The Strengths of Superstructure Modeled Divine Attributes and an Ultra-Infinite Bound.

Robert A. Herrmann, Ph. D.

21 JAN 2011. Last revision 5 NOV 2015.

Abstract: Since 1969, the superstructure has been the usual approach to nonstandard analysis. The method used to measure the strength of a GD-model Divine attribute, which is comparable to a human attribute, is, at least, partially expressible in terms of cardinality. In this paper, it is shown that for the non-atomic approach to superstructure construction such a cardinality does not have a set-theory upper bound when restricted to class theory. A one-step rational extension of class theory does yield an ultra-infinite bound. However, this is not the necessary generic infinite, a concept that can not be fully characterized.

1. The GD-model.

This article is a theological application of the Grundlegend-Deductive Model (GD-model). This mathematical model predicts that Divine attributes, which are comparable to human attributes, are infinitely stronger than the corresponding human attributes. There is a specific list of attributes that are termed as "Divine attributes." The phrase "higher-attribute" can, relative to context, refer to a Divine attribute. But, it can also refer to a more general list. The term "infinite," in this case, is not the "generic infinite." This article shows that the generic infinite concept is not fully describable via class theory. This article also contains some rather complex mathematics.

As is customary within mathematical modeling, there first are the intuitive notions relative to such a language L. There are various approaches. Robinson [12] employs basic set theory. In this approach, for a formal language, there is a set of individuals U and "That is to say, there are certain disjoint sets of individuals of U, of adequate cardinal numbers, that serve as brackets, commas, connectives ($\sim\ \wedge,$ and $\vee$), quantifiers ($\forall$ and $\exists$), variables, constants, relations and functions of L" [12, p. 90]. He apparently considers the symbols themselves as members of U. Further, "the terms and well-formed formula (wff) of L are also constitute subsets of U." Then Robinson goes back-and-forth between the intuitive notions and the formal: ". . . one-place relations $Q_v(x)$ 'x is an L-variable,' . . . , $Q_s(x)$ 'x is an L-sentence,' . . . ." Of course, these are the predicates that determine the sets. This, of course, is the customary approach, where sets are defined "informally" and then a set-theory is applied.

On the other hand, Mendelson [11, p. 28] mentions that the mathematical objects used in Mathematical Logic can be considered as abstract in character. Then the language aspects are obtained via an interpretation of the abstract relations.

In the Robinson approach, representations for how the wff are formed is not included. In the original Herrmann approach [4], an intuitive interpretation injection $\mathrm{i} \colon \mathrm{L} \to \mathbb{N}$ is introduced. Thus natural numbers represent members of L. This leads to an abstract representation via relations between the natural numbers. Then the construction of a word from an alphabet is represented by a set of equivalence classes $\mathcal{E}$ of partial sequences of the images of i. There is the usual back-and-forth informal to formal correspondences. For example, the injection i is often suppressed and one has a member of a set described by "$\mathrm{An}|||\mathrm{elementary}|||\mathrm{particle}|||\mathrm{k}'(\mathrm{i}',\mathrm{j}')|||\mathrm{with}|||\mathrm{total}|||\mathrm{energy}|||\mathrm{c}'+\mathrm{n}'$" when this actually is the natural number $\mathrm{i}(\mathrm{An}|||\mathrm{elementary}|||\mathrm{particle}|||\ \mathrm{k}'(\mathrm{i}',\mathrm{j}')|||\mathrm{with}|||\mathrm{total}|||\mathrm{energy}|||\mathrm{c}'+\mathrm{n}')$. Then under the *-transfer process, without the i, this appears as $\mathrm{An}|||\mathrm{elementary}|||\mathrm{particle}|||\mathrm{k}'(\mathrm{i}',\mathrm{j}')|||\mathrm{with}|||\mathrm{total}|||\mathrm{energy}|||\mathrm{c}'+\mathrm{n}'$, which actually is the extended standard member ${}^*(i(\mathrm{An}|||\mathrm{elementary}|||\mathrm{particle}|||\mathrm{k}'(\mathrm{i}',\mathrm{j}')|||\mathrm{with}|||\mathrm{total}|||\mathrm{energy}|||\mathrm{c}'+\mathrm{n}'))$ of ${}^*\mathbb{N}$. These notationally simplifications are used throughout [4].

In the new alterations to the entire Theory of Ultralogics [7], the Robinson approach is adjoined to the original and an additional set of equivalences classes $\mathcal{W}'$ is also employed. In this specific case, no intuitive injection i is used. Partial sequences also generate these equivalence classes. Each class contains partial sequences that represent the construction of a word via juxtaposition, but the images are members of the language itself, now denoted by W'.

