STRUCTURAL SIMILARITY BETWEEN THE ORDERED PAIRS OF PRIMES AND NON-PRIMES

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Dedicated to Someone Standing on the Shoulders of Giants

Abstract. There have been various approach to prove Goldbach's conjecture using analytical number theory. We go back to the starting point of this famous probelm and are able to show that the number of Goldbach partition is related to that of ordered pairs of non-primes. This proof is based on the world's first dynamic model of primes and can be a key to identify the structure of prime numbers.

1. Introduction

Although a number of studies have been conducted on prime number, many parts of primes remain unknown. Riemann hypothesis, twim prime problem and Goldbach's conjecture are typical examples of the famous unsolved problems. In 1742, Christian Goldbach suggested that every integer greater than 6 can be represented as a sum of three primes. This conjecture can be divided into two different statement: weak Goldbach's conjecture and strong Goldbach's conjecture. The weak Goldbach's conjecture states that every integer greater than 5 can be written as the sum of three primes[1]. Vinogradov showed that this conjecture holds for number larger than $3^{3^{15}}$ and this value is reduced to 3.33×10^{43000} by Chen. The strong Goldbach's conjecture states that every even integer greater than 2 can be written as the sum of two primes. Unlike the weak conjecture, much of this conjecture still remain unsolved. In this study, dynamic model for Goldbach partition is suggested and the structural similarity between the ordered pairs is shown as the first step for proving the strong Goldbach conjecture.

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2. Preliminaries: Goldbach's conjecture

Goldbach's conjecture is one of the oldest and famous unsolved problems in mathematics which states: Every even integer greater than 2 can be expressed as the sum of two primes. It can be easily expressed as follws.

$$2n = p_1 + p_2 \tag{1}$$

p_1 and p_2 are both primes for $\forall$ n $\geq$ 2, n $\in N$

This can be transformed as follows for using the geometric characteristics.

$$2n = (n - \alpha) + (n + \alpha) \tag{2}$$

$$(\exists \alpha \in Z \text{ s.t. } (n-\alpha) \text{ and } (n+\alpha) \text{ are both primes}$$
$$\text{and } 0 \leq \alpha < n \text{ for } \forall n \geq 2, n \in N)$$

This means that we have to show the existence of the integer α which makes both $(n-\alpha)$ and $(n+\alpha)$ primes. Also it can be interpreted that there are always two pair of primes which have same distance of α apart to n $\geq$ 2.

3. The main mathematical results

Let $\mathbf{G_n} = \{((n-\alpha),(n+\alpha)) \mid 0 \leq \alpha < n, \alpha \in Z\}$ be a set of given n. It is easey to classify elements in $\mathbf{G_n}$ into 4 groups: $a_n = (p,p)$, $b_n = (p,q)$, $c_n = (q,p)$ and $d_n = (q,q)$. Here p is prime nad q is not prime. Now, Goldbach's conjecture is equivalent with the following proposition.

$$n(a_n) \neq 0 \, for \, \forall n \geq 2 \, and \, n \in N \tag{3}$$

Since $\mathbf{G_n}$ and its 4 small groups of element varies with a value of n, now we will call $\mathbf{G_n}$ as a dynamic model of primes. It is necessary to observe a difference between $\mathbf{G_{n+1}}$ and $\mathbf{G_n}$ and 8 different flows (v_j) are introduced in Figure 1 where $\mathbf{G_n}$ is represented as a **Reactor n**. For representing the dynamic model in terms of each flow, the following function are used.

$$\pi(n) = \text{the number of prime } p \text{ which satisfies } p \leq n \tag{4}$$

$$\delta(n) = \begin{cases} 1, & \text{if n is prime} \\ 0, & \text{otherwise} \end{cases} \tag{5}$$

list numbers from 1 to 2n and fold;

