omega^2}{4\sin^2(Ka/2)}$ .

But as an approximation it is useful.

With  $X(t) = \frac{eE}{m\omega^2}$  and polarization  $P = -nex$  you receive  $P = -\frac{ne^2}{m\omega^2}E$ .

$$\text{And: } \varepsilon(\omega,0) = 1 + \frac{P}{\varepsilon_0 E} = 1 - \frac{ne^2}{\varepsilon_0 m\omega^2} = 1 - \frac{\omega_{EL}^2}{\omega^2} = 1 - \frac{\hbar t \cdot \omega_{EL}^2}{\hbar t \cdot \omega^2} = 1 - \frac{\hbar \omega_{EL}}{\hbar \omega}$$

In this equation  $\hbar\omega$  is the energy of the incident wave.

## 2.5 What means $\hbar\omega_{EL}$ and what is the maximum pseudogap?

In conventional superconductors the energy-exchange necessary for pair-formation is mediated by zero-point-modes of atomic rests. This theory demonstrated here supposes in the HTSC an intermediating of this energy-exchange by oscillations of Cu-3d-EL. Therefore the characteristic energy- scale is the energy, which the EL can absorb in form of oscillation-energy. In a gas of free electrons the magnitude of this energy can be determined easily: It is the well-known energy of plasma-oscillations. In a HTSC the Cu-3d-EL do not create a gas of free electrons. For a longer time it has been assumed, that in the HTSC the “Hubbard-model” in two dimensions is realized. This assumption is taken into account in this theory. Probably the oscillation-energy, which the Cu-3d-EL maximal can absorb, is identical with the so-called “Hubbard-energy” of the EL. Therefore it is assumed:

$$\hbar\omega_{EL} \equiv \frac{t^2}{U} = \mathbf{MPG} =$$

### Maximum Pseudogap

It is well-known, that the HTSC`s have a two-dimensional layer-structure. They show a strong anisotropy in their normal and superconducting characteristics parallel or crossover to the lattice-

layers. This two-dimensionality must be regarded in every approximation.

Thereby it is valid in HTSC:  $\varepsilon(\omega,0) = 1 - \frac{\hbar\omega_{EL}}{\hbar\omega} = \frac{t^2/U}{\hbar\omega}$ .

At this point it should be emphasized, that this is a pure assumption only, which cannot be deduced.

The author explicitly remarks, that this interrelationship only was guessed by himself. This

interrelationship between “Hubbard-Energy” and dielectrical function  $\varepsilon(\omega,0)$  will be discussed in

the following.

The letter “ $\mathbf{t}$ ” in the nominator does not mean a time, but means the energy of the plasma-oscillations of the EL, considering the statically shielding. As mentioned at the beginning, there exists a similarity between cuprates and a Wigner-crystal. Therefore the EL also can create a sort of “lattice-vibrations”. In case of this kinetic energy is valid:

$$t^2 = \hbar^2 \omega_p^2 \cdot \frac{q^2}{q^2 + k_{TF}^2} = \hbar^2 \cdot \frac{ne^2}{\varepsilon_0 m_{EL}} \cdot \frac{q^2}{q^2 + k_{TF}^2}$$

And the quantity  $\mathbf{U}$  in the denominator is the difference between the direct (“naked”) and the shielded collective Coulomb-interaction of the EL together.

Remark: The statically shielding is calculated for three dimensions, because it is generated from the Coulomb-interaction, and the long-ranging Coulomb-interaction is working in all three space-dimensions, that also means crossways to the lattice-layers.)

From all that results:

$$\text{MPG} = \frac{t^2}{U} = \hbar^2 \frac{\frac{ne^2}{\varepsilon_0} \cdot \frac{q^2}{q^2 + k_{TF}^2} \cdot \frac{\hbar^2}{m_{EL}}}{\frac{ne^2}{\varepsilon_0} \cdot \frac{q^2}{q^2 + k_{TF}^2} \cdot \frac{\omega_D^2}{\omega^2 - \omega_D^2}} = \frac{\hbar^2 q^2}{m_{EL}} \cdot \frac{\omega_D^2 - \omega^2}{\omega_D^2} = \frac{\hbar^2 q^2}{m_{EL}} \cdot \left(1 - \frac{\omega^2}{\omega_D^2}\right)$$

$$\text{MPG} \propto (1 - a \cdot M_{ion}) \quad [\text{Nm}]$$

$M_{ion}$  is the atomic mass and  $\mathbf{a}$  the lattice-constant. At once it is visible, that MPG is depending on the numbers of the neutrons in the nucleus and depending on the pressure. The result is an isotope-effect and a pressure-dependency of the MPG. As demonstrated in the following **chapter 3**. there exists a strong dependence of the critical temperatures on the

magnitude of MPG. This dependence can explain the isotope-effect (see **chapter 4.**) and the dependence of the Tc-values on pressure.

At last we take a glance to the magnitudes of MPG:

With  $q \approx 10^9$  and  $\frac{\omega^2}{\omega_D^2} \approx 0,36$  is received for  $\hbar\omega_{EL} \equiv \frac{t^2}{U} = \mathbf{MPG}$  a value of about  $6,4 \cdot 10^{-21}$  J.

This corresponds to 40 meV. In experiments values between 10 and 140 meV were found.

So this really is in the calculated magnitude.

## 2.6 What is the “variable” pseudogap PSL ?

As it is demonstrated above, it is valid:  $\varepsilon(\omega,0) = 1 - \frac{MPG}{\hbar\omega}$ .

If the energy of the incident wave is smaller than MPG, then the dielectrical function is negative. Also the conduction-electrons in a HTSC –almost holes– have wave-characteristics. As long as the kinetically energy (i. e. the Fermi-Energy) is smaller than MPG, so long  $\varepsilon(\omega,0)$  is negative, directing to an attractive interaction between the holes. Therefore at sufficient low temperatures hole-pairs are formed. At higher temperatures or in strong magnetic fields these pairs cannot exist, but in the DOS of the holes an energy gap will appear. That results from the total-reflection of incident waves with a frequency lower than the Eigen-frequency of the EL. (For a similar reason light in the visible region is reflected by metals.)

The cause of conducting the electrical current by HTSC for all that is, that the full total-reflection occurs only parallel to the connecting axis of nearest-neighbourd Cu-ions. Then only there are the EL, which can excited to vibrations. Other EL cannot interact, because possessing the same spin-position. An –although reduced- electrical current can flow apart of these connecting axes. Therefore an important experimental consequence is produced: pseudo-energy-gap and superconductivity are directional. Or, in a scientific expression, both possess d-wave symmetry. This so-called “Pseudo-Gap” only can exist in the presence of conduction-electrons (mostly holes). It does not exist from the first. Instead of this it is generated by the interaction between conduction-electrons (mostly holes) and **Cu-3d-EL**.

So this (variable) pseudogap appears in the system, respectively in the DOS, of the conduction-electrons (mostly holes). And nowhere else.

Explanation of  $\hbar\omega$ :

$\hbar\omega$  is the kinetic energy of the incidenting wave, in our case the energy of the holes. Because the pairing holes possess energies at the Fermi-Energy, one can equate the kinetic energy of

the holes with the Fermi-Energy:  $\varepsilon(\omega,0) = 1 - \frac{\hbar\omega_{EL}}{\hbar\omega} = 1 - \frac{MPG}{E_F}$ .

This expression will play an important role in the quantitative calculations in **chapter 3.5** and has consequences, verifiable by experiment:

a) If the Fermi-Energy is approaching zero, the pseudogap will reach its maximum value:

$$\hbar\omega_{EL} - E_F = \hbar\omega_{EL} - 0 = \hbar\omega_{EL} = MPG.$$

But that means, the maximum pseudogap has the value  $\hbar\omega_{EL}$ . Or, on the other hand,  $\hbar\omega_{EL}$  and the maximum pseudogap MPG are not the same (not identical), but they have the same value and the same dimension. MPG is defined as the pseudogap in the case of the Fermi-Energy zero, respective the doping of zero.

b) In the Cuprates also in the normal conducting state appears an energy gap of the magnitude  $\hbar\omega_{EL} - E_F = MPG - E_F$ . This energy gap  $MPG - E_F$  naturally means nothing else as the „Pseudogap“.

c) With increasing hole-concentration, standing for augmenting doping, the Fermi-Energy is raising, also. By that the (Pseudo-)energy-gap decreases and disappears final totally. When  $E_F$  is big enough, than  $\varepsilon(\omega,0)$  at first will be zero and will become positive at last. In the case of a positive  $\varepsilon(\omega,0)$  is resulting a stronger repulsion between the holes. In consequence the superconductivity either leaves with vanishing of the energy-gap or at moderate higher doping.

- d) The relevant energy-shell of pair-building is identical with  $MPG - E_F$ , which is the pseudogap (Isotope Effect!)
- e) At a doping of zero the pseudogap reaches their maximum value MPG.
- f) Because of collisions between holes and Cu-3d-EL the electrical resistance is increasing linearly with temperature. This will be explained in **chapters 6. to 8.**
- g) The pseudogap is of another origin as the “superconduction-gap”. In the HTSC`s this “superconduction-gap” appears besides the pseudogap.