The generic symbol $\underline{\mathcal{W}'}$ is used to represent either of these sets of equivalence classes. Further, of considerable importance is that when the abstract nonstandard model is being interpreted the composition of a "nonstandard word" $[f]$ is best determined by considering the values of the hyperfinite partial sequences in $[f]$. For the approach using a single set of atoms, the use of W' and $\mathcal{W}'$, or the use of only $\mathcal{E}'$ is a matter of individual choice, but what must be maintained is the consistent modeling process of going back-and-forth between the informal interpretations and corresponding formal symbolic expressions. This will not be the case for the generation of the superstructures in Section 2, where the mathematical model is composed of the consistent informal interpretations of the abstract formalism.

For the case where our set theory uses a basic set of atoms, the two disjoint sets W' and rational numbers $\mathcal{Q}$ of atoms are, usually, employed as the ground set $X_0 = \mathrm{W}' \cup \mathcal{Q}$ for superstructure construction [4, 7]. (NOTE: The rational numbers can be replaced with the natural numbers, integers, real numbers or other sets, as the case may be.) However, requiring the set theory employed to have atoms has been shown not to be necessary.

By a special coding [10, p. 57-58], any infinite set X is representable by a set Y of the same cardinality and Y preserves the necessary atomic properties needed for the construction of a superstructure. The coding is a bijection. It is standard practice that, in all applications, all specially defined relations on X are isomorphically impressed upon Y. The practical modeling requirement is that informal interpretations employed for members of a superstructure based upon X be exactly maintained for the corresponding Y isomorphic copies.

In the previous article on measuring intelligence, the notion $|\cdots|$ is employed. This symbol is used to indicate the "power of a set" compared to another set, where $|A| < |B|$, means there is a injection on A into B, but no injection on A onto B. And, $|A| = |B|$ means there is an injection on $|A|$ onto $|B|$ (i.e. a bijection). In this paper, the order $\leq$ [resp. $\geq$] is denote by $\preceq$ [rep. $\succeq$] [1] and the cardinality of a set is denoted by $\|\cdots\|$.

As first proposed, an alphabet is a finite set of symbols, images, and, by a coding, all human sensory information. The rational numbers can be considered as objects that are only members of a formal language. On the other hand, they can also be considered as members of an informal alphabet using notions such as Kleene's tick notation. If they are part of our informal alphabet, it is trivial to consider the informal rational numbers as part of the informal language. This gives a denumerable alphabet. Then as mentioned, each word is representable by a finite equivalence class $[\mathrm{f}] \in \underline{\mathcal{W}}'$ of partial sequences and a unique $\mathrm{f}_n \in [\mathrm{f}]$, where each $\mathrm{f}_n(i)$ is an alphabet symbol and the finite $n > 0$ represents the length of a word as intuitively written via the join operator. The length includes symbol repetitions. With or without intuitive rational number symbols, the set W' or employed $\underline{\mathcal{W}}'$ is denumerable. On the other hand, if members of $\mathbb{R}$ are to have symbol names in W', then the concept of the "extended language" of a higher cardinality is necessary.

2. A Superstructure Approach.

In what follows, an altered superstructure approach is used [7]. In general, for the foundationally conceived W', the superstructure ground set $X_0 \supset \mathrm{W}' \cup \mathbb{R}$ (or $\mathbb{R}$ is replaced with $\mathbb{N}$ or the set of rational numbers $\mathbf{Q}$). In accordance with the general principles of mathematical modeling, abstract mathematical objects represent entities identified within other specific disciplines. The properties associated with various mathematical relations between the representative entities represent the behavior of the entities being so identified. The set W' represents a <u>general</u> language concept as used in an intuitive model. The language contains symbols for various numbers, an ordinary alphabet, and, additionally, images, diagrams and digitally representable "virtual reality" human sensory information. Which additional features one employs depends upon the application. The intuitive monoid behavior displayed by the basic juxtaposition operator is reflected by a monoid relation defined on W'.

As shown in [10, p. 58-59], atoms are not necessary for proper superstructure application to nonstandard analysis. Hence, this non-atomic approach can be used for an infinite set X_0 of the appropriate cardinality and for a superstructure, where the "ground set" Y of the same cardinality as X_0 is obtained by the special coding of the members of X_0. Within the set-theory itself are the sets $\mathbb{N}$, $\mathcal{Q}$ and $\mathbb{R}$. Then, as mentioned, the bijection used for this coding impresses upon Y any necessary structure defined for X_0 or on subsets of X_0. For example, consider $\mathrm{W}' \cup \mathbb{R} \subset X_0$. Then the special coding bijection $k: X_0 \to Y$, yields disjoint $k[\mathrm{W}']$ and $k[\mathbb{R}]$. Further, all of the necessary relations and properties defined for W' and $\mathbb{R}$ are passed to Y via k.

