A formula which conducts to primes or to a type of composites that could form a class themselves

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Abstract. In this paper I present a very simple formula which conducts often to primes or composites with very few prime factors; for instance, for the first 27 consecutive values introduced as "input" in this formula were obtained 10 primes, 4 squares of primes and 12 semiprimes; just 2 from the numbers obtained have three prime factors; but the most interesting thing is that the composites obtained have a special property that make them form a class of numbers themselves.

Observation:

The numbers C = 3^3*(3^3 + n*10) + n*10, where n is a positive integer of the form 4 + 9*k, or in other words C = 2520*k + 1849, are very often primes or numbers with very few prime factors, composites that have certain very interesting properties. Let's see the case of the first 27 consecutive such numbers C; we will consider all 27 numbers but we will list them separatelly in three different lists: the case C is prime or square of prime, the case C is Coman semiprime and the case of the other numbers C (note that a Coman semiprime is a semprime p*q with the property that p – q + 1 is a prime or a square of prime; this is a class of numbers that I met it often in my research, for instance in the study of 2-Poulet numbers, many of these semiprimes having this property, but as well in the study of the prime factors of Carmichael numbers):

The case C is prime or square of prime:

: for k = 0 we have C = 43^2 where 43 prime;
: for k = 1 we have C = 4369 prime;
: for k = 2 we have C = 83^2 where 83 prime;
: for k = 3 we have C = 97^2 where 97 prime;
: for k = 5 we have C = 14449 prime;
: for k = 7 we have C = 19489 prime;
: for k = 11 we have C = 29569 prime;
: for k = 12 we have C = 32089 prime;
: for k = 16 we have C = 42169 prime;
: for k = 19 we have C = 223^2 where 223 prime;
: for k = 20 we have C = 52249 prime;
: for k = 23 we have C = 59809 prime;

:   for k = 25 we have C = 64849 prime;
:   for k = 26 we have C = 67369 prime.

The case C is Coman semiprime:

:   for k = 4 we have C = 79*151 and 151 – 79 + 1 = 73 prime;
:   for k = 6 we have C = 71*239 and 239 – 71 + 1 = 13^2, where 13 prime;
:   for k = 8 we have C = 13*1693 and 1693 – 13 + 1 = 41^2, where 41 prime;
:   for k = 13 we have C = 53*653 and 653 – 53 + 1 = 601 prime;
:   for k = 14 we have C = 107*347 and 347 – 107 + 1 = 241 prime;
:   for k = 15 we have C = 31*1279 and 1279 – 31 + 1 = 1249 prime;
:   for k = 24 we have C = 157*397 and 397 – 157 + 1 = 241 prime.

The other numbers C:

:   for k = 9 we have C = 19*1291 and 1291 – 19 + 1 = 19*67 and 67 – 19 + 1 = 7^2, where 7 prime;
:   for k = 10 we have C = 11*2459 and 2459 – 11 + 1 = 31*79 and 79 – 31 + 1 = 7^2, where 7 prime;
:   for k = 17 we have C = 23*29*67 and 23*29 – 67 + 1 = 601 prime, 29*67 – 23 + 1 = 17*113 where 113 – 17 + 1 = 97 prime and 23*67 – 28 = 17*89 where 89 – 17 + 1 = 73 prime;
:   for k = 18 we have C = 17*2777 and 2777 – 17 + 1 = 11*251 and 251 – 11 + 1 = 241 prime;
:   for k = 21 we have C = 11*13*383 and 11*13 – 383 + 1 = -239 prime in absolute value, 11*383 – 13 + 1 = 4201 prime, 13*383 – 11 + 1 = 4969 prime;
:   for k = 22 we have C = 59*971 and 971 – 59 + 1 = 11*83 and 83 – 11 + 1 = 73 prime;
:   for k = 27 we have C = 47*1487 and 1487 – 47 + 1 = 11*131 and 131 – 11 + 1 = 11^2, where 11 prime.

Note:

It can be seen that also “the other numbers C” have special properties; for instance, the semiprimes can be considered a kind of “extended Coman semiprimes” because of the iterative process that ends also in a prime or in a square of prime: let N = p1*q1; than p1- q1 + 1 = p2*q2 then p2 – q2 + 1 = p3*q3 and so on until is obtained a prime. On the other side, the numbers with three prime factors obtained p*q*r have the property that p*q – r + 1, p*r – q + 1 and q*r – p + 1 are primes or (extended) Coman semiprimes.