### 3. Quantitative description of formation of hole-pairs in HTSC:

In this capital will be derivated an equation for the quantitative connections between parameters of metals and characteristic features of superconducting.

**3.1 I)** We will begin with the self-consistency-equation of the BCS-theory<sup>7</sup>. This equation is applicable for all well-known superconductors, independent of the pair-building mechanism,

here also: 
$$\frac{V}{2} \cdot \sum_k \frac{1}{\sqrt{\varepsilon_k^2 + \Delta_{SC}^2(T)}} \cdot \tanh \frac{\sqrt{\varepsilon_k^2 + \Delta_{SC}^2(T)}}{2k_B T} = 1.$$

Thereby  $\varepsilon_k$  is the kinetic energy of the paired holes, measured by the Fermi-energy of the unpaired holes.  $\Delta_{SC}(T)$  doesn`t mean the pseudogap, but the “superconduction gap”, generated by the interaction of the pairs with themselves.

In accordance to BCS-theory<sup>7</sup> the matrix-element of the interaction of two holes is described

as: 
$$V_{kk}^{eff} = \frac{1}{Vol} \cdot \frac{e^2}{\varepsilon_0(q^2 + k_{TF}^2)} \cdot \frac{1}{\varepsilon(\omega,0)}.$$
 There is nothing new in our model.

II) But new is the expression of  $\varepsilon(\omega,0)$  [see chapter. 2.4, “variable”Pseudogap]:

$$\text{In the HTSC`s is valid: } \varepsilon(\omega,0) = 1 - \frac{MPG}{E_F} = 1 - \frac{E_F + PSL}{E_F} = 1 - \left(1 + \frac{PSL}{E_F}\right).$$

(The abbreviation PSL means “Pseudo-Lücke”. For better distinction between this variable pseudogap on the one hand and the maximum pseudogap MPG on the other, this borrow was taken from the German language.)

In the expression  $1 - \left(1 + \frac{PSL}{E_F}\right)$  the One in front of the parenthesis belongs of the repulsive

Coulomb-interaction. One can see that by setting PSL as zero. That means, in vacuum or in the model of the empty box without a pseudogap, the dielectrical function =  $+1(\cdot \varepsilon_0)$ .

Therefore the One in front of the parenthesis only can belong to the Coulomb-repulsion and the content of the parenthesis only can belong to the Coulomb-attraction. Therefore by the

parenthesis-expression  $\left(1 + \frac{PSL}{E_F}\right) = \frac{E_F + PSL}{E_F} = \frac{MPG}{E_F}$  the **attractive** part of the Coulomb-

interaction of the holes **only** will be described. This lengthy expression is necessary, because

in the dielectrical function in the Matricelement  $V_{kk}^{eff}$  of the holes the **attractive interaction**

**only** appears. The repulsive Coulomb-interaction has no influence on Tc. This is well-known as “Cooper-approximation”.

Therefore  $V_{kk}^{eff}$  is proportional to  $\frac{1}{\varepsilon(anz.)} = \frac{E_F}{MPG}$ .

III) Approximately is valid:  $\frac{1}{Vol} \cdot \frac{e^2}{\varepsilon_0(q^2 + k_{TF}^2)} \approx \frac{2E_F}{3N_{L\ddot{o}}}$ .

In this equation  $N_{L\ddot{o}}$  means the number of the holes and it is supposed, that  $q \ll k_{TF}$ .

Connecting II) and III), we resume  $V_{kk}^{eff} = \frac{1}{Vol} \cdot \frac{e^2}{\varepsilon_0(q^2 + k_{TF}^2)} \cdot \frac{1}{\varepsilon(attractive)} \approx \frac{2E_F}{3N_{L\ddot{o}}} \cdot \frac{E_F}{MPG}$

**3.2** Now we substitute  $V_{kk}^{eff}$  in the self-consistency-equation:

In the case of temperature =  $T_c$ , the “superconduction gap”  $\Delta_{SC}(T_c)$  becomes zero and the

following equation is valid:  $\frac{1}{2} \frac{(2/3)E_F^2}{N_{L\ddot{o}} \cdot MPG} \cdot \sum_k \frac{1}{\epsilon_k} \cdot \tanh\left(\frac{\epsilon_k}{2k_B T}\right) \propto 1$

Then it is necessary to pass over from summation to integration. Because it is integrated about the kinetic energy of the holes, we need the mean DOS of the holes:  $D(E_F) \propto \frac{3N_{L\ddot{o}}}{2E_F}$

$N/E$  ist the DOS of energetic twice-occupied s-states of free conduction electrons. But this approximation is useful, because the experimental known DOS's, divided through the number of holes are proportional to  $1/E$  also <sup>8et9</sup>.

By that we get:  $\frac{1}{2} \cdot \frac{E_F^2}{N_{L\ddot{o}} \cdot MPG} \cdot \frac{N_{L\ddot{o}}}{E_F} \cdot \int \frac{1}{\epsilon_k} \tanh\left(\frac{\epsilon_k}{2k_B T_c}\right) d\epsilon_k \propto 1$

For evaluating the integral, we set:  $\frac{\epsilon_k}{2k_B T_c} \equiv X$

### 3.3 Setting the limits of integrating:

In **chapter 2.6** it was shown, that  $\epsilon(\omega,0)$  is negative, exactly when the Fermi-energy is lower than the maximum pseudogap MPG. In the virtual pairing-state the energy of the holes is lying higher than  $E_F$  (because of the Pauli-principle!) and smaller than MPG. In all other cases the interaction either does not exist or is repulsive. Therefore one receives as integration-limits  $\pm(MPG - E_F)$ .  $\epsilon(\omega,0)$  becomes zero, that means, attracting and repulsing interaction are exactly compensated, when PSL goes to zero. Therefore  $T_c$  goes to zero, when PSL goes to zero. By that the relevant energy-shell will become still smaller with increasing doping. (Here a remarkable distinction to conventional SC appears: In conventional SC the relevant energy-shell has the fixed value  $\hbar\omega_D$ , independent of doping or Fermi-energy.) It is in the nature of such an approximation, that it is not always correct. But as a rule of thumb it is useful. The pseudogap is identical with the term  $MPG - E_F$ .

### 3.4 Fundamental Equation of HTSC

Made out this equation is: 
$$\frac{E_F}{2MPG} \cdot \int_{\left(\frac{MPG-E_F}{2k_B T_C}\right)}^{\frac{MPG-E_F}{2k_B T_C}} \frac{\tanh x}{x} dx \propto 1$$

It will be turned out, that in practical results is valid: For all doping the energy of pseudogap is always significant higher than  $k_B T_C$ . Than the worth of the integral =

$$2\left[\ln\left(\frac{MPG-E_F}{2k_B T_C}\right) - \ln 0,44\right] = 2 \cdot \ln\left(1,14 \frac{MPG-E_F}{k_B T_C}\right) \text{ and } \frac{E_F}{2MPG} \cdot \int_{\left(\frac{MPG-E_F}{2k_B T_C}\right)}^{\frac{MPG-E_F}{2k_B T_C}} \frac{\tanh x}{x} dx =$$

$$\frac{E_F}{2MPG} \cdot 2 \cdot \ln\left(1,14 \frac{MPG-E_F}{k_B T_C}\right) \propto 1$$

This can be transformed in:  $\text{Exp}(MPG/E_F) = 1,14 \left(\frac{MPG-E_F}{k_B T_C}\right)$ ,

respective  $k_B T_C = 1,14 \frac{MPG-E_F}{\exp\left(\frac{MPG}{E_F}\right)}$ .

Identifying the expression  $MPG-E_F$  as pseudogap, results:

$$k_B T_C \propto 1,14 \cdot \frac{\text{Pseudogap}}{\exp(MPG/E_F)}$$

This equation is the **Fundamental Formula of HTSC** (This eq. will be nominated as FF at now.) It is the fundamental quantitative relation between the parameters evaluated by experiment. She is the principal quantitative connection between the experimentally measurable parameters of the HTSC.

From this formula results an additional connection of interesting: As lower the densities of states (DOS) per hole are in the undisturbed state, as bigger has to be **MPG** and as higher will be the critical temperatures. There is a difference to the DOS at the edges of the pseudogap in the state with interaction. As higher **this** DOS are, as higher are the critical temperatures.

### 3.5 Comparison between theory and experiment

If the characteristic reference-energy of pair-building is identified as the energy of the pseudogap, **seven(!)** quantitative relations, which can be proved by experiment, are received.

**Annotation:** The quantity of  $MPG/E_F$  can be received with the assistance of the measured and the maximum pseudogap:

If  $MPG - E_F = \text{Pseudogap}$ , then  $E_F/MPG = 1 - (PSL/MPG) = 1 - \frac{PSL}{MPG}$ .

And: 
$$MPG/E_F = 1 / \left[ 1 - \left( \frac{PSL}{MPG} \right) \right]$$

Thereby was used the (not always correct) approach, that the pseudogap disappears, when the Fermi-energy is reaching the energy of the maximum pseudogap.