When one states that an image of a member of one of the equivalence classes in $\mathcal{W}'$ is an alphabet symbol "m," ones means that the image "represents" this symbol under the codings being employed. As with W', the intuitive monoid behavior displayed by the basic juxtaposition operator is reflected by a monoid relation defined on coded W' as well as on coded $\underline{\mathcal{W}'}$ [6]. This coding statement will be understood. Technically, under the rules for modeling, the intuitive language concepts can be directly associated with the coded entities via the mappings employed. However, when a consistent interpretation is made this technical aspect is not employed.

Let X' be any infinite set such that $\|X'\| \geq \|X_0\|$. Then $X' \succeq X_0$ and the injection from X_0 into X' can be used to pass to X' all the defined relations associated with W' and $\mathbb{R}$. In what follows, the same notation is used for each functionally obtained isomorphic copy. The set X' is coded to obtain the representation Y. Since there can be different X' that yield different Y, then rather then use the customary symbolism ${}^*Z$ to denote the monomorphism mapping applied to Z, ${}^{*_Y}Z$ is employed. This alteration in symbolism is used for ${}^{\sigma}$ as well. As usual, for the infinite set Y, the basic superstructures employed are $S(Y) = \bigcup\{X_i \mid i \in \mathbb{N}\}$, $X_0 = Y$, and $S({}^{*_Y}Y)$, $X_0 = {}^{*_Y}Y$. Recall, that for any $X \in S(Y)$, ${}^{\sigma_Y}X = \{({}^{*_Y}x \mid (x \in X) \wedge (x \in S(Y))\}$ and ${}^{\sigma_Y}$ embeds $S(Y)$ into $S({}^{*_Y}Y)$ and the ${}^{\sigma_Y}$ objects model the standard superstructure $S(Y)$. Since this is an altered approach to superstructure construction, the $x \in S(Y)$ in the definition of ${}^{\sigma_Y}$ appears necessary. This allows the same identification as used in [4] to be applied to these superstructures. This yields that for $x \in X_0$, ${}^{\sigma_Y}x = \emptyset$.

As for this identification, since for ground sets Y and ${}^{*_Y}Y$, $a \in Y$ yields that ${}^*a \in {}^{*_Y}Y$, then it is customary to write ${}^*a = a$. Since in each case, a member of $\underline{\mathcal{W}'_Y}$ is a finite set of finitely many entities, and each of these entities is itself a finite set that reduces to finitely many members of the ground set, then this identification procedure also holds for $[f] \in \underline{\mathcal{W}'_Y}$. That is, for ${}^*[f] \in {}^{\sigma_Y}\underline{\mathcal{W}'_Y}$, ${}^*[f] = [f]$. Further, the relation ${}^{*_Y}{<_Y}$ and operation ${}^{*_Y}{+_Y}$ relative to ${}^{*_Y}\mathbb{N}_Y$ are denoted as $<_Y$ and $+_Y$, since $<_Y$ and $+_Y$ defined on $\mathbb{N}_Y$ are considered as but restrictions of ${}^{*_Y}{<_Y}$ and ${}^{*_Y}{+_Y}$, respectively. These identifications are used throughout the appropriate portions of what follows.

Notice that the definition of an hyperfinite interval such as ${}^{*Y}[\mu, \nu],\ \nu, \mu \in {}^{*Y}\mathbb{N}_Y$ under the identification is the same as $[\mu, \nu]$. Further, it is well know that, in general, for any $\nu \in {}^{*Y}\mathbb{N}_Y - \mathbb{N}_Y$, and any $m \in \mathbb{N}_Y,\ \{x \mid (m \leq_Y x) \land (x \in \mathbb{N}_Y)\} \subset [m, \nu]$. This follows since for arbitrary $n \geq_Y m,\ n \in \mathbb{N}_Y,\ [m, n] \cap ({}^{*Y}\mathbb{N}_Y - \mathbb{N}_Y) = \emptyset$. Recall that the set ${}^{*Y}\mathbb{N}_Y - \mathbb{N}_Y$ is termed as a set of "infinite" numbers.

