ON A NEW FUZZY TOPOLOGICAL NON-LINEAR DIFFERENTIAL EQUATIONS

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Abstract

After the introduction of fuzzy set (FS) by Zadeh [15] in 1965 and fuzzy topology by Chang [2] in 1967, several researches worked on the generalizations of the notions of fuzzy sets and fuzzy topology. The concept of intuitionistic fuzzy set (IFS) was introduced by Atanassov [1] in 1983 as a generalization of fuzzy sets. In 1997 Coker [3] introduced the concept of intuitionistic fuzzy topological space. In this paper, the authors attempt to introduce an entirely new intuitionistic fuzzy theoretic and fuzzy topological concept. Also, the authors applied these new fuzzy ideas to the equations of a line segment, circle and ellipse and yielded a new family of second order non linear differential equations.

Keywords: Equation a line segment, of circle & ellipses stretches , new intuitionistic fuzzy concept, intuitionistic fuzzy topology, intuitionistic fuzzy topological non linear differential equations.

MSC: 51H15, 54A10, 54A40, 54B99, 54G99, 65M99, 00A69, 03B52, 03E72, 03E75, 34G20, 35K99, 35L99, 35M99, 35Q99, 35R99, 37B99, 46A99, 46B20, 46B99, 51H99, 51L99, 51M99, , 54F65

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1. Introduction of new fuzzy set theoretic and new fuzzy topological concept

Topology is rubber sheet geometry. If we stretch a rubber sheet circle, it will become an ellipse. Topologically circle and its stretched ellipse are equal. If we go on stretching an ellipse, ultimately it will become a straight line. Topology is the study of how spaces are organized, how the objects are structured in terms of position. It also studies how spaces are connected.

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Topology has sometimes been called rubber-sheet geometry, because in topology of 2 dimensions, there is no difference between a circle and a square (a circle made out of a rubber band can be stretched into a square). Choose a thin elastic rubber sheet. Describe an ellipse on it.**[Figure 1]** Go on stretching as long as possible**.[Figure2]** In figure the stretched ellipse of figure 1 more or less becomes equal to a line segment. If we prolong the stretching process, the ellipse of figure 1 will become a line segment. **See figure 3**

So, we have topologically deformed a circle in to an ellipse and an ellipse in to a straight line. Here an abstract idea appears that if we contract a line segment, it may become an ellipse and a circle. Further studies to be devoted in this topic may explore new results. In the above conducted experiment, we have not violated the topological stretching rule**.[1-5]** So, logically our experiment is consistent.

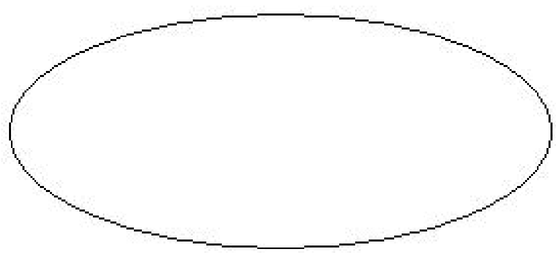

Figure 1

Figure 2

A **Figure 3** **B**

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2. Formulation of new family of functions by applying the authors new fuzzy concepts

Differential equations are the heart of science and technology. Most of the mathematical formulation of physical phenomena are in non linear.[6-12].If we create differential equations for our topological experiment, it will be in non linear. The would be equations will unlock many mathematical information. And the applications to science and technology will answer many current problems.

While a fuzzy set gives the degree of membership of an element in a given set, an Atanassov's intuitionistic fuzzy set gives both a degree of membership and a degree of non-membership. Many concepts in fuzzy set theory were also extended to intuitionistic fuzzy set theory, such as intuitionistic fuzzy relations, intuitionistic L-fuzzy sets, intuitionistic fuzzy implications, intuitionistic fuzzy logics, the degree of similarity between intuitionistic fuzzy sets, intuitionistic fuzzy rough sets. Atanassov's intuitionistic fuzzy sets as a generalization of fuzzy sets can be useful in situations when description of a problem by a (fuzzy) linguistic variable, given in terms of a membership function only, seems too rough.

In our new concept, instead of objects, degrees of memberships and non memberships, the notion of circles are assumed. In figures 4, 5 and 6 there are three circles. These are the objects of our new sets. In figure 4, the three circles are connected. The centers of these three circles lie on a line segment. If we stretch this, ultimately all these three circles will coincide with a line segment. If we repeat this experiment with figures 5 and 6, we will yield the same result. When these circles lie arbitrarily, we will get three distinct line segments, if we stretch. Let us assume that {0,1} represent our new sets. Here 0 is the circle. Let 0.1 is the first stretch. Let 0.2 is the second stretch. And let 0.3 is the third stretch. And 1 (one) represents the line segment of the stretched circles. This relation is the new fuzzy membership concept. For intuitionistic fuzzy, we can add the non membership of this relation. From this new idea, a new sub filed of point set topology may be created.