#### I) The critical temperatures and their dependence on doping

In 3.4 it was derived the Fundamental Formula for HTSC:

$$k_B T_C \propto 1,14 \frac{PSL}{\exp(MPG/E_F)} = 1,14 \frac{MPG - E_F}{\exp(MPG/E_F)} \quad (6)$$

(Remark: In the case of some HTSC this simple equation does not succeed.

In case of  $YBa_2Cu_3O_7$ , (YBCO), the following formula is valid:

$$k_B T_C \propto 1,14 \frac{PSL + MPG/3}{\exp\left(\frac{3 MPG}{2 E_F}\right)}$$

In the case of YBCO this equation was taken for calculations. Why the YBCO shows a deviation is not yet known.)

Considering the Fundamental Formula for calculating the critical temperatures, you verify, that the  $T_c$ 's do not depend on doping like a parabola, but similar to a parabola. That means, with increasing doping,  $T_c$  also is increasing in the beginning, to go down at more higher doping. The maximum of  $T_c$  will be reached at  $E_F/MPG = 0,62$ . That means, if superconductivity appears on doping  $\leq 0,27$ , the maximum is reached at 0,17. Above a discrete doping (that implies above discrete Fermi-energies) all Cuprates only show normal-state-conduction at all temperatures. (Physical explanation see under **2.6 “variable” Pseudogap c**).

These forecasts of theory are confirmed by experiments (till now). The following tables allow a comparison between theory and measured quantities.

## II) The relation between PSL and $T_c$

Following the Fundamental Equation, we find out:  $\frac{2PSL}{k_B T_C} \propto 1,75 \cdot \exp\left(\frac{MPG}{E_F}\right)$ .

Here are presented tables for **I**) and **II**):

Table I. Critical temperatures and the relation between PSL (= Pseudogap) and  $T_c$  for **Bi2212**

$$MPG \text{ (extrapolated)} = 90 \text{ meV}^{10-12}$$

Doping	$\approx 0,105$	0,12	0,13	0,16	0,18	0,19	0,2	0,21	0,26*
Measured <b>PSL</b> [meV]	49	46,4	44	36	35,5	31	39,3	25	26
$MPG/E_F$ (calc., see above)	2,2	2,064	1,96	1,67	1,65	1,525	1,775	1,38	1,48
$T_c$ (calculated) [K]	72	78	82	90	90	89	88	83	60
$T_c$ (experimental) [K]	64	80	85	90	88	86	79	76	57
$\frac{2PSL}{k_B T_C} = 1,75 \exp(MPG/E_F)$ , calculated	15,8	13,8	12,4	9,3	9,1	8,0	10,3	7,0	7,7
$\frac{2PSL}{k_B T_C}$ (experimental)	17,5	13,2	12,0	10,0	9,35	8,6	11,3	8,7	10,6

**Lit.:**<sup>10</sup>

Author: Hans Chr. Haunschild, born Ffm 20. 2. 1964

Table II. Critical temperatures and the relation between PSL (= Pseudogap) and Tc for **LSCO**

$$\text{MPG}(\text{extrapolated}) = 38 \text{ meV}^{10,12}$$

Doping	0,04	0,05	0,06	0,075	0,08	0,086
Measured PSL [meV]	33	31,5	30	27,5	27	26
MPG/ $E_F$ (calc., s. above)	7,6	5,85	4,75	3,62	3,46	3,17
Tc (calculated) [K]	0,2	1,2	3,4	9,7	11,3	14,5
Tc (experimental) [K]	[Null] <sup>13er18</sup>	[Zero] <sup>15</sup>	8 <sup>14</sup> 5,5 <sup>15</sup> 8,5 <sup>15</sup>	21,2 <sup>13</sup> 24 <sup>16</sup> 19 <sup>15</sup>	19,6 <sup>17</sup> 29,7 <sup>18</sup>	22,25 <sup>17</sup>
$\frac{2Psl.}{k_B T_C} = 1,75 \exp\left(\frac{MPG}{E_F}\right)$	3500	608	202	65	56	42
$\frac{2Psl.}{k_B T_C}$ (experimentell)	$\infty$	$\infty$	103	31	27	30

Doping	0,09	0,1	0,105	0,11	0,113	0,115	0,12
Measured PSL [meV]	25	23	22,75	22,5	22,3	22,25	22
MPG/ $E_F$ (calc., s. above)	2,9	2,53	2,5	2,45	2,42	2,41	2,375
Tc (calculated) [K]	17,8	24	25	25,6	26,2	26,4	27
Tc (experimental) [K]	27,5 <sup>14</sup> 29,3 <sup>16</sup> [16 <sup>15</sup> ]	29,2 <sup>16</sup> 26,2 <sup>19</sup>	27,8 <sup>14</sup>	26,1 <sup>20</sup> [13,9 <sup>21</sup> ] 29,1 <sup>16</sup>	29,6 <sup>13</sup>	27,3 <sup>20</sup>	30,2 <sup>20</sup> 29,4 <sup>16</sup>
$\frac{2PSL}{k_B T_C} = 1,75 \exp(MPG/E_F)$	32	22	21,3	20,3	19,7	19,5	18,8
$\frac{2PSL}{k_B T_C}$ (experimental)	22	21	22	20	18,5	19,9	17,75

Doping	0,125	0,13	0,14	0,15	0,16	0,17	0,18
--------	-------	------	------	------	------	------	------

Measured PSL [meV]	21,5	21	20,5	16,6	14	11,75	11
MPG/ $E_F$ (calc., s. above)	2,3	2,235	2,17	1,775	1,58	1,44	1,4
Tc (calculated) [K]	28,4	30	31	37	38	35	35,6
Tc (experimental) [K]	28,4 <sup>22</sup> 27,8 <sup>23</sup>	34,4 <sup>16</sup>	36,9 <sup>16</sup> 40 <sup>12</sup> 35 <sup>24</sup>	34,5 <sup>22</sup> 33,5 <sup>25</sup> 28,6 <sup>19</sup> 38,0 <sup>23</sup> 37,0 <sup>14</sup> 36,5 <sup>13</sup> 33,0 <sup>27</sup> 31,5 <sup>18</sup> <b><math>\Phi = 34,5\text{K}</math></b>	39,0 <sup>26</sup>	34 <sup>15</sup>	40 <sup>12</sup> 35 <sup>24</sup>
$\frac{2PSL}{k_B T_C} = 1,75 \exp(\text{MPG}/E_F)$	18,35	16,4	15,3	10,3	8,5	7,4	7,1
$\frac{2PSL}{k_B T_C}$ (experimental)	18,35	14,6	11,2	11,2	9,5	7,5	6,2

Doping	0,188	0,2	0,225	0,24	0,25	0,263	0,3
Measured PSL [meV]	8,5	7	5	4	3	Ca. 2	Zero
MPG/ $E_F$ (calc., s. above)	1,3	1,226	1,15	1,12	1,086	1,06	Ca. 1
Tc (calculated) [K]	31,0	27,2	21	17,3	13,4	9,2	Zero
Tc (experimental) [K]	34,4 <sup>13</sup>	[18,9] <sup>19</sup> 33,5 <sup>15</sup> 30,4 <sup>18</sup>	23 <sup>13</sup> 25 <sup>12</sup>	19 <sup>18</sup>	10,1 <sup>21</sup>	8 <sup>13</sup>	[15,2 <sup>19</sup> ]
$\frac{2PSL}{k_B T_C} = 1,75 \exp(\text{MPG}/E_F)$ , calculated	6,35	6	5,5	5,4	5,2	5	4,8 (theor. value)
$\frac{2PSL}{k_B T_C}$ (experimental)	6,65	5,8	5,8	5,6	[9,2]	5,8	[1,5]

**Literature:** The values of pseudogaps are from <sup>11er12</sup>.

Experimental Tc-values in brackets are "freak values", resp. "runaways", probably measuring faults. Take note of the phenomenon, that a "plateau-region" between the dopings of 0,1 and 0,13 is predicted in theory. This predicted "plateau-region" is confirmed by experiment.

### III) The magnitude of the Pseudogap in dependence of doping

In the case of doping zero ( $E_F = 0$ ) is valid:  $MPG - E_F = MPG - 0 = MPG$ . Following this theory the pseudogap must reach his maximum -defined as MPG- by zero doping.

This assumption is confirmed by experiment<sup>8,9</sup>.

The maximal Tc is reached at: 
$$E_F = \frac{\sqrt{5}-1}{2} MPG \approx 0.618 \text{ MPG}.$$

This is exactly the formula of the “Golden Section”, which cannot be only a coincidence. That means, when superconduction appears at doping  $\leq 0.27$ , the theoretical Tc-maximum will be reached at a doping of  $0.62 \cdot 0.27 = 0.167$ . The most HTSC have their Tc-maximum at a doping of 0.16. That is not valid in the case of YBCO and PCCO. For this both the second eq. mentioned in **3.5 I**) must be taken. In this cases the Tc`s reach their maxima at  $E_F \approx 0.85$  MPG, which is confirmed exactly by experiment<sup>8,9</sup>.