Theorem 1. *Consider any infinite set X' of the appropriate cardinality that contains, at least, an isomorphic copy $\mathbb{N}_{X'} \subset X'$ of the natural numbers. Let Y be the coded representation for X' and ${}^*\mathcal{M}$ a $\|S(Y)\|^+$-saturated model contained in $S({}^{*Y}Y)$. Then $\|S(Y)\| \leq \|{}^{*Y}\mathbb{N}_Y\| = \|A\|$, for any infinite internal set $A \in S({}^{*Y}Y)$.*

Proof. Consider infinite ground set $Y = X_0 \in S(Y)$. From saturation, there exists a hyperfinite F_0 such that ${}^{\sigma Y}X_0 \subset F_0 \subset {}^{*Y}X_0$. (Note: Each hyperfinite sets is an internal set.) From [1], $X_0 \preceq {}^{\sigma Y}X_0$. Since F_0 is hyperfinite, then there exists some $\nu_0 \in {}^{*Y}\mathbb{N}_Y - \mathbb{N}_Y$ such that for the segment $[0, \nu_0],\ [0, \nu_0] \simeq F_0$. Thus, $X_0 \preceq [0, \nu_0]$.

Now consider X_1. Then, in like manner, there exists some $\nu_1 \in {}^{*Y}\mathbb{N}_Y - \mathbb{N}_Y$ such that $X_1 \preceq [0, \nu_1]$. Consider $[\nu_0 +_Y 1, \nu_0 +_Y \ \nu_1 +_Y 1]$. Then using $f(x) = x +_Y \nu_0 +_Y 1,\ 0 \leq x \leq \nu_1,\ [0, \nu_1] \simeq [\nu_0 +_Y 1, \nu_0 +_Y \nu_1 +_Y 1] \succeq X_1$ and $[0, \nu_0] \cap [\nu_0 +_Y 1, \nu_0 + \nu_1 +_Y 1] = \emptyset$. Let $n_0 = 0,\ \nu_0 = m_0, n_1 = m_0 +_Y \ 1,\ m_1 = m_0 +_Y \ \nu_1 +_Y \ 1$. Then, since $m_0 <_Y n_1,\ [n_0, m_0] \cap [n_1, m_1] = \emptyset$. Suppose that for $k > 0,\ k \in \mathbb{N}$ there is a nonempty finite set of intervals $\{[n_i, m_i], \mid (0 \leq i \leq k) \land (i \in \mathbb{N}) \land (n_i \in {}^{*Y}\mathbb{N}_Y - \mathbb{N}_Y) \land (m_i \in {}^{*Y}\mathbb{N}_Y - \mathbb{N}_Y) \land (\forall j((j \in \mathbb{N}) \land (0 \leq j < i \leq k) \to (m_j <_Y n_i))\}$ and $[n_i, m_i] \succeq X_i,\ 0 \leq i \leq k$.

Consider X_{k+1}. Then there is a $\nu_{k+1} \in {}^{*Y}\mathbb{N}_Y - \mathbb{N}_Y$ such that $[0, \nu_{k+1}] \succeq X_{k+1}$. Let $n_{k+1} = m_k +_Y 1,\ m_{k+1} = m_k +_Y \ \nu_{k+1} +_Y 1$. Then $[n_{k+1}, m_{k+1}] \succeq X_{k+1},\ m_j <_Y n_i,\ 0 \leq j < i \leq k+1,\ j, i \in \mathbb{N}$. Thus, $[n_p, m_p] \succeq X_p,\ 0 \leq p \leq k+1$ and $\{[n_p, m_p] \mid (0 \leq p \leq k+1) \land (p \in \mathbb{N})\}$ is a set of disjoint intervals. Hence, by induction, there is a set of disjoint intervals $\{[n_k, m_k] \mid k \in \mathbb{N}\}$ such that $[n_k, m_k] \succeq X_k,\ k \in \mathbb{N}$. Consequently, $\bigcup\{X_k \mid k \in \mathbb{N}\} = S(Y) \preceq \bigcup\{[n_k, m_k] \mid k \in \mathbb{N}\}$. (This comes from the fact that a surjection $f: A \to B$ reduces to an injection $g: A \to B$ and that the $[n_k, m_k]$ are disjoint.) But, $\bigcup\{[n_k, m_k] \mid k \in \mathbb{N}\} \subset {}^{*Y}\mathbb{N}_Y$. Therefore, $S(Y) \preceq {}^{*Y}\mathbb{N}_Y$. Hence from [1, Corollary 10, p. 365], $\|S(Y)\| \leq \|{}^{*Y}\mathbb{N}_Y\|$. For any infinite internal set $A \in S({}^{*Y}Y)$, from [13, p. 38, 0.4.4], it follows that $\|S(Y)\| \leq \|{}^{*Y}\mathbb{N}_Y\| = \|A\|$. (Considering [1], I can find no reason why this statement is not valid for a superstructure constructed using the coded method presented in [10].) This completes the proof.

