Black Holes have no Interior Singularities

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Abstract

The singularity problem is arguably the most troubling result of Einstein's General Relativity, if not of all physics. This pathology contradicts the laws of thermodynamics and quantum mechanics. It is also refuted by observations confirming the existence of Hawking's Radiation. Previous solutions to the problem have utilized a regime which keeps General Relativity intact, except at the singularity point, at which the classical spacetime is bridged by a discrete quantum one. Here I propose a simple solution to the gravitational, spherical black hole. Specifically, I show that a simple *Newtonian Relativity* theory yields a black hole size which equals the Schwarzschild radius. For a supermassive black hole at a galaxy center, the theory yields a simple expression for the mechanics of the host galaxy. The derived solution predicts that a black hole has *no singularity at the interior*, and that it is part of a binary system with a *naked singularity* located at redshift z = $2^{-\frac{1}{2}} \approx 0.7071$, suspected to be a quasar with extreme velocity offsets or an active galactic nucleus.

No less important, it is shown that the proposed theory, while being intimately related to Newtonian mechanics, is consistent with quantum mechanics and with ΛCDM cosmology.

Keywords: Black holes, Singularity, Schwarzschild radius, Relativity, General Relativity.

Black Holes - A brief History

The name "Black Hole" was coined by John Wheeler in in 1964, but the possibility of its existence within the framework of Newtonian physics was conjectured in 1784 by John Michell, who argued that there might be an object massive enough to have an escape velocity greater than the velocity of light [1]. Twelve years later Simon Pierre LaPlace also predicted the existence of black holes. Laplace argued that "...[It] is therefore possible that the largest luminous bodies in the universe may, through this cause, be invisible." [2]. A better understanding of black holes, and how gravity and waves intermingle, had to wait until 1915, when Albert Einstein delivered a lecture on his theory of General Relativity to the German Academy of Science in Berlin. Within a month of the publication of Einstein's work, Karl Schwarzschild, while serving in the German Army on the Russian front, solved Einstein's field equations for a non-rotating, uncharged, spherical black hole. For a star of a given mass, M, Schwarzschild found the critical radius R (= $\frac{2\,G\,M}{c^2}$, where G is the gravitational constant and c is the velocity of light), at which light emitted from the surface would have an infinite gravitational redshift, and thereby infinite time dilation. Such star, Schwarzschild concluded, would be undetectable by an external observer at *any* distance from the star. Nonetheless, the Schwarzschild metric suffers from a singularity point in spacetime, such that any matter or wave intering the event horizon is predcted to be thrown out, through the singularity, to an undefined spacetime. .

Our understanding of the processes involved in the evolution and decay of black holes was is largely due to quantum mechanical and thermodynamic theories. Early in 1974 Stephen Hawking predicted that a black hole should radiate like a hot, non-black ("gray") body [5]. Hawking's theory of black holes, and the discovery of the Hawking Radiation, confirmed the generalized second law of thermodynamics of Jacob Bekenstein, stating that the sum of the black-hole entropy and the ordinary thermal entropy outside black holes cannot decrease. According to this prediction, black holes should have a finite, non-zero and non-decreasing temperature and entropy [6].

The first X-ray source, widely accepted to be a black hole, was Cygnus X-1 [7]. Since 1994, numerous black holes, of different sizes and redshifts, were detected by the Hubble Space Telescope, and by other space-crafts and extremely large ground telescope [see e.g., 8, 9]. We now know that black holes exist in two mass ranges. Small ones of ($M \lesssim 10\ M_\odot$) ($M_\odot$, solar mass), believed to be the evolutionary end points of the gravitational collapse of massive stars, and supermassive black holes of $M \gtrsim 10^6\ M_\odot$, responsible for the powering of quasars and active galactic nuclei (AGN) [10, 12]. Supermassive black holes, residing at the centers of most galaxies, are believed to be intimately related to the formation and evolution of their galaxies [10- 14].

Pathology and Previous Solutions

As mentioned above, the solution of Einstein's field equations [3, 4] yields a critical hole radius of $R = \frac{2\,GM}{c^2}$. However, Schwarzschild's solution suffers from a serious pathology, since it predicts a singularity where the fabric of spacetime is torn, causing all matter and radiation passing the event horizon to be ejected out to an undefined space-time, leaving the black hole empty. Thus, in violation of thermodynamics and quantum mechanics [e.g., 14-15], we end up with an enormously massive black body with and zero entropy and temperature. In fact, Many believe that the black holes' (and the Big Bang) singularities, mark a breakdown in General Relativity. Nonetheless, it is perhaps because GR is a major pillar of current cosmology, the singularity problem is usually tolerated or ignored.

Attempts to solve the singularity problem in quantum mechanics include the analyses carried out by Ashtekar and others [16-17] who proposed a loop quantum gravity model [18-19] which avoids the singularities of black holes and the Big Bang. Their strategy was to utilize a regime which keeps

General Relativity intact, except at the singularity point, at which the classical spacetime is bridged by a discrete quantum one. Although the solution is mathematically difficult, its strategy is simple. It begins with a semi-classical state at large late times ('now'), and evolves it back in time, while keeping it semi-classical till one encounters the deep Planck regime near the classical singularity. In this regime it allows the quantum geometry effects to dominate. As the state becomes semi-classical again on the other side, the deep Planck region serves as a quantum bridge between two large, classical space-times [16].

The Proposed Solution

Here I propose another solution to the black holes problem. As a case study, I consider here a spherical supermassive, gravitational black hole. The solution proposed is based on a new relativity theory, termed *Newtonian Relativity* (NR). First I present the theory, and then I use it to solve the black holes' paradoxes encountered by General Relativity. Specifically, I show that a simple relativity theory, based on two plausible and well accepted axioms, yields a solution to the black hole size which equals the Schwarzschild radius, but *with no singularity at the interior*. The proposed solution portrays black holes as "gray", extremely dense, and gigantically gravitational objects, which absorb, but also emit radiation. The theory is in agreement with quantum and thermodynamics predictions that an event horizon area of each black hole and its entropy do not decrease [e.g., 6, 14, 15, 20], but could rather increase. Moreover, the proposed solution predicts that black holes are part of binary systems, with a naked singularity at redshift z = $2^{-\frac{1}{2}} \approx 0.7071$, suspected to be a quasar with extreme velocity offsets or an active galactic nucleus (AGN).