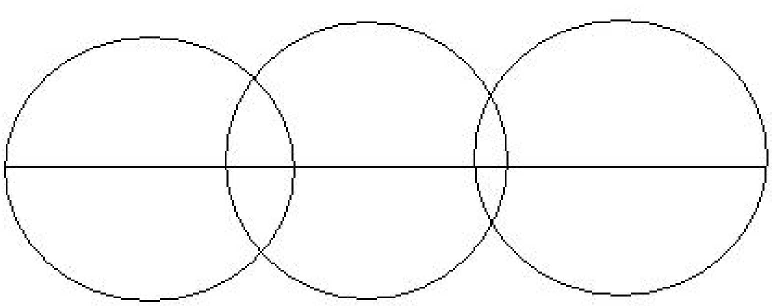

Figure – 4

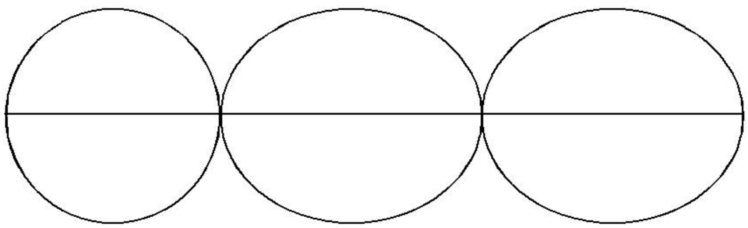

Figure – 5

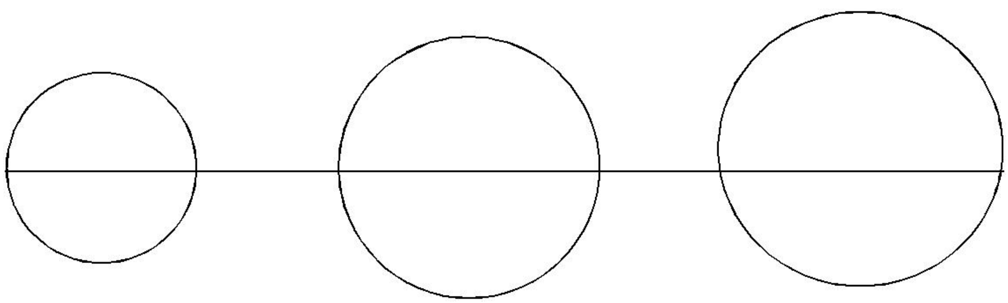

Figure 6

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Results:

Equation of circle with center at (h, k) and radius r is

$$(x-h)^2+(y-k)^2=r^2 \quad (1)$$

Equation of a line is given by,

$$y = mx + c \quad (2)$$

Diffentiating w.r.t 'x',

$$dy/dx = m$$

$$d^2y/dx^2= 0 \quad (2a)$$

Since the center (h, k) lies on the line y = mx + c, we get

$$K = mh + c$$

$$C = k- mh \quad (3)$$

Sub., (3) in (2)

$y= mx + k- mh$

$y- k = m(x-h)$ (4)

Sub., (4) in (1), we get

$(x-h)^2+(m(x-h))^2=r^2$

$(x-h)^2(1+m^2) =r^2$

$(x-h)^2= r^2 /(1+m^2)$ (5)

Expanding (5), $x^2 + h^2 – 2xh = r^2/ 1 + m^2$ (5a)

i.e $r^2 = x^2 + h^2 – 2xh + m^2x^2 + m^2h^2 – 2m^2hx$ (5b)

Diff w.r.t ‘x’,

$2rdr/dx=2x- 2h+2m^2x-2m^2h$

i.e., $rdr/dx= x- h+m^2x-m^2h$

$rdr/dx= x- h+m^2x-m^2h$

Again diff w.r.t ‘x’,

$rd^2r/dx^2+(dr/dx)^2= m^2$

$rd^2r/dx^2+(dr/dx)^2- m^2 = 0$ (6)

Discussion

It is well known that differential equations dominate all the field of science, technology and arts. They have explored beautiful and mar plus phenomena in science and technology. To cite one example, differential equations unlocked so many wonderful ground breaking results in physics and cosmology. The great Physicist Albert Einstein used to jovially comment time and again that differential equations entered into Physics as a maiden servant but became a mistress. Einstein is correct in his remarks. His field equations are in non-linear partial differential equations. Stephen Hawking and Roser Penrose studied Einstein’s field equations and found that even time has origin and end. Also they have proved that this universe came into being after the

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big bang. Lemaitre brilliantly told that the universe is expanding. Einstein field equations predicted that about gravitational waves, curvature of the space time, geodetic effect, frame tracking, time dilation, gravitation lenses, black holes, dark matter and dark energy. All these predictions of the field equations except gravitational waves have been experimentally verified. Even after 100 years of the publication of Einstein's non-linear partial differential equations, the studies and probing are still going on. But still there are some challenging questions in this field. When we apply these equations into quantum physics, these equations becomes incompatible. The authors sincerely believe that the non-linear partial differential equations that we have formulated above may be become a major clue and a tool to study this problematic problem. Equation (6) is a topological non-linear differential equation. The solutions would be mill stones both in topology and non-linear dynamics. Also this may be useful to study the burning problems of theoretical physics.

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