From Theorem 1, with $\mathcal{Q}_Y$ in place of $\mathbb{N}_Y$, $\|S(Y)\| \leq \|{}^{*Y}\mathbb{N}_Y\| = \|{}^{*Y}\mathcal{Q}_Y\| = \|{}^{*Y}Y\| = \|{}^{*Y}\mathbf{L}_Y\|$. Thus, the cardinality of each ${}^{*Y}\mathbf{L}_Y$ is rather "large" compared to $\|\mathbf{L}_Y\|$. Further, $\|\mathbb{N}_Y\| \leq \|Y\| < \|S(Y)\| \leq \|{}^{*Y}\mathbb{N}_Y\|$.

The language L, the rule of inference PR and all other standard entities are modeled in $S(Y)$ by $\mathbf{L}_Y$ and $\mathbf{PR_Y}$ **etc.** Further, in general, the attributes and the "greater than, better than, stronger than" order are also modeled by members of $S(Y)$ [4, Section 4.4]. For denumerable $\mathbf{L}_Y$, $\|\mathbf{L}_Y\| = \|\mathbb{N}_Y\| = \|Q_Y\| = \|\mathcal{E}_Y\| = \|\mathbf{PR_Y}\|$, etc.

Let $\mathbb{N}_{X_1'} \subset S({}^{*Y}Y) = X_1'$. Then $\|S(Y)\| \leq \|{}^{*Y}\mathbb{N}_Y\| < \|S(Y_1)\| \leq \|{}^{*Y_1}\mathbb{N}_{Y_1}\|$. If one considers cardinalities as a measure of a type of "size," then nonstandard ${}^{*Y_1}\mathbb{N}_{Y_1}$ is considerably greater in "size" [14] than ${}^{*Y}\mathbb{N}_Y$.

For a given infinite Y, the higher-language is ${}^{*Y}\mathbf{L}_Y$ and $\|{}^{*Y}\mathbf{L}_Y\| = \|{}^{*Y}\mathbb{N}_Y\|$. Thus, given any such higher-language ${}^{*Y}\mathbf{L}_Y$, there exists a higher-language ${}^{*Y_1}\mathbf{L}_{Y_1}$ such that $\|{}^{*Y}\mathbf{L}_Y\| < \|{}^{*Y_1}\mathbf{L}_{Y_1}\|$. Consequently, for such superstructure obtained higher-languages, there is no upper bound in the terms of cardinality.

3. Strengths of Divine Attributes.

In the proof of Theorem 4.4.1 [4], b is a basic attribute such as "intelligent." A member of C_b is the coded form of "very, very, . . . , b." It is shown in the proof that for any superstructure $S(Y)$ and a corresponding attribute **b** and any $\nu \in {}^{*Y}\mathbb{N}_Y - \mathbb{N}_Y$ there is an ultraword c, a **higher-attribute**, such that c is greater than or better than (i.e. ${}^{*Y}<_B$) any ${}^{*Y}w \in {}^{\sigma Y}C_b$. Under the identification $w \in C_b$.

Due to the construction of $\underline{\mathcal{W}}'_Y$, the form of this $c \in {}^{*Y}\underline{\mathcal{W}}'{}_Y - \underline{\mathcal{W}}'_Y$ can be determined. (Note: Recall that when the notation $\underline{\mathcal{W}}'_Y$ and elements are considered, the original intuitive injection i, if employed, is supressed and understood relative to the set of equivalence classes $\mathcal{E}'$.) In particular, $c = [g]$ and each member of $[g]$ is an internal function. There is a unique function $f \in [g]$ and $\nu \in {}^{*Y}\mathbb{N}_Y - \mathbb{N}_Y$ such that $f\colon [0,\nu] \to {}^*T$, ${}^*T = {}^{*Y}\mathbb{N}_Y$, and $f(0) = \mathrm{b} \in \mathrm{W}'_Y$, $f(j) = \mathrm{very}, |||$, where $1 \leq j \leq \nu$. This follows since, for each $0 \leq n \in \mathbb{N}_Y$, there is a $w = [h] \in C_b$, and a unique $k \in [h]$ such that $k\colon [0,n] \to T$, $k(0) = \mathrm{b}$ and $k(j) = \mathrm{very}, |||)$, where $1 \leq j \leq n$. These unique functions, when restricted to $[1,\nu]$ and $[1,n]$, respectively, are the "counting" functions, where the number of embedded "very," strings is directly related to the "size" of the intervals $[1,n]$ and $[1,\nu]$. The cardinality of $[1,n]$ can be symbolized as "n." And, if $n = 0$, then $[1,n] = \emptyset$ and $\|[1,n]\| = 0$.