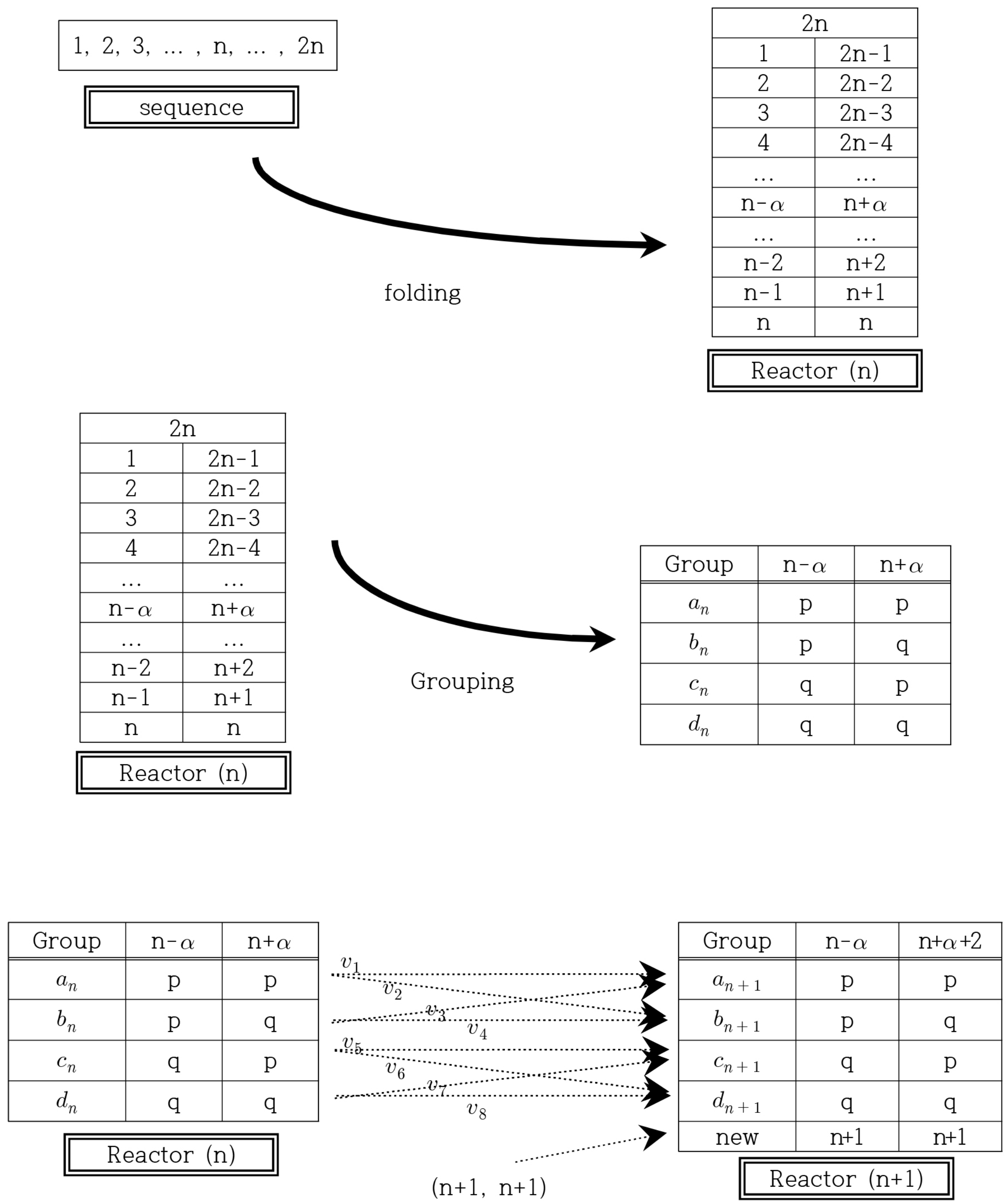

FIGURE 1. Dynamic model of ordered pairs

Now it is possible to present the dynamic model of primes in terms of v_j and given function.