### IV) The relation between the maximum pseudogap and that value of pseudogap, found at optimal doping

If  $\frac{E_F(T_C^{\max})}{MPG} = 0,62$ , then is  $MPG - E_F(T_C^{\max}) = 0,38 \text{ MPG}$ . (8)

From the relation:  $MPG - E_F(T_C^{\max}) = 0,38 \text{ MPG} = \text{pseudogap at } T_C^{\max}$ , results:

$$MPG = \frac{PSL(T_C^{\max})}{0,38} = 2,63 \cdot PSL(T_C^{\max}). \quad (9)$$

But that causes, the maximum pseudogap is 2,63 times higher than the pseudogap at optimal doping.

Table III. Relations of pseudogaps

Substance	LSCO	Bi2212	Bi2223	Hg1201	Hg1212	Hg1223	PCCO
$PSL(T_C^{\max})$ [meV]	14	36	45	33	50	46	11 (?)
Experimental							

$\frac{PSL(T_C^{\max})}{0,38} = \text{MPG}$ calculated [meV]	37	95	118	87	132	120	29
MPG, extrapolated from experiment [meV]	38	90	110	97	123	131	23

Substance	YBCO	$YBa_2Cu_4O_8$	NdCCO	Tl2201
$PSL(T_C^{\max})$ [meV] Experimental	22	24,4	4,6	35,5 $37^{15}$
$\frac{PSL(T_C^{\max})}{0,38} = \text{MPG}$ calculated [meV]	58	64	12	93 $97^{15}$
MPG, extrapolated from experiment [meV]	92	81	18,5	100

All values without <sup>15</sup> are from <sup>10et12</sup>.

### V) Coherence between $T_c$ and pseudogap at optimal doping

$$k_B T_C^{\max} = 1,14 \cdot PSL(at T_C^{\max}) \cdot \exp(-1/0,62) = 1,14 \cdot 0,2 \cdot PSL(T_C^{\max})$$

Then  $T_C^{\max}$  is the value of the pseudogap [measured in meV] multiplied with 2,64. (10)

Table 5. Coherence between  $T_c$  and pseudogap at optimal doping

Substance	LSCO	Bi2212	Bi2223	Hg1201	Hg1212	Hg1223	PCCO
$T_C^{\max}$ experiment.[K]	37	90	111	97	123	131	23
$PSL(T_C^{\max})$ [meV] Experimental	14	36	45	33	50	46(measuring fault ?)	11 (?)
$\frac{T_C^{\max} [K]}{PSL(at T_C^{\max}) [meV]}$	2,64	2,5	2,47	2,9	2,5	2,85	2,1

Substance	YBCO	$YBa_2Cu_4O_8$	NdCCO	Tl2201
$T_C^{\max}$ experim.[K]	92	81	18,5	90
PSL( $T_C^{\max}$ ) [meV]	22	24,4	4,6	35,5
Experimental				$37^{15}$
$\frac{T_C^{\max} [K]}{PSL(at T_C^{\max}) [meV]}$	4,2	3,3	4,0	2,54
				$2,43^{15}$

**Lit.** <sup>10et15</sup>

**VI) The maximum  $T_C$ , measured in K, has the same numerical value like the maximum pseudogap, measured in meV.**

From **V)** results, that  $T_C^{\max}$  [in K] =  $2,64 \cdot PSL(T_C^{\max})$  [in meV]

From **IV)** results, that MPG [meV] =  $2,63 \cdot PSL(T_C^{\max})$  [meV]

In consequence of the results of **IV) and V)** results, that  $T_C^{\max}$  [in K] =  $\frac{2,64}{2,63} \cdot MPG$  [in meV].

That means, with increasing MPG (by pressure, for example),  $T_C^{\max}$  also increases.

Table V. Ratio between MPG and  $T_C^{\max}$

Substance	LSCO	Bi2212	Bi2223	Hg1201	Hg1212	Hg1223	PCCO
$T_C^{\max}$ experiment.[K]	37	90	111	97	123	Ø =131	23
MPG [meV]	37	90	Not	deter-	min-	ed	20 (?)
$T_C^{\max} /MPG$	1,0	1,0					1,15

Substance	YBCO	$YBa_2Cu_4O_8$	NdCCO	Tl2201
$T_C^{\max}$ experiment.[K]	92	81	18,5	90
MPG [meV]	unknown	71	unknown	100
$T_C^{\max} /MPG$		1,14		0,9

**Literature:** <sup>10</sup>

**VII) Relationship between energy-gap of superconduction  $\Delta_{SC}(T=0)$   
and critical temperature  $T_c$ .**

About the presupposition, that the energy  $\varepsilon_k$ , transferred between the holes is situated in the area of the pseudogap, results from BCS-theory<sup>7</sup>:

$$\text{Superconducting-energy-gap} = \Delta_{SC}(T=0) = \frac{\text{Pseudogap}}{\sinh[1/D(E_F)V]} = \frac{\text{Pseudogap}}{\sinh[MPG/E_F]}$$

$$\text{Und: } 2\Delta_{SC}(T=0)/k_B T_c = \frac{2 \cdot \text{Pseudogap}}{\sinh[MPG/E_F]} \cdot \frac{\exp(MPG/E_F)}{1,14 \cdot \text{Pseudogap}} \approx \frac{2 \cdot 2 \exp(MPG/E_F)}{1,14 \exp(MPG/E_F)} \approx 3,5.$$

Since in the Cuprates the d-wave-pairing appears, it is necessary to multiplie this value with 1,21306<sup>28</sup>. In consequence appears a value of 3,52\*1,21306 = 4,27. This value of 4,27 is valid for underdoped substances. In case of overdoped substances results a value of 4,0\*1,21306 = 4,85. That means, with augmenting doping  $\Delta_{SC}$  increases circa about a seventeenth. Measured are values between 4 and 6<sup>29et30</sup>, and in fact the experimental mean-values are augmenting with increasing doping about a seventeenth. On Tl2201 seems to exist a recognizable deviation: Possible here are measured values between 8 and 11<sup>30</sup>.

### 3.6 Evaluation

Theory and experiment are in satisfying accordance. Distinct deviations between theoretical predictions and experimental measurements appear at the Yttrium-compounds and at NdCCO. For these classes of substances accordance between theory and experiment is available, if there are used modified formulas of the FF. This theory not only makes qualitatively, but quantitatively exactly right predictions. That is so more surprising, if it is bored in mind, how strong simplifications are included. An unequivocal relation between the pseudogaps, Maximum Pseudogap, doping and superconductivity is shown.

#### IV. Explanation of the Isotope-Effects (IE) of the HTSC

For completing, the isotope-effect in HTSC should be discussed.

That the HTSC at all show an IE, is a sure hint, that phonons are playing an important role in the pair-building.

Probably (see **chapter 2.6** in this work) the magnitude of the maximum pseudogap MPG is depending on the atomic-mass-number of Cu- and oxygen. For proving this assumption, it's really necessary to examine the PSL as a function of the atomic mass. In sight of the big variableness of the values of the single pseudogaps this way doesn't make sense. That is why we will go now an indirect way and compare the critical temperatures of Bi2212 and LSCO. Indeed, this two substances are not distinguishing in their atomic-mass-numbers, but in the magnitudes of their MPG's. If the Tc's really are depending on the MPG's and the latter are depending on the atomic masses, then every other variation of MPG should generate similar consequences for the Tc-values like the variation of the atomic-masses. That means, an augmentation of MPG will produce similar results as a reducing of the atomic-mass-numbers. So is telling the theory. Wether this prediction is holding, this can be ascertained by concerning together the critical temperatures of Bi2212 and LSCO:

$$\frac{T_c(\text{Bi2212})}{T_c(\text{LSCO})} = \frac{PSL(\text{Bi2212})}{PSL(\text{LSCO})} \cdot \exp\left(\frac{MPG(\text{LSCO})}{E_F} - \frac{MPG(\text{Bi2212})}{E_F}\right) \quad (14)$$

Taking this equation, results Table 6):

Doping	$\frac{PSL(Bi2212)}{PSL(LSCO)}$	$\frac{MPG}{E_F}(LSCO) - \frac{MPG}{E_F}(Bi2212)$	$\frac{T_C(Bi2212)}{T_C(LSCO)}$ theoretical	$\frac{T_C(Bi2212)}{T_C(LSCO)}$ experimental
0,06*	2,0	4,75 -3,0 = 1,75	11,5*	unknown
0,08*	2,0	3,46-2,5 = 0,96	5,2*	unknown
0,105	2,154	2,5-2,2 = 0,3	2,9	2,3
<b>0,12</b>	<b>2,11</b>	<b>2,533-2,064 = 0,47</b>	<b>3,4</b>	<b>2,7</b> <b>(rel. Max.)</b>
0,13	2,1	2,235-1,96 = 0,14	2,4	2,47
<b>0,16</b>	<b>2,67</b>	<b>1,5-1,67 = -0,17</b>	<b>2,25</b>	<b>2,3</b> <b>(rel. Min.)</b>
0,18	3,23	1,4-1,64 = -0,25	2,5	2,35
0,19	3,65	1,3-1,525 = -0,225	2,9	2,5
0,2	5,6	1,226-1,775 = -0,55	3,2	2,47
0,21*	4,0	1,2-1,38 = -0,18	3,34*	2,6
0,25*	5,8	1,0875 -1,241 = -0,1557	5,0*	unknown

PSL und Tc`s with \* are unknown partially, the values are estimated.