Theory

The theory, here applied to inertial systems, is based on the following two postulates:

1. Information regarding physical entities is translated from one laboratory to another via light and electromagnetic waves of equal velocity (information postulate).
2. The laws of physics are the same in each internal system, and at sufficiently low velocity, these laws are described by Newtonian mechanics (invariance postulate).

The information postulate is justified by the necessity, dictated by quantum mechanics, to specify the methods and devices used in observations, including the medium by which information is translated from one frame of reference to another. As prescribed by Niels Bohr, accounts of the experimental arrangements, and of the results of the observations, must be expressed in unambiguous language with suitable application of the terminology of classical physics [21]. In fact one should also postulate that the measurement methods used in all laboratories are the same, but this requirement could be considered as a default. Note that the same laws of physics could be derived for any information carrier velocity. Nevertheless, applying the theory to cosmology, and to high velocities and energies, makes the velocity of electromagnetic waves the only practical choice.

The first axiom implies that information sent from a body, when it is at distance *d* from the observer's laboratory, will reach the observer with time dilation of $\Delta t = \frac{d}{c}$, where *c* is the velocity of the wave signal relative to the observer. Notably, this consequence of the information postulate does not require that the body is in relative motion with respect to the observer. The fact that a light ray propagating from the sun reaches Earth in about eight minutes and 20 seconds depends on the distance between Earth and the sun and not on the fact that they are in relative motion with respect to each other. The same applies to wave travel between any two bodies.

The second axiom, although in agreement with Special relativity's first axiom, is more specific regarding the nature of laws which should apply to all systems. The specification that the observations of physical realities at low velocities should accord with classical physics is in agreement with the subjective Copenhagen interpretation of Quantum Theory [22-23], according to which wave functions are mere mathematical objects that allow us to calculate probabilities of future events. In other words, quantum states are interpreted here in accordance with the classic argument by Albert Einstein [24-25], that is as states of knowledge [26], rather than as states of objective reality [27].

The requirement that at low velocities the laws of physics are classical Galileo-Newton laws has a profound implication on the strategy used in the proposed theory. Not only must we expect that all laws should converge at low velocities to the laws of classical mechanics; we must also require that in considering different frames of reference, any relativistic effect should be *uniquely* a function of relative velocities. Although the case treated here concerned inertial systems, we expect the same proposition to hold for non-inertial systems, with non-zero acceleration vector $\vec{a}$. The possibility of such extension is guaranteed by the equivalence principle, positing that gravitational forces, if present, could be replaced by an equivalent acceleration vector, $\vec{a}$, from which the relative velocities at any time *t* could be determined. The advantage of this approach over General Relativity is twofold: On the one hand, it disentangles the force of gravity from relativity in space and time, allowing a commonsensical integration between the relativity of inertial (special) and non-inertial (general) systems, such that the laws of the former could be obtained from the laws of the latter by setting $\vec{a} = 0$. Second, unlike General Relativity, the present approach maintains a smooth and natural continuity between relativistic and classical (non-relativistic) physics.

In essence, the proposed theory is a relativity theory of large, fast moving systems. Nonetheless, its two postulates are anchored, philosophically, on basic principles of quantum mechanics. In addition, although not stated as a third postulate, it is accepted that small, Planck's scale, systems, are best described as quantum states of knowledge and thermodynamics, and are best analyzed by quantum mechanical and thermodynamic models. In other words, NR should be viewed as an alternative to GR in describing the large scale dynamics, while at Planck scales, the system dynamics are best described by quantum mechanics.

Transformations

The derivation of the theory's transformation is straightforward and simple. To derive the time transformation, consider two frames of reference, F and F', which depart from each other along the x axes (in F), with constant relative velocity v. Without loss of generality, assume that at $t_1 = t_1' = 0$, x = $x' = 0$. For an observer in F, at time t sec. in F (t_1' in F'), the systems should be at a distance $d = v\,t$. Now assume that exactly at time t_1' in F' (t in F) a light wave signal is sent from the point $x = d$, indicating that F' has just passed this point. If the velocity of the wave signal, as measured by an observer in F, is c, then the information, indicating the arrival of F' at point $x = d$, will be delayed by $\frac{d}{c} = \frac{v\,t}{c}$. Thus we can write:

$$t = t' + \frac{v\,t}{c} = t' + \beta \qquad \text{...... (1)}$$

where $\beta = \frac{v}{c}$

Or:

$$\frac{t}{t'} = \frac{1}{1-\beta} \qquad \text{...... (2)}$$

Note that Eq. (2) is similar to the Doppler Formula, except that the Doppler Effect describes red- and blue-shifts of waves propagating from a departing or approaching wave source, whereas the result above describes the time transformation of moving bodies. Also note that $\frac{1}{1-\beta}$ is *larger than one* (time dilation) if F and F' *depart* from each other, and *smaller than one* (time contraction) if they *approach* each other.

Similar derivations of the distance, mass density and kinetic energy yields the transformations depicted in Table 1 (see section A in the supporting material).

Table 1

Transformations

Physical Term	**Relativistic Expression**	
Time (sec)	$\frac{t}{t'} = \frac{1}{1-\beta}$	…. (3)
Time (round trip)	$\frac{t}{t'} = \frac{2}{1-\beta^2}$	….(4)
Distance (m)	$\frac{x}{x'} = \frac{1+\beta}{1-\beta}$	….(5)
Mass density (kg/m^3)	$\frac{\rho}{\rho'} = \frac{1-\beta}{1+\beta}$	….(6)
Kinetic energy (Joule)	$E = \frac{1}{2}\, m_0\, c^2\, \beta^2 \frac{1-\beta}{1+\beta}$	….(7)
Newton's Second Law	$F = \frac{1-2\beta-\beta^2}{(1+\beta)^2}\, m_0 a$	….(8)

Black holes in Newtonian Relativity

Figure 2 depicts a schematic representation of a supermassive black hole with mass M and radius *r* residing at a center of its host galaxy. The figure shows three particles at different distances from the center of the black hole.