First, it is obvious that n($\mathbf{G_n}$)=n, the overall balance equation can be obtained as follows.

(6)
$$n(\mathbf{G_n}) = n(a_n)+n(b_n)+n(c_n)+n(d_n) = v_1+v_2+v_3+v_4+v_5+v_6+v_7+v_8 = n$$

Second, the number of prime p such that $1 \leq p \leq n$ is only found in the left part of ordeered pairs in a_n and b_n, we can easily have the additional balance equation.

$$n(a_n) + n(b_n) = v_1 + v_2 + v_3 + v_4 = \pi(n) \tag{7}$$

The number of prime p such that n $\leq p \leq 2n$ is only in the right part of ordered pairs in a_n and c_n and here we get the balance equation for primes which are in right part of ordered pairs in $\mathbf{G_n}$.

$$n(a_n) + n(c_n) = v_1 + v_2 + v_5 + v_6 = \pi(2n) - \pi(n) + \delta(n) \tag{8}$$

Finally, when $\mathbf{G_n}$ is changed to $\mathbf{G_{n+1}}$ as n increases by one, we can observe the change of each flows and set the balance equation for primes which are in right part of ordered pairs in $\mathbf{G_{n+1}}$.

$$v_1 + v_3 + v_5 + v_7 + \delta(n+1) = \pi(2n+2) - \pi(n+1) + \delta(n+1) \tag{9}$$

Since we have to show $n(a_n) \neq 0$ for $n \geq 2$, it is necessary to observe Δa_n.

(10)
$$\Delta n(a_n) = n(a_{n+1})-n(a_n) = (v_1+v_3+\delta(n+1))-(v_1+v_2) = v_3-v_2+\delta(n+1)$$

Similarly, Δd_n can be obtained.

(11)
$$\Delta n(d_n) = n(d_{n+1})-n(d_n) = (v_6+v_8+1-\delta(n+1))-(v_7+v_8) = v_6-v_7+1-\delta(n+1)$$

If we combine the Eq.10 and Eq.11, the relation between Δa_n and Δd_n can be represented as a function of n and v_j.

$$\Delta n(a_n) = \Delta n(d_n) + v_3 - v_2 + v_7 - v_6 + 2\delta(n+1) - 1 \tag{12}$$

To convert $(v_3 - v_2 + v_7 - v_6)$ to a function of n, we can use Eq.8 and Eq.9. Then Eq.12 can be reduced as follows.

$$\Delta n(a_n) = \Delta n(d_n)+\delta(2n+1)+\delta(n+1)-\delta(n)-1 = \Delta n(d_n)+\eta(n) \tag{13}$$

Here $\eta(n) = \delta(2n+1) + \delta(n+1) - \delta(n) - 1$.
Since both n and $(n+1)$ cannot be primes except $n = 2$, it is easy to find the value of $\eta(n)$ based on the observation of 6 cases of possible when n $\geq$ 3.

$$\eta(n) = \begin{cases} 1 & \text{if 2n+1, n+1} \in P \text{ and n} \in Q \\ 0 & \text{if 2n+1, n} \in Q \text{ and n} \in P \text{ or if 2n+1} \in P \text{ and n+1, n} \in Q \\ -1 & \text{if 2n+1, n+1, n} \in Q \text{ or if 2n+1, n} \in \text{P and n+1} \in Q \\ -2, & \text{if 2n+1, n+1} \in \text{Q and n} \in P \end{cases}$$

This relation can be seen in table I and it can be found that the similar shape in graph of $n(a_n)$ and $n(d_n)$ is due to the function $\eta(n)$. Figure 2 represents the graph of $n(a_n)$ and $n(d_n)$. Although the $n(d_n)$ has values greater than $n(a_n)$, when we focus on $\Delta n(a_n)$ and $\Delta n(d_n)$, the relation $\Delta n(a_n) - \Delta n(d_n) = \eta(n)$ holds for all n $\geq$ 3. The similar tendency of the dark band in both $n(a_n)$ and $n(d_n)$, which is generated from the bottom to the top, can be explained by this.