More directly, there is no bijection Θ from any set of the form $[1,\nu]$ onto $[1,n]$. For if there is, then since internal $[1,n] \subset [1,\nu] \neq [1,n]$ the restriction $\Theta|[1,n]$ is an injection on $[1,n]$ onto a proper subset of $[1,n]$. However, no such mapping exists [14]. Consequently, $[1,n] \prec [1,\nu]$ and $\|[1,n]\| < \|[1,\nu]\| = \|{}^{*Y}\mathbb{N}_Y\| = \|{}^{*Y}L_Y\| > \|S(Y)\|$ since $[1,\nu]$ is infinite. This gives additional guidance since ${}^*<_B$, the stronger than ordering, only states that $w\,{}^*<_B\, c$ since $n < \nu$. This applies to an attribute b associated with any biological entity in any universe (even a countably infinite collection of universes with the attribute as a combined attribute) that can be qualified by the "very," strengthening. How should the strength of an attribute be measured? For superstructures as a class of objects, does it correspond to the hyperreal numbers or to the cardinality of hyperfinite intervals?

Within $S({}^{*Y}Y)$, there are two (and many more) higher-attributes c_1 and c_2 in the language ${}^{*Y}\mathbf{L}_Y$ such that for each $w \in C_b$, $w\ {}^{*Y}{<_B}\ c_1 {}^{*Y}{<_B}\ c_2$. For the ordering ${}^{*Y}{<_B}$, the c_1 attribute has the measure ν for the segment $[1,\nu]$ and c_2 has the measure $\mu > \nu$ and μ corresponds to $[1,\mu]$. However, $\|[1,\nu]\| = \|[1,\mu]\| = \|{}^{*Y}\mathbb{N}_Y\|$ since $[1,\nu]$ and $[1,\mu]$ are infinite and internal hyperfinite sets. Hence, within the set-theory in which $S({}^{*Y}Y)$ is an member, it is only necessary to consider one of these infinite measures for the results stated in [4].

The external cardinalities employed, for a specific structure $S({}^{*Y}Y)$, do not have the same properties as the greater than ordering, ${}^{*Y}{<_Y}$. For any structure $S({}^{*Y}Y)$, there is no $\nu \in {}^{*Y}\mathbb{N}_Y$ that is a ${}^{*Y}{<_Y}$ upper bound. But, such subtle words as c_ν, c_μ exist for each $\nu,\mu \in {}^{*Y}\mathbb{N}_Y - \mathbb{N}_Y$. So, in this case, using **any** $\nu \in {}^{*Y}\mathbb{N}_Y - \mathbb{N}_Y$, the ${}^*{<_\mathrm{B}}$ has no upper bound within $S({}^{*Y}Y)$. But, when ν is replaced by $[1,\nu]$, then, viewed externally, $\|[1,\nu]\|$ is an upper bound *relative to the structure* $S({}^{*Y}Y)$.

For arbitrary superstructure $S(Y)$, applying the above procedure to the superstructure $S(Y_1)$, it follows that for $\nu \in {}^{*Y}\mathbb{N}_Y - \mathbb{N}_Y$ and $\mu \in {}^{*Y_1}\mathbb{N}_{Y_1} - \mathbb{N}_{Y_1}$ that $\|[1,\nu]\| < \|[1,\mu]\|$. Further, there is no set of all superstrutures as here constructed. For if $\mathcal{S}$ is the set of all substructures, then $\mathcal{S}$ is a set of infinite sets and $\bigcup \mathcal{S}$ is an infinite set and can be used, as above, to construct a superstructure not a member of $\mathcal{S}$. Since $S(Y)$ is arbitrary, then no matter how a superstructure's ground set Y is obtained there is another superstructure that verifies that there is no upper bound for $\|[1,\nu]\|$, where ν is an infinite number. Thus the greater than ordering ${}^*{<_\mathrm{B}}$, as it relates to the external cardinality, at present, also has no $<$ upper bound that can represent the strength of a higher-Divine-attribute for class of all superstructures thus far being considered. This follows since for every infinite number ν in any superstructure, ${}^*C_b$ contains an ultraword c that contains $\|[1,\nu]\|$ "very, $|||$" symbol-strings. Relative to the external cardinality, $\|{}^{*Y}\mathbb{N}_Y\|$ represents the external strength of the superstructure specific Divine attributes.