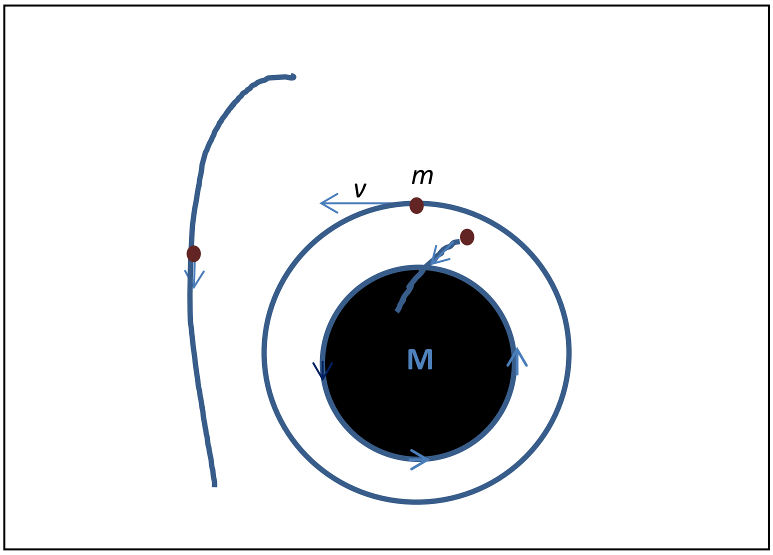


Figure 2 Three particles near a black hole

As depicted in the figure, the more distant particle will be deflected towards the black hole, but will escape it due to its large distance, and continue its travel in space. In contrast, the closest particle to the black hole will experience a strong enough gravitational force to cause its absorption into the black hole. Now consider the third particle, which travels tangentially to the hole's radius vector. Such particle could be a baryon or wave quanta entrapped at a critical distance, ensuring that it rotates around the black hole or a baryon or a Hawking Radiation quanta hovering just at the hole's event horizon. For such particle, the acceleration $|\vec{a}|$ supporting a uniform radial motion with radius R should satisfy:

$$a = |\vec{a}| = \frac{v^2}{r} = \frac{c^2}{R}\beta^2 \qquad \text{… (9)}$$

A relativistic derivation of Newton's Second Law (see Section B in SM) yields:

$$F = \frac{1-2\beta-\beta^2}{(1+\beta)^2}\ m_0\ a \qquad \text{…. (10)}$$

Substitution the value of *a* from Eq. 9 in Eq. 10 yields:

$$F = \frac{1-2\beta-\beta^2}{(1+\beta)^2}\ m_0\ a = \frac{1-2\,\beta-\beta^2}{(1+\beta)^2} m_0\ \frac{v^2}{r} = m_0\ c^2\ \frac{1-2\,\beta-\beta^2}{(1+\beta)^2}\beta^2\,\frac{1}{r} \qquad \text{…. (11)}$$

Using Newton's general law of gravitation, we get:

$$G\,\frac{m_0\ M}{r^2} = m_0\ c^2\ \frac{1-2\,\beta-\beta^2}{(1+\beta)^2}\beta^2\,\frac{1}{r} \qquad \text{…. (12)}$$

Solving for *r* yields:

$$r = \frac{G\ M}{c^2}\ \frac{(1+\beta)^2}{1-2\,\beta-\beta^2}\ \beta^2 \qquad \text{….. (13)}$$

Assuming spherical symmetry, Eq. 13 describes the dynamics of the host galaxy as a function of velocity. For a light photon ($\beta = 1$), we have:

$$r\,((\beta = 1) = R = \frac{2\ G\ M}{c^2} \qquad \text{….. (14)}$$

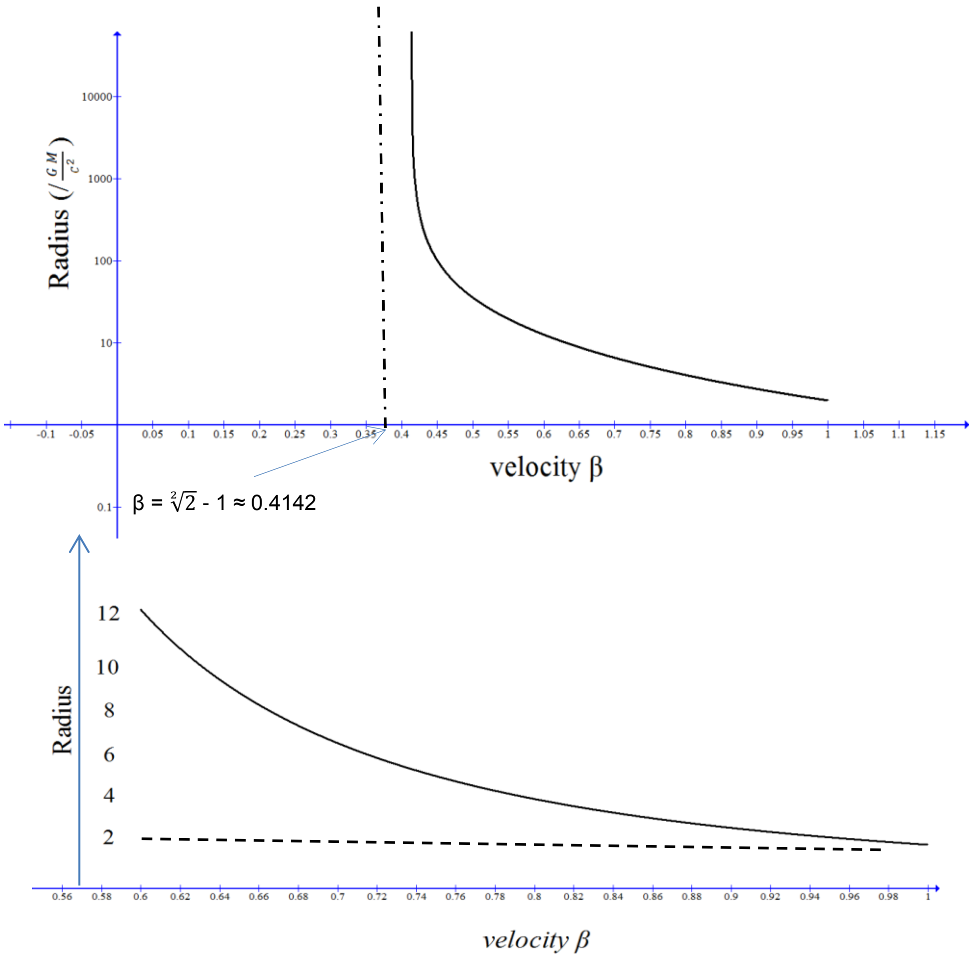
Which exactly equals the Schwarzschild radius, *but with no singularity in the hole's interior*. Interestingly, the solution (Eq. 13) has a naked singularity at β satisfying:

$$1 - 2\,\beta - \beta^2 = 0 \qquad \text{.... (15)}$$

Solving for β, we have:

$$\beta = \sqrt[2]{2} - 1 \approx 0.4142 \qquad \text{….. (16)}$$

Fig. 3a depicts the radius *r* as a function of β, and Fig. 3b shows an enlargement of the figure in the range β = 0.6 -1.