Table 1. Relation between ordered pairs of primes and non-primes

n	$\Delta n(a_n)$	$\Delta n(d_n)$	$\Theta(n)$
3	0	1	-1
4	1	1	0
5	-1	0	-1
6	1	0	1
7	0	2	-2
8	0	0	0
9	0	0	0
10	1	1	0
11	0	1	-1
12	0	0	0
13	-1	1	-2
14	1	1	0
15	-1	-1	0
16	2	2	0
17	0	2	-2
18	-2	-3	1
19	1	3	-2
20	1	1	0
21	-1	-1	0
22	1	1	0
…	…	…	…
1442	40	41	-1
1443	-50	-50	0
1444	20	21	-1
1445	14	15	-1
1446	-29	-29	0
…	…	…	…
4995	-167	-166	-1
4996	-4	-3	-1
4997	157	158	-1
4998	-156	-156	0
…	…	…	…

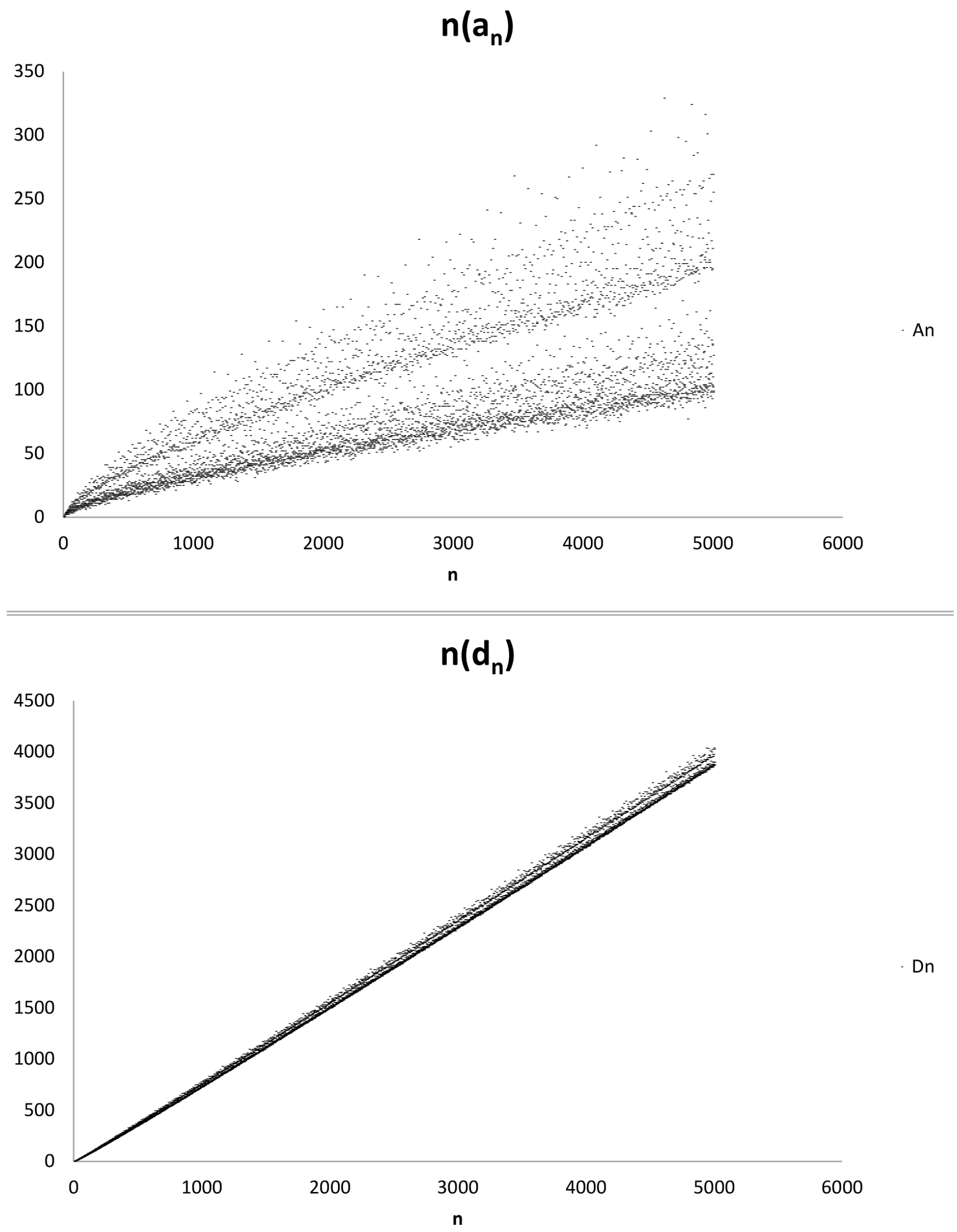

FIGURE 2. Graphs of $n(a_n)$ and $n(d_n)$

Likewise, the relation between $n(b_n)$ and $n(c_n)$ can be derived easily.

$$\Delta n(b_n) = v_3 - v_2 = -\Delta n(a_n) + \delta(n+1) \tag{14}$$

$$\Delta n(c_n) = v_7 - v_6 = -\Delta n(d_n) - \delta(n+1) - 1 \tag{15}$$

From Eq.13, Eq.14 and Eq.15,

$$\Delta n(b_n) = \Delta n(c_n) - \delta(2n+1) + \delta(n+1) + \delta(n) = \Delta n(c_n) + \theta(n) \tag{16}$$

Here $\theta(n) = -\delta(2n+1) + \delta(n+1) + \delta(n)$.
Since both n and $(n+1)$ cannot be primes except $n = 2$, it is easy to find the value of $\theta(n)$ based on the observation of 6 cases of possible when n $\geq$ 3.

$$\theta(n) = \begin{cases} 1 & \text{if 2n+1, n} \in Q \text{ and n+1} \in P \text{ or if 2n+1, n+1} \in Q \text{ and n} \in P \\ 0 & \text{if 2n+1, n, n+1} \in Q \text{ or if 2n+1, n} \in P \text{ or if 2n+1, n+1} \in P \\ -1 & \text{if 2n+1} \in P \text{ and n, n+1} \in Q \end{cases}$$

This relation can be seen in Figure 3 and it can be found that the similar shape in graph of $n(b_n)$ and $n(c_n)$ is due to the function $\theta(n)$. When we focus on $\Delta n(b_n)$ and $\Delta n(c_n)$, the relation $\Delta n(b_n) - \Delta n(c_n) = \theta(n)$ holds for all n $\geq$ 3. The similar tendency of the dark band in both $n(a_n)$ and $n(d_n)$, which is generated from the top to the bottom, can be explained by this.