Consider the class $\mathcal{C}$ of all $\|{}^{*Y}\mathbb{N}_Y\|$. This class of all such characterizing cardinalities can conceptually be considered as a type of generic "ultra-infinite bound." By one simple extension of the order concept as it relates to cardinality, the class $\mathcal{C}$ does have a corresponding "order" notion. If $\alpha,\ \beta$ are cardinal numbers, then $\alpha < \beta$ if an only if $\alpha \in \beta$ [1]. Although this would not correspond to actual cardinal number ordering since $\mathcal{C}$ is not a cardinal number, from the class theoretic viewpoint, each $\|{}^{*Y}\mathbb{N}_Y\| \in \mathcal{C}$. This, of course, only establsihes the rationality of this concept. This result may further increase comprehension once an individual has accepted the existance of such a Divine entity.

Wilder [14] discusses cardinality in terms of the intuitive notion of "size." The superstructure method leads to an higher-language. For the set theory being employed, the cardinality measure gives but a partial measure as to the "size" of an higher-language. There is no set-theoretic upper bound for this size notion when additonal superstructures are considered. These "sizes" also correspond to the members of $\mathcal{C}$. This rationally yields a generic "size" notion for an ultimate higher-language notion.

Notice that a formal proof can be represented in W' by a single finitely long word $\mathrm{w} \in \mathrm{W}'$. The number "$n$" of steps in a formal proof, or more generally as produced by the deduction algorithm, is a measure for the number of deductions. This number is determined by a specific member of the equivalence class of partial sequences associated with $\mathrm{w} \in \mathrm{W}'$. It is known that, at least for a formal predicate language, that there exists for each $n \in \mathbb{N}$ a formal theorem that requires, at least, n steps to deduce. Considering propositional deduction relative to W' and properly characterizing this fact leads to the prediction that, for each $\nu \in {}^{*Y}\mathbb{N}_Y - \mathbb{N}_Y$, there is higher-form of deduction characterized by the "step number" ν.

4. The Generic Infinite.

Consider a predicate P(x). There are various comprehensible linguistic expressions where the parameter can be expanded in an inductively increasing way. Let $\mathrm{P}(x) =$ "I think about x." Then we have the expanding collection of statements, (1) "I think about *my thinking*." (2) "I think about *my thinking about my thinking*." (4) "I think about *my thinking about my thinking about my thinking*." Etc. This produces a "logical regress," a sequence of meaningful expressions that has no resolution. One should not dwell upon such regresses. These types of regresses also occur when various theological notions are discussed [5].

Consider the notion of *physical reductionism.* Suppose one states that physical objects are compose of x-tons. "Then x-tons are composed of xx-tons, Then xx-tons are composed of xxx-tons. . . . Then x^n-tons are composed of x^{n+1}-tons. . . ." This also forms a logical regress. Such a regression is stopped by simply restricting the language used. In pure quantum field theory, reductionism is essentially stopped by not allowing something more fundamental to generate the fields. That is, not allowing such a language to be employed.

Consider the notion of *physical expansionism.* Suppose one states that physical objects are compose of X-tins. "Then X-tins are contained in XX-tins, Then XX-tins are contained in XXX-tins. . . . Then X^n-tins are contained in X^{n+1}-tins. . . ." This also forms a logical regress. For various atheistic science communities, expansionism is stopped by not allowing a selected cosmology to be generated by any process that is not part of cosmologies technical language. No statements are allowed that even in the slightest manner would lead one to conclude that a specific universe is not "all there is and all there ever will be," so to speak.

Class terminology is a mathematical expansionism concept that is terminated relative to the language allowed. Should the Wider notion of the size of a set [14] be extended to the size of a class? Does not the class of all cardinal numbers intuitively carry the idea of a size notion? From the class theoretic viewpoint the answer is no. Although the symbol string $\alpha \in \mathcal{C}$ is allowed changing this to $\alpha < \mathcal{C}$ is not allowed. Any expansion of the technical concept of the infinite is stopped if the notion is restricted to class theory. Of course, the term "ultra-infinite" is a one-step addition to the language and it has not been further extended. However, relative to theology, a type of one step expansion is introduced. This occurs when the "omni" notion is applied.