Figures 3a-3b Black hole radius as function of velocity

Expressing the radius *r* in Eq. 13 as a function of the redshift yields:

$$r = \frac{G\,M}{c^2}\;\frac{z^2(1+2z)^2}{(1+z)^2\,(1-2z^2\,)} \qquad \text{....(17)}$$

With corresponding redshift of:

$$z = \frac{\beta}{1-\beta} = \frac{1}{\sqrt[2]{2}} \approx 0.7071 \qquad \text{..... (18)}$$

The above results are quite interesting. They imply that a non-rotating black hole, with Schwarzschild radius R, is part of a binary system with a naked singularity. The picture drawn by the solution is of a black hole at relatively high redshift, paired with a quasar with extreme velocity offsets or an active galactic nucleus (AGN), at redshift z ≈ 0.7071.

Eq. 17 has some peculiar properties. For z = φ ≈ 1.618 we have *r* = z, which indicates a Golden Ratio symmetry. In addition, *r* has a minimum of r_m ≈ 1.5867, at redshift z ≈ 2.0782.

The present results confirm with an ΛCDM model with $\Omega_m = \frac{1}{3}$ and $\Omega_\Lambda = \frac{2}{3}$. Fig. 4 depicts the radius *r* as a function of redshift for a range of M - 10^3M. together with results adopted from [28] for a cosmology of H_0 = 70 km s^{-1} Mpc^{-1}, Ω_M = 0.3 and Ω_Λ = 0.7 for intermediate and massive black holes. Comparison of the two figures reveals a remarkable similarity between the predictions of the two models.

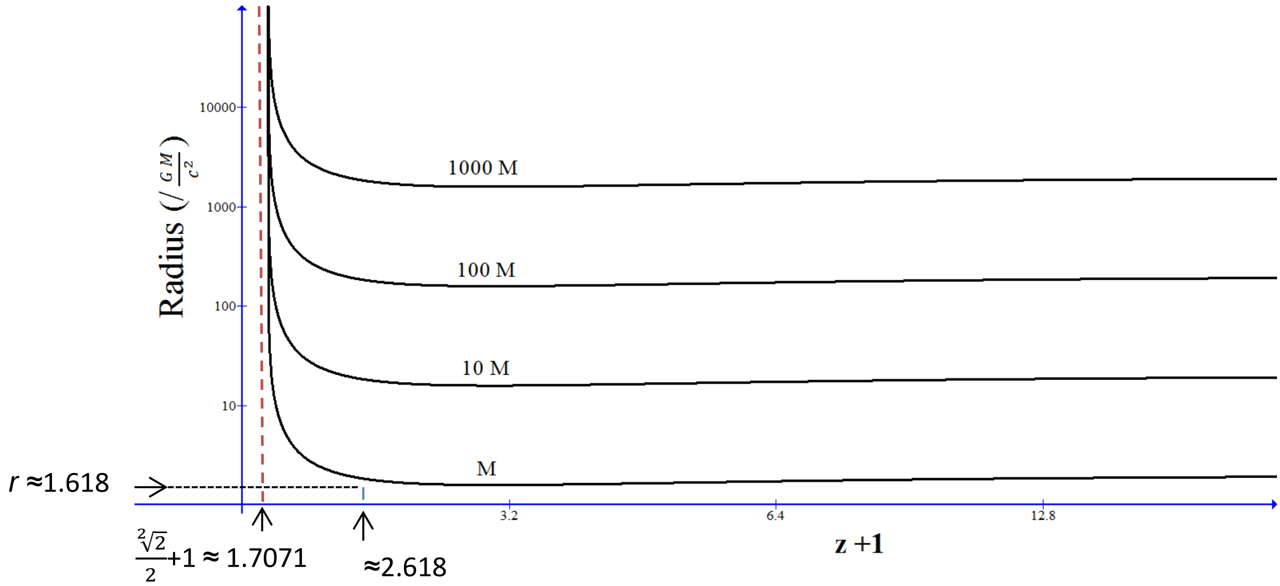


Figure 4 Radius as function of redshift

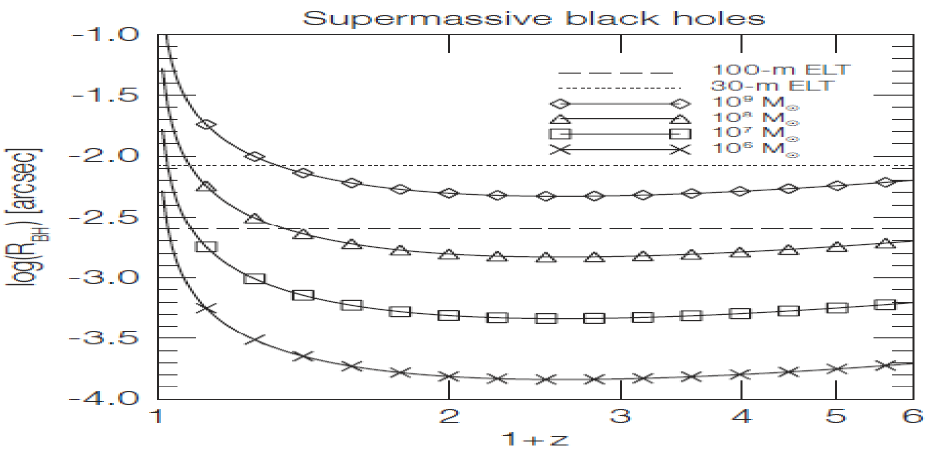


Figure 5 Intermediate and massive black holes for Ω_M = 0.3, Ω_Λ = 0.7 (Source: Hook, 2005)

Area

Assuming a spherical hole, its area A as a function of its mass could be expressed as:

$$A = 4\,\pi\, R^2 = 4\,\pi\,\left(\frac{2\,G\,M}{c^2}\right)^2 = 16\,\pi\,\frac{G^2}{c^4}\,M^2 \qquad \text{....(19)}$$