**[Lit.: chapter 3.5, tables I) und II) in this work]**

Considering the big differences between the magnitudes of the the pseudogaps of Bi2212 and LSCO the variableness of these magnitudes can hardly falsify the results. The relations of the experimental Tc`s are consistent in the framework of measuring-accuracy. One could expect such a result considering the success of the theory till now. It is more interesting, that the relation of Tc in dependence of doping demonstrates the same behaviour as the IE in dependence of doping: Table 7) is showing the experimental OIE in LSCO:

Doping	Pseudogap [meV]	MPG/ $E_F$	$-\Delta T_C$ [%] experimental	$T_C(O^{16})$ [K]	$-\Delta T_C$ [K]
0,06	30	4,75	12,5 <sup>20</sup>	8,0	1,0

<b>0,075</b>	<b>28</b>	<b>3,8</b>	<b>3,9<sup>13et30</sup> (rel. Min.)</b>	<b>20,1</b>	<b>0,76</b>
0,08	27	3,46	5,5 <sup>17</sup>	19,6	1,1
0,086	26	3,17	5,3 <sup>17</sup>	20,0	1,07
0,09	24,5	2,9	5,1 <sup>32et20</sup>	24,4	1,6
≈0,105	23,75	2,67	5,75 <sup>20;21;33</sup>	28,5	1,7
0,11	23,5	2,62	6,1 <sup>33et20</sup>	26,1	1,6
≈0,114	23,3	2,585	7,7 <sup>13;20;33</sup>	29,6 u. 20,1	1,9 u. 1,46
0,12	23	2,88	7,6 <sup>31;33</sup>	25,4	2,25
<b>0,125</b>	<b>22</b>	<b>2,375</b>	<b>9,4<sup>22;23</sup> (rel. Max.)</b>	<b>27,6</b>	<b>2,0</b>
0,13	21	2,235	6,4 <sup>32</sup>	29,8	2,0
0,14	20,5	2,17	1,85 <sup>32</sup>	32,4	0,6
0,15	16,6	1,775	1,9 <sup>13;20-23;32</sup>	37	0,7
<b>0,166</b>	<b>12</b>	<b>1,46</b>	<b>1,4<sup>35</sup> (rel. Min.)</b>	<b>35</b>	<b>0,5</b>
0,175	11,4	1,43	1,85 <sup>32</sup>	16,2	0,3
0,19	8	1,27	0,6 <sup>13</sup>	34,4	0,21
0,2	7	1,266	3,9 <sup>21</sup>	28,4	1,1
0,225	5	1,15	1,0 <sup>13</sup>	23	0,23
0,25	3	1,086	3,0 <sup>21</sup>	10,1	0,3
0,263	Ca. 2	1,06	2,75 <sup>13</sup>	8,0	0,22

The values of the pseudogaps see **chapter III. in this work**

In the diagrams belonging to the IE`s, are drawn:

- a) experimental ratio between the Tc`s of Bi2212 and LSCO and
- b) experimental OIE of LSCO in %

Considering the diagrams the qualitative similarity of the slopes a) and b) is striking. Every time in both slopes depending on  $p \approx 1/8$  appears a relatively maximum and depending on optimal doping every time a relatively minimum. There are absolutely minima at optimal doping every time. In the overdoped region both slopes are increasing steep with increasing doping.

In the strong underdoped region ( $p = 0,06$ ) the experimental OIE-values are extremely raising. That is also predicted by this theory.

These remarkable accordances can demonstrate, that an augmentation of atomic mass is producing a reduction of MPG and lower critical temperature by this way.

That the ratio between  $T_c(\text{Bi2212})$  and  $T_c(\text{LSCO})$  is increasing much faster than the OIE of LSCO is no tragedy: That is the consequence of the fact, that the ratio of the MPG`s of this both substance-classes is much higher (bigger) than the mass-ratio between  $O^{18} / O^{16}$ .

Beginning at optimal doping, both slopes are increasing with increasing and with decreasing doping. Indeed, both slopes do not reach the value of zero at any time, although there is pretended the contradiction in many textbooks (compendia).

Comparing theory and experiment it is established, that the predictions of theory are satisfied in general. The prediction of a decreasing maximum pseudogap MPG with increasing atomic weight is reliable, resulting in an Isotope Effect.

## **5. Influence of impurities upon the superconducting parameters**

Nearly with discovering of HTSC it was established, that the substitution of copper by paramagnetic ions suppresses the  $T_c$ -values. This could be expected by the well-known theories of superconductivity at this time. But it was not provided for a stronger sinking of critical temperatures by substitution of copper by unmagnetically zinc-ions. There are no hints for changing the (maximum) pseudogap by impurities. By that a decreasing of the pair-density by impurities can be supposed. Quantitatively: The impurities increase the argument of the exponential-function in the denominator of the FF. If this assumption is true, than with

increasing doping the influence of the impurities is decreasing. Someone very similar we got to know upon the IE. In accordance with the FF is valid:  $T_c = 1,14 \cdot PG \cdot \exp(MPG/E_F)$ .

This simple equation only is valid **without substitutions**. With substituting impurities is

$$T_c = 1,14 \cdot P_{sl} \cdot \exp\left(F \cdot \frac{MPG}{E_F}\right).$$

In this formula is  $F > 1$  and proportional to the portion of impurities. It is in the nature of such an approximation, that the worth of  $F$  depends on the type of impurities, too.

The result is:  $\frac{T_{c1}}{T_{c2}} = \exp\left[(1-F) \frac{MPG}{E_F}\right]$ . If we point out  $1-F = f$ , we get:

$$T_{c2}/T_{c1} = \exp[-f(MPG/E_F)]. \quad (15)$$

The size of “**f**” is an other for every metal (and probably for every class of substances, too) and must searched by experiment. In contrary the size of “**f**” should not depend on doping.

For the class of LSCO-substances the following tabulas will show the quantitative comparison between theory and experiment:

**Zink:**  $\underline{La_{1,85}Sr_{0,15}Cu_{(1-x)}Zn_xO_4}$ :

$MPG/E_F = 1,6$                       Parameter = 4 und  $F = 1 + \frac{\%Zn}{4}$                       Therefore is valid:

$$T_{c2}/T_{c1} = \exp\left[(1-F) \frac{MPG}{E_F}\right] = \exp\left[\left(1 - 1 - \frac{\%Zn}{4}\right) \frac{MPG}{E_F}\right] = \exp\left(-\frac{1,6}{4} \cdot \%Zn\right)$$

Zn-portion in %	$\frac{1,6}{4} \cdot \%Zn$	Tc2/Tc1 Theory	Tc2/Tc1 Experiment
0,4	0,16	0,85	1,0
0,8	0,32	0,73	0,76
1,2	0,48	0,62	0,63

1,6	0,64	0,53	0,51
2,2	0,9	0,4	0,36

**Lit.** <sup>36et37</sup>

**Nickel:**

$\underline{La_{1,8}Sr_{0,15}Cu_{(1-x)}Ni_xO_4}$        $MPG/E_F = 1,6$       Parameter = six

**und**  $\underline{La_{1,8}Sr_{0,2}Cu_{(1-x)}Ni_xO_4}$        $MPG/E_F = 1,25$       Parameter = six

	for opt.	doping		for over-	Doping
Ni-portion in %	Tc2/Tc1 Theory	Tc2/Tc1 Experiment		Tc2/Tc1 Theory	Tc2/Tc1 Experiment
0,5	0,88	0,92		0,9	0,91
1,0	0,77	0,8		0,81	0,82
1,5	0,67	0,69		0,73	0,74
2,0	0,59	0,59		0,66	0,65

**Lit.** <sup>36et37</sup>

**Iron:**

$\underline{La_{1,85}Sr_{0,15}Cu_{(1-x)}Fe_xO_4}$        $MPG/E_F = 1,6$       Parameter = 3,2.

Therefore is valid:

$Tc2/Tc1 = \exp\left(-\frac{1,6}{3,2} \cdot \%Fe\right) = \exp(-0,5 \text{ multiplied with the \% -portion of Fe})$

**and**  $\underline{La_{1,8}Sr_{0,2}Cu_{(1-x)}Fe_xO_4}$        $MPG/E_F = 1,25$       Parameter = 3,2.

Here is valid:  $Tc2/Tc1 = \exp\left(-\frac{1,25}{3,2} \cdot \%Fe\right) = \exp(-0,4 \text{ multiplied with the \% -portion of Fe}).$

Fe-portion in %	Tc2/Tc1 Theory	Tc2/Tc1 Experiment		Tc2/Tc1 Theory	Tc2/Tc1 Experiment
0,3	0,86	0,9		0,89	0,94
0,5	0,78	0,82		0,82	0,85

0,7	0,71	0,7		0,76	0,81
0,8	0,67	0,66			unknown.
0,9	0,64	0,53			unknown
1,0	0,61	0,45 <sup>38</sup>		0,68	0,69

**Lit.** <sup>36-38</sup>

### **Result:**

The theory predicts a linear decrease of critical temperatures by increasing portions of impurities. The dependency of the optimal doped HTSC (and probably of the underdoped, too) is bigger than the dependency of the overdoped. Again a nearly quantitative agreement between theory and experiment exists.