The logic employed by science-communities is the classical logic as studied within Mathematical Logic. All such logic is based upon various forms of languages. For the GGU-model a general language is employed. Within the logic employed, the term "all" ("every") has no truth-value unless it is restricted to a specific collection of objects. But, in general the "all" associated with the "omni" notion is not so restricted. This "all" concept does not appear to be fully describable via any form of rationally presented language. The atheist presents classically stated "omni" related theological questions in their attempts to discredit the God-concept. In all known cases, it is the restricted notion of "all" that leads to rational difficulties. The unrestricted "omni" notion shows that the questions asked have no meaning relative to this Divine aspect.

As here presented the intuitive concept of the *generic infinite*, relative to the strength of Divine attributes, only has a partial descriptive bound - the ultra-infinite. The generic infinite is a concept that falls into the category of a concept that is some-how-or-another known by the human mind. But, we are unable to present a full linguistic description for its properties. "I know what it is but I can't find the words that fully describe it." Or as Eccles and Robinson state it:

> [W]e have ideas in mind that have no relationship to linguistic expression and may never be expressed [2, p. 118].

The existence of such concepts counters the philosophic claim that the "essence" of a concept is linguistically expressible.

Thus, for various none-abstract models, using the statement that a higher-attribute is "infinitely greater than" the same attribute displayed by an entity in the sense of the greater than ordering has but a "partial" set-theoretic measure. Stating without qualification that a specific higher-attribute is "infinitely greater than" a similar attribute should be considered as a <u>generic</u> characteristic. Further, infinitely strong Divine-attributes can be further described. **A higher-Divine-attribute is "stronger than," and, indeed,**

beyond any modern mathematical means to measure its strength. And no attribute is intuitively stronger than such a higher-Divine-attribute.

.

The actual notion of the generic infinite does not appear as a specific word-form in the oldest extant versions of the Bible. Linguistically, the **rule of reciprocation** needs to be applied to Biblical statements. This rule states that comprehension of a collection of words does not come from the meaning of a single word, but that words in a statement alter their meanings relative to the "neighboring" words and conversely. When this rule is applied additional comprehension is gleaned from the informational content of the collection of words employed. From this, further detailed descriptions can be properly made.

Unfortunately, since the Vulgate and except for one translation, this law cannot be properly applied to the known Bible translations. Such translations as KJV, NIV, etc. contain chosen word meanings, for the oldest Greek, that tend to force doctrinal concepts upon the reader. The Concordant Version of the Bible was specifically constructed so as to avoid such special word choices. It allows for the proper application of the law of reciprocation to Bible passages via fixed and strict literal meanings. An example of the application of the rule of reciprocation is to be found in reference [A].

References

[1] Abian, A., The Theory of Sets and Transfinite Arithmetic, A. B. Saunders Co., Philadelphia, PA, (1965).

[2] Eccles, J. and D. A. Robinson, The Wonder of Being Human, Our Brain and Our Mind, The Free Press, New York, 1984.

[3] Henson, C. W. The isomorphism property in nonstandard analysis and its use in the theory of Banach Spaces, J. Math. Logic, 39(4):717-731.

[4] Herrmann, R. A., The Theory of Ultralogics Parts I and II (1978-1993) http://arxiv.org/abs/math/9903081 http://arxiv.org/abs/math/9903082 or www.raherrmann.com/books.htm

[5] Herrmann, R. A., The Omni Concept, http://raherrmann.com/omni.htm

[6] Herrmann, R. A., Nonstandard Ultra-logic-systems Applied to the GGU-model, http://vixra.org/abs/1308.0125

[7] Herrmann, R. A., An Alteration in the Foundations of the Theory of Ultralogics, http://vixra.org/abs/1406.0100

[8] Herrmann, R. A., Science Declares Our Universe IS Intelligently Designed, Xulon Press, Fairfax, VA (2002),

[9] Jech, Thomas J., Lectures in Set Theory with Particular Emphasis on the Method of Forcing, Lecture Note in Math # 217, Springer-Verlag, New York, 1971.

[10] Loeb, P. A. and Wolff, M. (ed) "Nonstandard Analysis for the Working Mathematician," Kluwer Academic Pub., Boston, (2000).

[11] Mendelson, E., "Introduction to Mathematical Logic," Wadsworth & Brooks/Cole, Advanced Book and Software, Monterey, CA. (1987).

[12] Robinson, A., On languages which are based upon non-standard arithmetic, Nagoya Math. J., 22(1963):83-117.

[13] Stroyan, K. D. and J. M. Bayod, Foundations of Infinitesimal Stochastic Analysis, North-Holland, NY, (1986).

[14] Wilder, R. L. Introduction to the Foundations of Mathematics, John Wiley & sons, Inc, New York (1967).

Additional Reference

[A] Herrmann, R. A., Genesis 1:1 - 2:1, http://vixra.org/abs/1312.0122