Alternatively, the mass of the black hole as a function of its surface area is:

$$M = \frac{c^2}{4G} \sqrt[2]{\frac{A}{\pi}} \qquad \text{....(20)}$$

Kinetic Energy

Assuming a uniform black hole, the kinetic energy density at a distance r > R from the center of a black hole (see table 1) is given by:

$$e = \frac{1}{2}\rho_0 \, c^2 \, \beta^2 \frac{1-\beta}{1+\beta} \qquad \text{....(21)}$$

Which could be written in redshift z as:

$$e = \frac{1}{2}\rho_0 \, c^2 \, \frac{z^2}{(1+z)^2 \, (2z+1)} \qquad \text{....(22)}$$

As shown in Fig. 5 the ratio $\frac{e}{\frac{1}{2}\rho_0 \, c^2}$ as function of z is normal like. Strikingly, the point of maximum, obtained by deriving the term in Eq. 22 with respect to z and equating the result to zero, is equal to:

$$z_m = \frac{\sqrt[2]{5}+1}{2} = \varphi \approx 1.618 \qquad \text{....(23)}$$

Where φ is the famous Golden Ratio. The max value of $\frac{e}{\frac{1}{2}\rho_0 \, c^2}$ is given by:

$$\frac{e}{\frac{1}{2}\rho_0 \, c^2} = \left(\frac{1}{\varphi}\right)^5 \approx 0.09016994 \qquad \text{....(24)}$$

Strikingly, the obtained maximum is equal to L. Hardy's probability of entanglement (29-30).

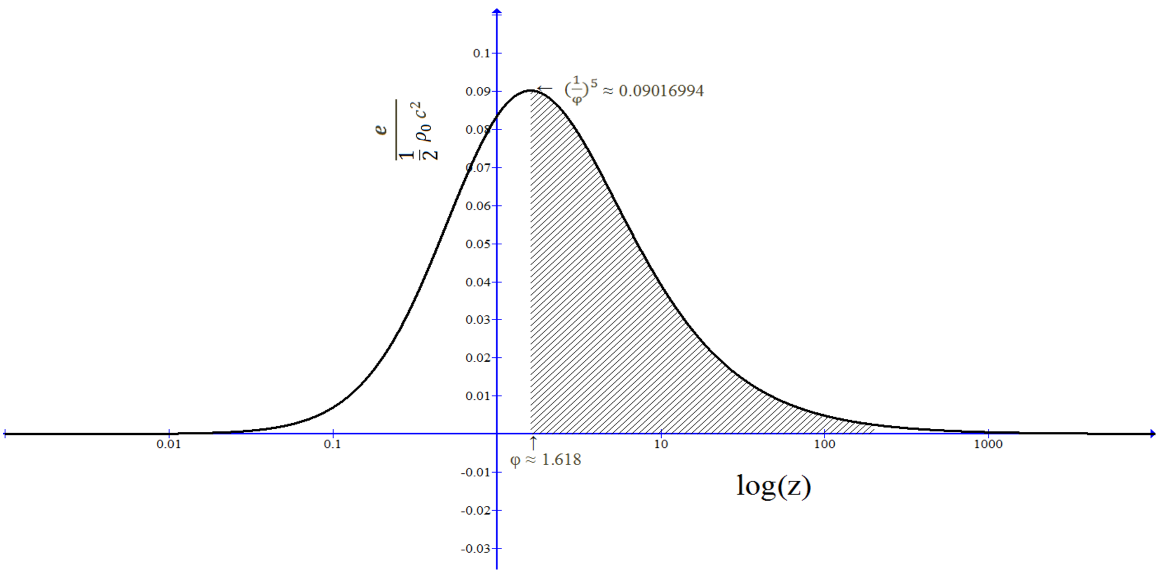


Figure 6 Kinetic Energy as a function of redshift

Summary and Concluding Remarks

The present paper describes a simple [(1)] relativity theory, termed Newtonian Relativity. The theory is based on two plausible postulates: That information translation, by light or other electromagnetic waves, suffers from time dilation, and that at low velocities all the relativistic terms reduce to the classical Galileo-Newton terms. Based on the above two axioms, I derived new relativistic terms for time, distance, mass density, kinetic energy, as well as a relativistic modification of Newton's Second Law. Application of the theory to non-rotating, purely gravitational, spherical black holes yielded the following prediction:

1. The radius of a black hole is equal to the is equal to the Schwarzschild radius (R= $\frac{2\,G\,M}{c^2}$).

2. At the *interior* of the hole, the solution has *no singularity*.

3. At the *exterior* of the hole, the solution has *one naked singularity* at redshift z = $\frac{1}{\sqrt[2]{2}}$ ≈ 0.7071.

4. The energy density (normalized by $\frac{1}{2}\,\rho_0\,c^2$) is predicted to reach its peak at redshift z = φ ≈ 1.68, where φ is the Golden Ratio. This is a novel testable prediction, since it implies the existence of extreme galactic activity located in the neighborhood of z= φ.

5. For a spherical black hole, the area A of the event horizon is proportional to the square of its mass (A = 16 π $\frac{G^2}{c^4}\,M^2$).

6. A gravitational black hole is a part of a binary system with an extreme galactic activity located at redshift z = $2^{-\frac{1}{2}}$ ≈ 0.7071, suspected to be a quasar with extreme velocity offsets, or a weaker active galactic nuclei (AGN). This prediction is supported by many observations [e.g., 31-33]. For example, Steinhardt et al. [32] reported the the discovery of a Type 1 quasar, SDSS 0956+5128, with extreme velocity offsets at redshifts z = 0.690, 0.714, and z 0.707.

For an observer located at the exterior of a galaxy, assuming radial symmetry, the picture emerging from the present analysis and from previous results reported in [34] is sketched in Fig 7. The galaxy is centered at supermassive black hole with Schwarzschild radius, located at z >> 1. For 0 ≤z≤ 0.5 the galaxy is dominated by baryons and kinetic energy and a z > 0.5 it is dominated by dark matter and dark energy.

Footnote 1: If we believe that the rules of nature should be simple ones and if Occam's razor principle is taken seriously, then the mathematical simplicity of the theory, rendering it comprehendible by high school science students, if not taken as an asset, should, at least, not be taken as a liability.