Diagrams to these tables you can find in the supplement.

In contrary to copper the zinc possesses a closed 3d-shell. Therefore the substitution with zinc-ions prevents the “conditional-hopping-interaction” of the EL, leading to a decreasing of density of pairs. Naturally the reduction of density of pairs leads to lower critical temperatures.

The explanation of reducing the density of pairs by paramagnetic ions is a part of the BCS-theory and must not discussed here.

## **6. The temperature-dependency of normal state resistance of HTSC**

It is well-known [**Lit. see chapter 7.**] that in overdoped HTSC the normal state resistance is increasing **linear** with temperature over a large range of temperature –especially down to the

lowest temperatures. This behaviour cannot be explained by the conventional models of electron-phonon- or electron-electron-interactions. In the following a possible explanation of this unusual behaviour will be presented:

In a totally periodical lattice no electrical resistance appears. The electrical resistance always is generated by a disturbance of the periodicity of lattice. Thereby in HTSC must appear disturbances whose number or whose scattering cross-section is increasing linearly with increasing temperature. In a metal the density of charge-carriers, which are thermally excited above a discrete energy, is increasing linearly with temperature. It's nearby, searching this extra-ordinary temperature-dependence of resistance in this connection.

Therefore it is reasonable to search for the extraordinary temperature-dependence in this context.

As mentioned in **chapter 2.2**, in the HTSC the EL (not the holes) do not behave as part of the electron gas but behave similarly to particles of a lattice. These electrons, which are stimulated about the higher band edge behave similarly to **irregularly** distributed lattice-sites. These irregularly distributed "lattice-sites" disturb the periodicity of lattice. Since the holes are not only particles, but also are waves, they are scattered at these lattice deformations. By that the quasi-impulse  $\Delta k\hbar$  is translated upon the total crystal, this means upon all Cu-3d-EL in the crystal. Therefore these thermal stimulated EL act as disturbances of the periodicity of lattice –that means of scattering centres for the holes.

As the mass of all EL is bigger than the mass of one hole, it is a nearly elastic collision. Translating of energy is negligible. In the last resort, the hole will be scattered into a state with similar energy (about  $E_F$ ), but another (that means a negative) wave-vector. But that means that electrical energy will be converted in warmth, because the translated quasi impulse  $\Delta k\hbar$  will be absorbed by the whole crystal. By that, no charge can be transmitted.

That means that the charge, which had been transported by the hole, reverses its direction of motion. This process is noted as electrical resistance. Due to the Pauli-principle only these

Cu-3d-EL, which are nested in the vicinity or above the upper edge of the band can temporarily take energy and linear momentum. Cu-3d-EL in lower levels of the band do not come into consideration as targets for the holes.

**In the HTSC the interaction between holes and thermal excited EL gives rise to an electrical resistance.**

Although the scattering is nearly elastic, the total angle of impact between hole and pushed EL is always  $90^\circ$ . This results from the conservation of energy and impulse: In the beginning the hole strikes with the EL and after that the EL translates energy and impulse upon the lattice. By that the total angle of impact between hole and pushed EL always has a size of  $90^\circ$  (classical physic).

Therefore the hole is scattered in an angle smaller than  $90^\circ$ . This small angle scattering between holes and EL only can exist, if one of the partners of scattering (in our case hole or EL) was excited about the band edge before scattering. Otherwise no push is coming about, since the hole would forced to be scattered in already occupied states.

The densities of EL and holes, which are thermally excited about the band edges, are both growing linearly with temperature. Naturally the sum of both densities also grows linearly with temperature. From this fact the equation for the electrical resistance in the HTSC includes the well-known linear therm.

The elastically interaction of holes with thermal excited EL in the HTSC is leading to an electrical resistance, increasing **linear** with temperature.

In conventional metals there is a pure electron gas, whose constituents cannot be excited to (“lattice site”)-oscillations because of absence of restoring force. Therefore the mechanism of resistance –described above- cannot exist in conventional metals.

The density of EL which are thermally excited above the Cu-3d-band increases linearly with increasing temperature. Accordingly the density of scattering frequency and because of that

the normal state resistance also increases linearly with increasing temperature, what is in agreement with experiments. [**Literature at chapter 7.**]

This argumentation assumes that the interaction between mobile holes and immobile but excited EI does not depend on temperature and that the density of state of the EI does not depend on their energy. The well-known interaction between holes and phonons is more than a factor of hundred smaller than the explained interaction between holes and bound EI and does not matter.

For completeness it should be remarked, that this consideration are not available for metallic hydrogen and for palladium-hydrogen-systems. In such systems the protons are installed at fix lattice sites and can be identified by that. Therefore as many as you like of protons can occupy the same state. The conduction electrons of these substances can be scattered on every proton irregularly installed in the lattice. Therefore results a T-independent RRR and no linear T-dependent resistivity can be observed.

## **7. Numerical calculation of temperature-dependence of the normal state resistivity**

### **7.1 Calculation of the density of thermal above the Fermi-energy stimulated EI.**

In this section we will make a quantitative calculation of the normal state resistivity.

In a certain subject the number (not the density) of thermal excited EI. is:

$$N_{th} = \int_{E_F}^{E-E_F} f(E)D(E)dE$$

In a two-dimensional crystal the electronic density of states  $D(E)$  does not depend on the energy of the states of electrons. Therefore  $D(E)$  does not depend on carried thermal energy or of temperature. That is why  $D(E)$  can be taken before the integral:

$$N_{th} = D(E) \int_{E_F}^{E-E_F} \frac{dE}{\exp\left(\frac{E-E_F}{k_B T}\right) + 1}$$

We now take  $E_F$  as zero point of energy and define  $E/k_B T = X$ . Now it is possible to write:

$$N_{th} = D(E) k_B T \int_0^{\infty} \frac{dx}{e^x + 1} = D(E) k_B T [x - \ln(1 + e^x)]_0^{\infty}$$

The limitation of the integral is received by the following considerations: At very low temperatures  $x$  will reach infinite quantity. For high temperatures  $x$  will reach zero.

The DOS of a two-dimensional electron gas is:  $D(E) = \frac{m_e A}{\hbar^2 \pi}$

(In reality this formula is acceptable only for s- and not for d-electrons. The real DOSes are larger and additionally depending on the density of d-El. in the ground state. But to show that the normal state resistivity really can be brought about by this described mechanism, this primitive formula is sufficient.)

By that  $N_{th} \approx 0,7 \frac{m_e A}{\hbar^2 \pi} k_B T$ .

And the density  $n_{th}$  of thermal excited El.:

$$n_{th} = \frac{N_{th}}{A} = 0,22 \frac{m_e}{\hbar^2} k_B T = 2,5 \cdot 10^{14} \cdot \left(\frac{T}{K}\right) \cdot m^{-2}. \quad (16)$$

Therefore the density of thermal super  $E_F$  excited El. the theoretical value of  $2,5 \cdot 10^{14}$  El. on square meter and Kelvin. By that the density of excited El. in a  $CuO_2$ -area of a HTSC depends only on temperature, but not on the electronic density in the ground state.

## 7.2 Probability of interaction between holes and EL

As explained in section 6., the thermal excited EL are disturbing the periodicity of the lattice site by which is generated an electrical resistivity. With the results of the density of thermally excited EL in section 7.1 it is now possible to make a numerical calculation of the resistivity generated by the hole-EL.-scattering:

$$\rho = \frac{m_{\text{hole}}}{n_0 e^2 \tau} \quad \frac{1}{\tau} = \frac{v_F}{l} \quad \text{and} \quad l = \frac{1}{\frac{n_{\text{ofll.}}}{\text{meter}} \cdot n_{\text{th}} \sigma_0}$$

Because of calculation of  $n_{\text{th}}$  in  $m^{-2}$ , it was necessary to convert in  $m^{-3}$ . Therefore it was multiplied with the number of lattice-layers per meter (n. of ll./meter).

### Intermediate notice:

$n_0$  is the concentration of holes in the dimension  $m^{-3}$  and  $\tau$  is the relaxation rate, that is the time between two collisions. Meant are the collisions between the mobile holes and the immobile EL.  $v_F$  is the Fermi-velocity. As the holes can only move in two directions of space, it is necessary to calculate  $v_F$  for a two-dimensional electron gas. That is why

$$v_F = \frac{\hbar}{m_{L\delta}} \sqrt{2\pi \frac{N}{A}}$$

$n_{\text{th}}$  means the density of thermally excited d-EL. at the Cu-sites (not the density of the holes) in two-dimensional metals.  $\sigma_0$  is the cross-section for interactions between the mobile holes and the thermally excited, but localized EL.