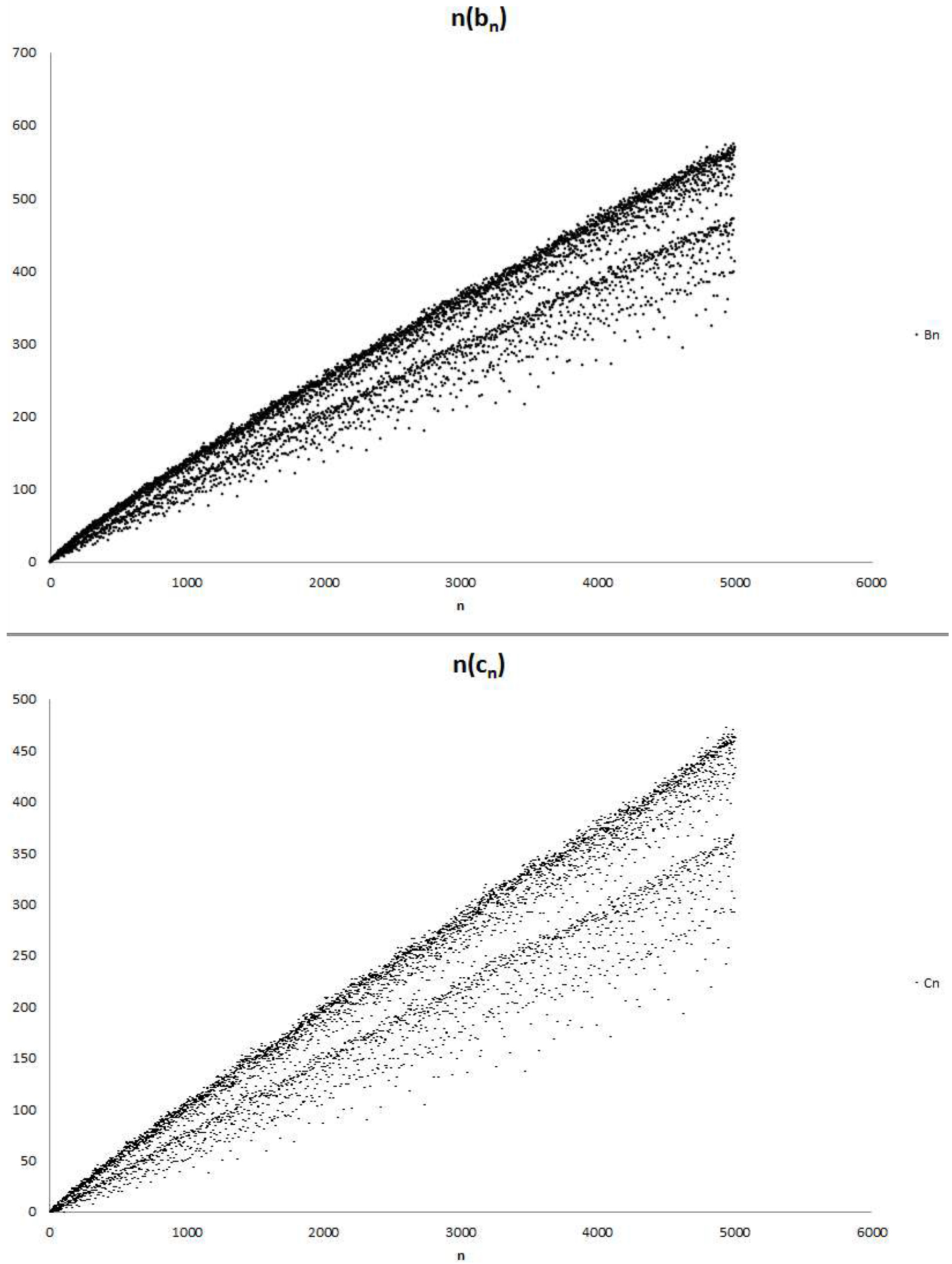

FIGURE 3. Graphs of $n(b_n)$ and $n(c_n)$

4. Conclusions

We have shown that there is a structural similarity between the ordered pairs in $\mathbf{G_n}$. Goldbach partition, $n(a_n)$, and the number of expression for even number as the sum of the two composite number, $n(d_n)$ have a similar structure which can be desribed as $\Delta n(a_n) = \Delta n(d_n) + \eta(n)$. From this, it is possible to explain the similar tendency in the graph of both $n(a_n)$ and $n(d_n)$ quantitatively. Meanwhile, it can be also found that the number of Goldbach partition has a special periodicity based on the suggested dynamic model. This will be discussed further in next paper.

References

[1] KR. Kanchu and S. Kak, *On Randomness of Goldbach Sequences.* arXiv:1207.6307

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