Notwithstanding, the density distribution of kinetic energy reaches its peak at z ≈ 1.618, the point of quantum criticality. The nearby exterior of the black hole is gray. The rim of the event horizon is luminous, due to trapped highly energized waves (Cosmic rays, X-rays, light), bursting out as Hawking Radiation when acquiring superluminal escape velocities. The interior of the black hole is dark.

The above galaxy mapping provides several testable predictions. One interesting prediction concerns the domination of baryonic matter and kinetic energy vs. dark matter and dark energy in a galaxy. A second prediction concerns the normal shaped distribution of kinetic energy in the galaxy, centered at redshift equaling the Golden Ratio. A third prediction concerns the quadratic relationship between the surface area of a black hole's event horizon and its mass (Eq. 19).

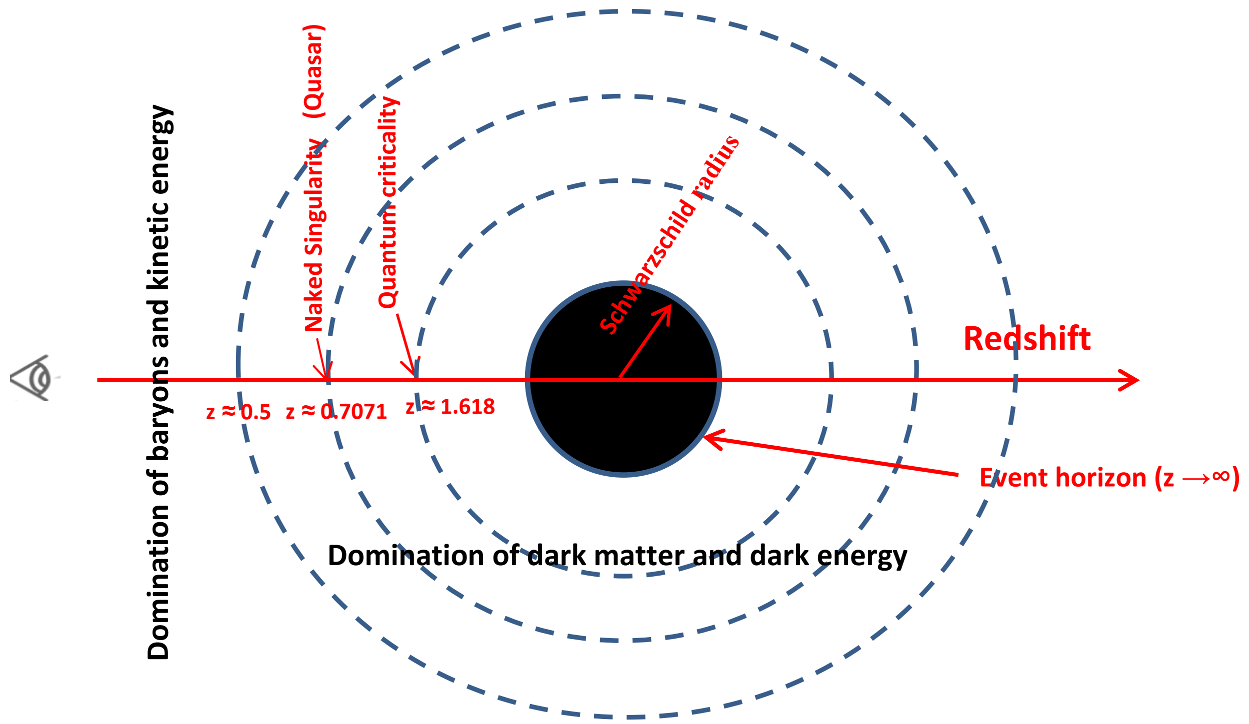


Figure 7 Illustration of the predicted black hole region

The luminous halo at the event horizon is comprised of light photons and quasi-luminal particles, like neutrinos, with critical velocity that supports a circular motion on the rim of the event horizon.

Obviously, the present analysis and the conclusions drawn above suffer from oversimplifications. It assumed radial motion whereas the dynamics of most galaxies are quasi-elliptical and spiral. It also assumed that the black hole has no electric charge and that except for the gravitational force of the black hole, all other forces, including the electromagnetic force and the gravitational forces of quasars and active galactic nuclei (AGNs) are negligible.

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Supporting Material for:

R. Suleiman, "Black Holes have no Interior Singularities"

A. Derivation of the distance, mass density and energy transformations

A1. ***Distance***

Consider the two frames of reference in Figure a1. Assume that a body moving in the +x direction arrives at time t_1 in F (t'_1 in F') to the point x_1 in F (x'_1 in F') and continues to point x_2 in F (x'_2 in F') at which it arrives at time t_2 in F (t'_2 in F'). Assume further that the body's arrival at each point is signaled by a light pulse sent in the –x direction to two observers, one stationed at the point $x = 0$ in F and another stationed at point $x' = 0$ in F', and that the light signals travel with velocity c relative to F.

The signal indicating the body's arrival at x'_1 reaches the observer stationed at $x' = 0$ at time t'_1 which equals:

$$t'_1 = \frac{x'_1}{c+v} \quad \text{.... (a1)}$$

Where v is the velocity of F' relative to F, and c is the velocity of light as measured in F.

Similarly, the time t'_2 indicating the body's arrival at x'_2 is given by:

$$t'_2 = \frac{x'_2}{c+v} \quad \text{.... (a2)}$$

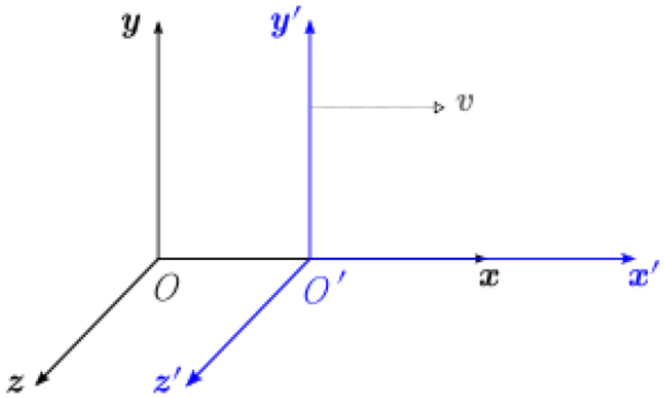

Figure a1. Observers in two reference frames moving with velocity v with respect to each other

The time arrivals in F at x_1 and x_2 are given, respectively, by: $t_1 = \frac{x_1}{c}$ and $t_2 = \frac{x_2}{c}$. Thus, we can write:

t_2 - t_1= $\frac{x_2 - x_1}{c}$ …. (a3)