With  $\sigma_0 = \frac{2 \cdot 10^5}{v_F^4}$  (see **chapter 6.3**) results for reluctance:

$$\frac{d\rho}{dT} = \frac{m_{L\delta}}{e^2 \cdot \frac{N}{A} \cdot \frac{Z.d.GS}{\text{meter}}} \cdot v_F \cdot \frac{Z.d.GS}{\text{meter}} \cdot n_{\text{th}} \cdot \frac{2 \cdot 10^5}{v_F^4}$$

With  $v_F^3 = \frac{\hbar^3}{m_{L\ddot{o}}^3} (2\pi)^{3/2} \cdot \left(\frac{N}{A}\right)^{3/2}$  the final result for resistivity is:

$$\rho = \frac{m_{L\ddot{o}}}{n_0 e^2} \cdot \frac{1}{\tau} = \frac{m_{L\ddot{o}}}{n_0 e^2} \cdot \frac{\hbar \sqrt{2\pi \cdot N / A}}{m_{L\ddot{o}}} \cdot n_{th} \cdot \frac{Z.d.Gs.}{meter} \cdot \sigma_0 \quad (17)$$

### Example:

In  $YBa_2Cu_3O_{6,9}$   $1,7 \cdot 10^9$  current-carrying lattice-layers are paralleled<sup>39</sup>.

. The density of holes amounts to, what is (numerical):  $\frac{N}{A} = \frac{4,6 \cdot 10^{27} m^{-3}}{1,7 \cdot 10^9 m^{-1}} = 2,7 \cdot 10^{18} m^{-2}$ .

$n_{th}$  multiplied by the number of lattice-layers per meter:  $2,5 \cdot 10^{14} m^{-2} K^{-1} \cdot 1,7 \cdot 10^9 m^{-1}$ .

From this results as conductivity  $\rho = 1,57 \cdot 10^9 \frac{\Omega}{meterK} \sigma_0$ .

With  $\sigma_0 = 3,9 \cdot 10^{-18} m^2$  (For calculation of cross-section  $\sigma_0$  look at 5.3) results an increase of resistivity of ca.  $6 \cdot 10^{-9} \Omega m/K = 0,6 \mu\Omega cm/K$ . On assumption, that shielding reduces the resistivity an increase of **0,1  $\mu\Omega cm/K$**  results. Measured are values between 0,25 and  $1,46 \cdot 10^{-9} \mu\Omega cm/K$ <sup>40-45</sup>.

In the middle, this is about  $0,85 \mu\Omega cm/K$ . Theory and experiment are not in agreement as well. The very strong simplifications made by this model are a possible reason. In the case of LSCO better agreements between theory and experiment are received (look at VIII, Table 11.). It should be remarked, that doping of  $CuO_2$ -planes of the same substance with  $2,24 \cdot 10^{17}$  Cobaltions per squaremeter leads to a RRR of ca.  $270 \mu\Omega cm$ <sup>46</sup>.

Therefore  $2,5 \cdot 10^{14}$  excited EL per squaremeter and Kelvin should make an increase of resistivity of  $0,3 \mu\Omega cm/K$ . This result makes confidence into the actual insure calculation.

### **7.3 Calculation of collision cross-section $\sigma_0$ .**

$\sigma_0$  will be calculated with the following equation from plasmaphysics:

$$\sigma_0 = \frac{\pi Z^2 e^4}{(4\pi \epsilon_0)^2 m_{L\delta}^2 v_F^4} = 2 \cdot 10^5 v_F^{-4} \text{ [squaremeters]} \quad (18)$$

$v_F$  is the Fermi-velocity (in two dimensions), because only the holes on the Fermi-surface can be scattered.  $Z$  is equal one (charge of holes) and  $m_{L\delta}$  is the mass of holes (= mass of electrons).

The attentive reader will find fault with something:

1. This equation only considers the large-angle-scattering.
2. In this formula a cross-section is calculated. Consequently in a two-dimensional hole-gas a perimeter ought to be calculated. But the interaction between holes and El. will be produced by the Coulomb-interaction. Although the holes cant leave their lattice-layers, nevertheless they are in interaction with the El from neighbouring lattice-layers and will be shielded by themselves.
3. The Coulomb-interaction of El. with the holes reduces the cross-section of collision. In this work will be used as an approximate calculation the theory of Thomas-Fermi. It`s possible to deduce this theory indeed for two dimensions. But this is senseless, because the Coulomb-shielding exists also cross to the lattice-layers. The shielding reduces the resistance to about 15%.

## **8. The temperature-dependence of normal state resistivity of some HTSC at low temperatures for example.**

Summarizing all just terms factors for calculation of resistivity, results the following equation:

$$\frac{d\rho}{dT} = 7,27 \cdot 10^{45} \cdot \left(\frac{N}{A}\right)^{-2,5} \mu\Omega cm / K. \quad (19)$$

Apart from shielding, which has to be considered for each HTSC separately, this formula is valid for all HTSC, independently of the chemical composition, the stoichiometry or density of holes. In any case, if the approximation of quasi-free holes is acceptable and conventional scattering mechanisms are only playing a subordinate part. Therefore the formula above is very suitable for proving the usability of the represented theory. With this formula for  $\rho$  the plot of the measured resistivity values against  $(N/A)^{-2,5}$  will make a straight line with an ascending gradient of  $7,35 \cdot 10^{45}$ . Additional regarding the shielding (shielding factor see

**chapter 7.3 c)**, then we should get a straight line with the ascending gradient of

$$\frac{7,35}{6} \cdot 10^{45} \approx 1,2 \cdot 10^{37} \Omega m^6 K^{-1}. \quad (20)$$

With this equal were set up the following schedules. In all cases the hole concentration was calculated with<sup>37</sup>.

Table 11.  $d\rho/dT$  in dependence on the density of holes, by example of LSCO.

Sr-doping x	N/A [ $10^{18} m^{-2}$ ]	$(N/A)^{-2,5}$ [ $10^{-46} m^5$ ]	$\rho$ [ $\mu\Omega cm / K$ ] Calculated without shielding	$\rho$ [ $\mu\Omega cm / K$ ] Calculated with shielding	$\rho$ [ $\mu\Omega cm / K$ ] experimental
0,1	0,7	24,4	18	3,0	$2,2^{47}$
0,12	0,84	15,5	11,4	1,9	$2,0^{47}$
0,15	1,05	8,85	6,5	1,1	$1,3^{47}$ ; $1,4^{48}$ ; $1,4^{49}$ ; $0,8^{49}$ ; $1,3^{18}$
0,17	1,19	6,5	4,78	0,8	$0,9^{48}$
0,20	1,4	4,3	3,16	0,53	$1,0^{49}$ ; $0,8^{18}$
0,22	1,54	3,4	2,5	0,42	$0,45^{48}$
0,24	1,68	2,73	2,0	0,33	$0,9^{18}$
0,25	1,75	2,46	1,8	0,3	$0,66^{51}$ ; $0,7^{51}$
0,3	2,1	1,565	1,15	0,2	$0,63^{49}$ ; $0,43^{18}$

Table 12.  $d\rho/dT$  for the substance  $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+x}$  (=Bi2212)

Oxygen doping x	N/A [ $10^{18} \text{ m}^{-2}$ ]	$(N/A)^{-2,5}$ [ $10^{-46} \text{ m}^2$ ]	$\rho$ [ $\mu\Omega\text{cm}/K$ ] calculated with shielding	$\rho$ [ $\mu\Omega\text{cm}/K$ ] experimental
0,2135	0,73	22	2,7	2,6
0,217	0,74	21,2	2,6	2,3
0,22	0,75	20,5	2,5	1,83
0,24	0,82	16,4	2,0	1,5
0,25	0,85	15	1,84	1,3
0,255	0,87	14	1,7	1,17
0,26	0,887	13,5	1,65	1,08
0,27	0,92	12,3	1,5	0,8

The calculated resistivities already regard the shielding by a factor of six. Experimental values from <sup>52</sup>.

Table 13. Comparison between theory and experiment:

Class of substances	$\frac{\Delta\rho}{\Delta(N/A)^{-2,5}}$ [ $10^{37} \Omega\text{m}^6 K^{-1}$ ] Experimental	$\frac{\Delta\rho}{\Delta(N/A)^{-2,5}}$ [ $10^{37} \Omega\text{m}^6 K^{-1}$ ] theoretical
LSCO	1,1	1,2
Bi2212	1,26	1,2

According to the formula for  $\rho$ , the outline of resistance values in  $\mu\Omega\text{cm}/K$  against  $(N/A)^{-2,5}$  should make a straight line with an ascending gradient of  $1,210^{37} \Omega\text{m}^6 K^{-1}$ .

The deviations between theory and experiment are lower than 10%.

Regarding the amounts on has to take into account that an explicit theoretical and simplified model was taken as a basis. In addition to that, measured values differ between authors.

Nevertheless this model can describe the unusual dependence of the normal state resistivity qualitatively true and quantitatively well.