And:

t'_2 - t'_1= $\frac{x'_2 - x'_1}{c+v}$ …. (a4)

From Eqs. a3 and a4 we have:

$\frac{x_2 - x_1}{x'_2 - x'_1} = \frac{c+v}{c} \frac{t_2 - t_1}{t'_2 - t'_1} = (1 + \beta) \frac{t_2 - t_1}{t'_2 - t'_1}$ … (a5)

Substituting the time transformation from Eq. 2 (see main text) in Eq. a5 and defining x = $x_2 - x_1$ and $x' = x'_2 - x'_1$, the distance transformation could be written as:

$\frac{x}{x'} = \frac{(1+\beta)}{(1-\beta)}$ …. (a6)

The relative distance $\frac{x}{x'}$ as a function of β, together with the respective relative distance according to *SR* (in dashed black), are shown in Fig a2. While *SR* prescribes that irrespective of direction, objects moving relative to an internal frame will contract, *CR predicts that a moving object will contract or expand, depending on whether it approaches the internal frame or departs from it.*

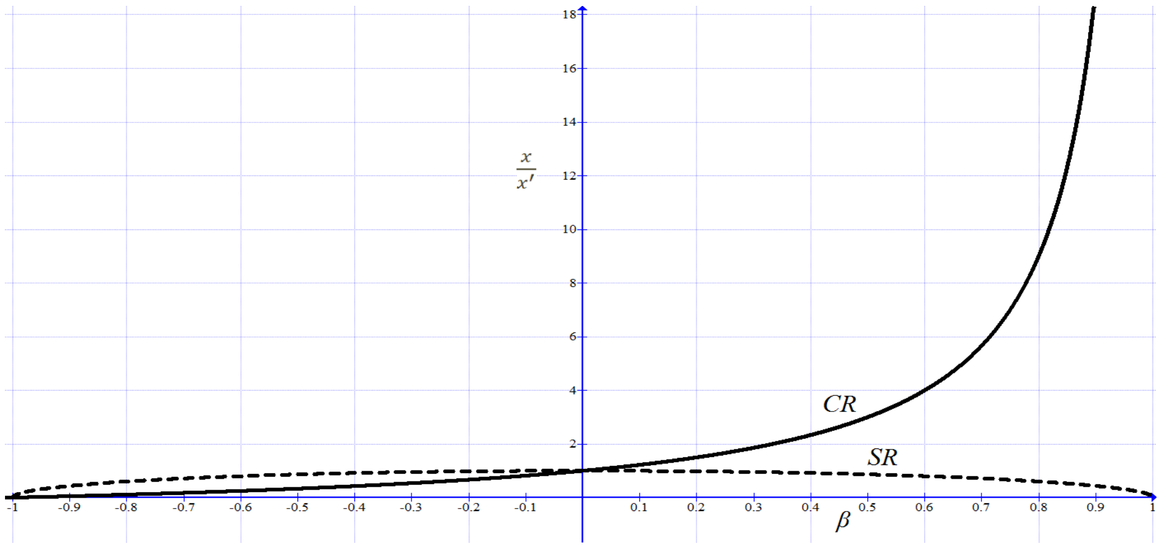

Figure a2. Distance transformation for the one-way trip. The dashed line depicts the corresponding prediction of SR

A.2 *Time and Distance in the Round-Trip*

For the *round trip* from *F* and back, synchronization of the start time is not required. For this case, using Eq. (2), the total relative time is given by:

$t = \vec{t} + \overleftarrow{t} = (\frac{1}{(1-\beta)} + \frac{1}{(1+\beta)})t' = (\frac{2}{1-\beta^2})\, t'$ ….(a7)

Or,

$$\frac{t}{t'} = \frac{2}{1-\beta^2} \qquad \text{.... (a8)}$$

Similarly, using Eq. (a6), the distance transformation for the round trip is given by:

$$x = \left(\frac{(1+\beta)}{(1-\beta)} + \frac{(1-\beta)}{(1+\beta)} \right) x' \qquad = \frac{2\beta}{1-\beta^2}\ x' \qquad \text{.... (a9)}$$

Or:

$$\frac{x}{x'} = \frac{2\beta}{1-\beta^2} \qquad \text{... (a10)}$$

The relative time and distance as functions of β in the round trip are depicted in figures a3 and a4, respectively. The dashed lines depict the corresponding predictions of SR. Note that for the round trip the results of CR and SR are qualitatively similar, except that the time dilation predicted by CR is larger than that predicted by SR. For small β values, the two theories yield almost identical results. Conversely, while SR predicts distance *contraction*, CR predicts distance *expansion*.

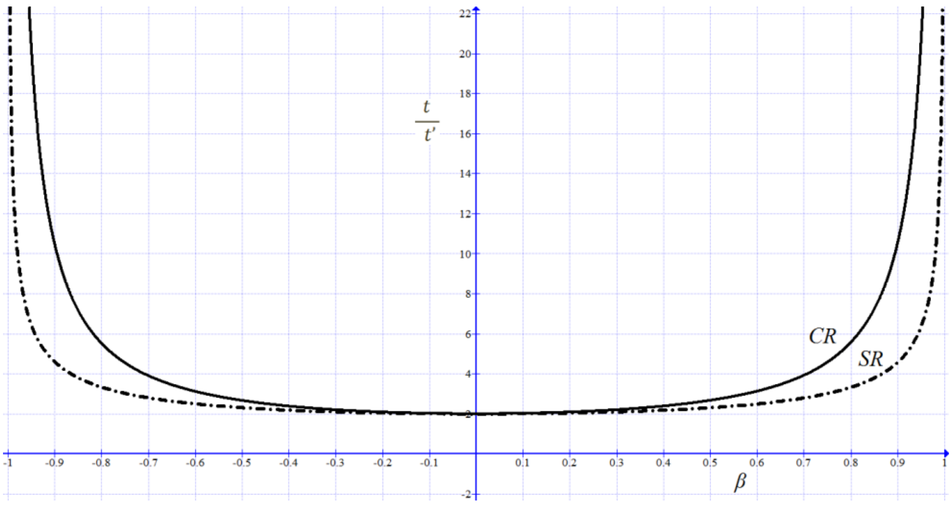

Figure a3. Time transformation for the round trip. The dashed line depicts the corresponding prediction of SR