## **9. The heat conduction in HTSC**

### Preliminary remark:

The following explanations belong to the normal- and not to the SC-state.

**9.1** The model of electrical resistivity which was explained in sections **6.-8.** also is applicable to thermal resistance. Hence it follows that the thermal resistance must increase with the density of the thermally excited EL, that means likewise linearly with temperature. As the electronic contribution to thermal conduction likewise proportional follows to the density of the thermally excited EL, that means by a linear temperature law, the thermal stipulated increasing of density of thermal conducting particles and the increasing of thermal resistance must counterbalance each other (in the normal state).

But that mean, that the hole-induced part of thermal conduction in HTSC should show a constant quantity. This quantity is independent of temperature. A deviation only will occur at low temperatures: At low temperatures the thermal-resistance will be generated by defects of lattice and impurities. Therefore in the low-temperature region the thermal conductivity should increase with increasing temperature like in normal metals. In the temperature-region between 0 and 30 K the thermal conductivity should increase linearly with temperature and then reach its constant maximum value. This temperature-region is mentioned explicit, because normal metals show a different behaviour: In normal metals at temperatures which

are not too low the electrical and thermal resistivity is dominated by scattering of conduction-electrons at phonons, leading to the phenomenon of a maximum thermal conductivity in the just mentioned temperature-region.

In accordance to these considerations in many HTSC the thermal resistance is dominated by interaction (that means scattering) between EL and holes. In consequence, between 10 and 30 K the heat conduction **does not** decrease with increasing temperature.

Meanwhile this prediction is confirmed. Up to 30 K the heat-conductivity of HTSC is increasing step. At higher temperatures it is nearly independent of temperature. (This does not work in the SC-state, naturally.)

From the formula of heat-conduction in two dimensions by movable charge-carriers:

$$K_h = \frac{\pi^2 n k_b^2 T}{3 m_h v_F} \cdot l \quad \text{and with } l = \frac{1}{\frac{Z \cdot d \cdot G_s}{\text{meter}} \cdot n_{th} \sigma_0} \text{ results}$$

$$K_{L\ddot{o}} = \frac{\pi^2 n k_b^2 T}{3 m_{\ddot{o}} v_F} \cdot \frac{v_F^4}{2 \cdot 10^5 n_{th} \cdot Z \cdot d \cdot G_s / \text{meter}} \cdot \text{shieldingfactor} \cdot$$

$$\text{With } v_F = 2,9 \cdot 10^{-4} \cdot \sqrt{\frac{N}{A}} \text{ results} \quad K_{L\ddot{o}} = 33,6 \cdot 10^{-45} \cdot \left(\frac{N}{A}\right)^{2,5} \frac{W}{mK} \cdot \text{shieldingfactor} \cdot$$

$$= \frac{\pi^2 n k_b^2 T}{3 m_h v_F} \cdot \frac{v_F^4}{2 \cdot 10^5 n_{th} \cdot Z \cdot d \cdot G_s} \text{ and } v_F = 2,9 \cdot 10^{-4} \cdot \sqrt{\frac{N}{A}}$$

$$\text{results } K_h = 3,36 \cdot 10^{-46} \cdot \left(\frac{N}{A}\right)^{-2,5} \cdot \quad (21)$$

Table 14. Thermal conduction coefficient in various HTSC

Substance	N/A [ $10^{18} m^{-2}$ ]	$(N/A)^{+2,5}$ [ $10^{+45} m^2$ ]	$K_{L\ddot{o}}$ [W/mK] calc., without shielding	$K_{L\ddot{o}}$ [W/mK] calc., with shielding	$K_{L\ddot{o}}$ [W/mK] experimental value
LSCO, x =0,17	1,19	1,55	0,5	2,5	4,5 <sup>15</sup>
LSCO, x =0,2	1,4	2,3	0,8	3	Circa 3 <sup>53</sup>
YBCO6,3					1,9 <sup>54</sup>
YBCO6,53					2,3 <sup>54</sup>
YBCO6,7	1,36	2,2	0,7	3	5,0 <sup>54</sup> ; 2,5 <sup>55</sup>
YBCO6,77	1,83	4,5	1,5	4	2,9 <sup>54</sup>
YBCO6,9	2,7	12	4,0	8,0	8,0 <sup>53</sup> ; 2,5 <sup>54</sup> ; 3,5 <sup>56</sup> ; 4,0 <sup>57</sup> ;

On heat-conduction also exists convergence between theory and experiment, if it is considered, that the experimental values show a big variation and the phonons are also contributing to the heat-conduction.

For  $Bi_2Sr_2CaCu_2O_{8-y}$  the thermal-conduction-coefficient also has a small temperature-dependence<sup>58</sup>.

### 9.3 Wiedemann-Franz-law<sup>59</sup> and Lorenz-number

The theoretical relation between heat-conduction and electrical conductivity is:

$$L_0 = \frac{K_{L\ddot{o}}}{\sigma T} = 3,36 \cdot 10^{-46} \left( \frac{N}{A} \right)^{2,5} \cdot 7,35 \cdot 10^{37} \cdot \left( \frac{N}{A} \right)^{-2,5} W\Omega K^{-2} = 2,47 \cdot 10^{-8} W\Omega K^{-2}.$$

For  $YBa_2Cu_3O_{6,9}$  was measured:  $L_0 = 10,5 \cdot 10^{-8} W\Omega K^{-2}$  (53). bzw.  $5 \cdot 10^{-8} W\Omega K^{-2}$  (46).

It is characteristic, that these measured quantities do not depend on temperature, corresponding to the represented theory. Unfortunately for other HTSC no values are available. Oddly the value of the Lorenz-number is the double as in conventional metals. It is

not yet possible, to answer, why. Also the values of  $L_0$  only show a small dependence of temperature, what is also predicted by theory.

#### 9.4 Result

In conventional metals between 10 and 30 K you receive no solid Lorenz-number, but a so-called “Lorenz-relation”, starting decreasing with increasing temperature, than reaching a minimum and then increasing again<sup>60</sup>. **This conclusion is very important:**

It **proves** that in the HTSC electrical resistance and thermal resistance (of the holes) are created by processes with the same effectiveness **at every temperature**. By that both types of collision processes are of the same effectiveness, an elastically or nearby elastically scattering is necessary. In HTSC the thermal and the electrical resistance come about by elastically scattering. That is established by obeying the Wiedemann-Franz-law at all temperatures. From this reason, inelastically scattering can be excluded. Therefore the scattering of holes comes about by elastically scattering processes with a collision cross-section  $\sigma_0$  or a frequency increasing linearly with temperature. Scattering of conduction-electrons (in HTSC almost holes) always is inelastic and by that is not useful as an explanation. Attempting to explain this result by dynamical elastic scattering of holes at bosons, the density of this bosons also has to increase at the lowest temperatures linearly with the temperatures. (In HTSC the resistance is increasing linearly with T at the lowest temperatures.) That is possible only, if the characteristic exciting-energy of this bosons is going to zero. That is sizeable unrealistic, but thinkable. But if the excitement-energy is in the near of zero, the matrix-element of the hole-boson-coupling goes to zero. And with a coupling-constant near zero the electrical resistance also is going to zero, at every temperature.

The extra-ordinary temperature-dependence of resistivity cannot be explained by an interaction with bosons. Then the only explanation is the elastic interaction with fermions.

The number or the collision cross-section  $\sigma_0$  of these fermions also has to increase linearly with the temperature. The one and only fermions in HTSC with these presuppositions are the thermal excited CU-3d-EL. It is significant, that these types of electrical and thermal resistivity and the anomalies in the WF-law only exist in the HTSC. Indeed, in the heavy-fermion –superconductors the resistivity at low temperatures increases (nearly) with the temperature, but it is proved, that this scattering is inelastic<sup>61</sup>.

In principle a scattering of holes at ordered areas in the crystal is possible. But then the ordering in the crystal must increase with temperature, what is in contradiction to the second main theorem of thermodynamic. It only rests the solution, that the holes are scattered at the thermal excited Cu-3d-EL.

This does not prove the coming about of superconductivity by interactions between holes and Cu-3d-EL, but it is obvious.

## **10. Conclusion**

**10.1** It was demonstrated in **sections 6. to 9.**, that electrical resistance and heat resistance in HTSC are created by interaction between mobile holes and localized Cu-3d-electrons. High-temperature-superconductivity is explained by a closely related mechanism. Furthermore the mechanism presented in this abstract also can explain all the other experimental results qualitatively and quantitatively. This theory of HTSC is closed, quantified and able to explain the phenomenas connected with the cuprate-based superconductivity.

The here presented relations between experimental verified metal-parameters and the superconductivity are totally more quantitative than all, which was offered till now by the theory of superconductivity.

Apart from that the theory is unusually, there is no doubt on her correctness.

It should be remarked for completeness, that this mechanism is also appreciable to electron-doped HTSC. But it is true, that the interaction between negative loaded Cu-3- $d_{x^2-y^2}$ -EL and conduction-electrons is weaker than with positive loaded holes, leading to lower critical temperatures.

## 10.2 Acknowledgements

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**Diagrams: see extra-files**