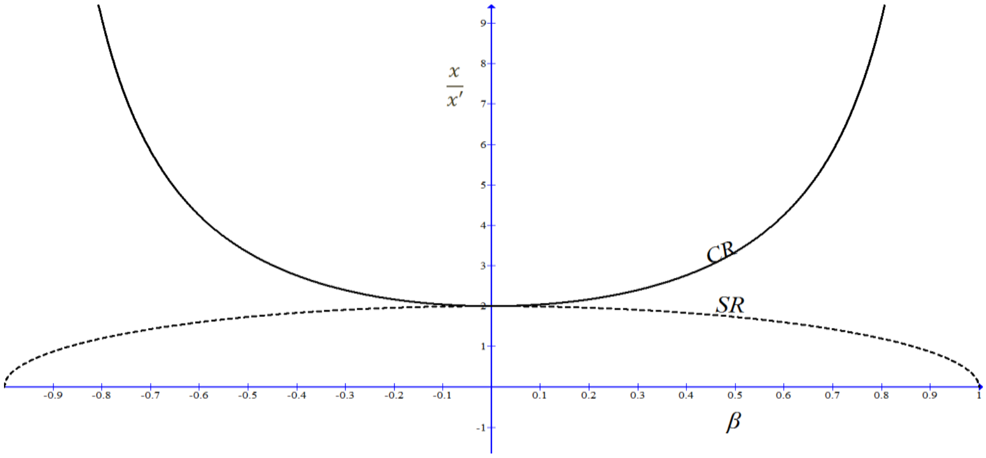

Figure a4: Distance transformation for the round trip. The dashed line depicts the corresponding prediction of SR

A3 mass density and kinetic energy

Consider the two frames of reference F and F'shown in Figure a4. Suppose that the two frames are moving relative to each other at a constant velocity v. Consider a uniform cylindrical body of mass m_0 and length of l_0 placed in F' along its travel direction. Suppose that at time t_1 the body leaves point x_1 ($x_1{}'$ in F') and moves with constant velocity v in the +x direction, until it reaches point x_2 ($x_2{}'$ in F') in time t_1 ($x_2{}'$ in F'). The body's density in the internal frame F' is given by: $\rho' = \frac{m_0}{A\, l_0}$, where A is the area of the body's cross section, perpendicular to the direction of movement. In F the density is given by: $\rho = \frac{m_0}{Al}$, where l is the object's length in F. Using the distance transformation (Eq. 6a) l could be written as:

$$l = \frac{1+\beta}{1-\beta}\, l_0 \qquad \text{… (a11)}$$

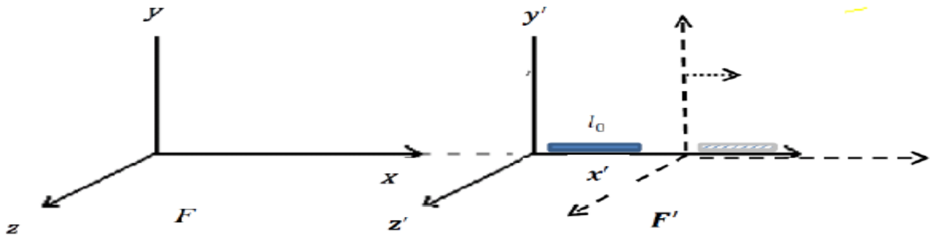

Fig. a4. Two observers in two reference frames, moving with constant velocity v with respect to each other

Thus, we can write:

$$\rho = \frac{m_0}{Al} = \frac{m_0}{A\ l_0\,(\frac{1+\beta}{1-\beta})} = \rho'\ (\frac{1-\beta}{1+\beta}) \qquad \text{….. (a12)}$$

Or,

$$\frac{\rho}{\rho_0} = \frac{1+\beta}{1-\beta} \qquad \text{….. (a13)}$$

The kinetic energy of a *unit of volume* is given by:

$$E = ½\, \rho\, v^2 = ½\, \rho_0 \frac{(1-\beta)}{(1+\beta)}\, v^2, \qquad \text{....(a14)}$$

or**:**

$$E = \frac{1}{2}\, \rho_0\, c^2\ \frac{(1-\beta)}{(1+\beta)}\ \beta^2 \qquad \text{….. (a15)}$$

And the kinetic energy for a body of mass m_0 is given by:

$$E = \frac{1}{2}\, m_0\, c^2\, \frac{(1-\beta)}{(1+\beta)}\, \beta^2 = \frac{1}{2}\, E_0\, \frac{(1-\beta)}{(1+\beta)}\, \beta^2 \qquad \text{….. (a16)}$$

where $\beta = \frac{v}{c}$ and $E_0 = m_0\ c^2$. For β →0 (or v << c) Equation x3 reduces $\rho = \rho_0$, and the kinetic energy expression (Eq. a16) reduces to Newton's expression $E = \frac{1}{2} \rho_0\, v^2$. Figures a5 and a6, respectively, depict the relativistic mass density and energy as a function of β.

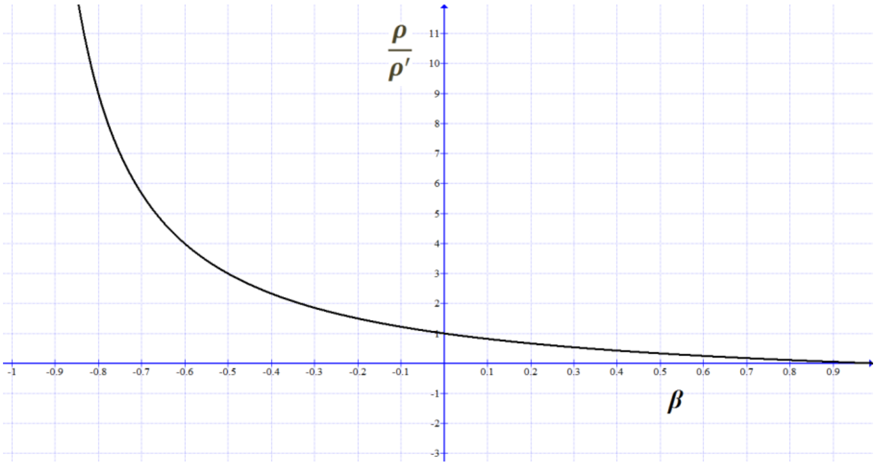

Figure a5. Mass density as a function of velocity

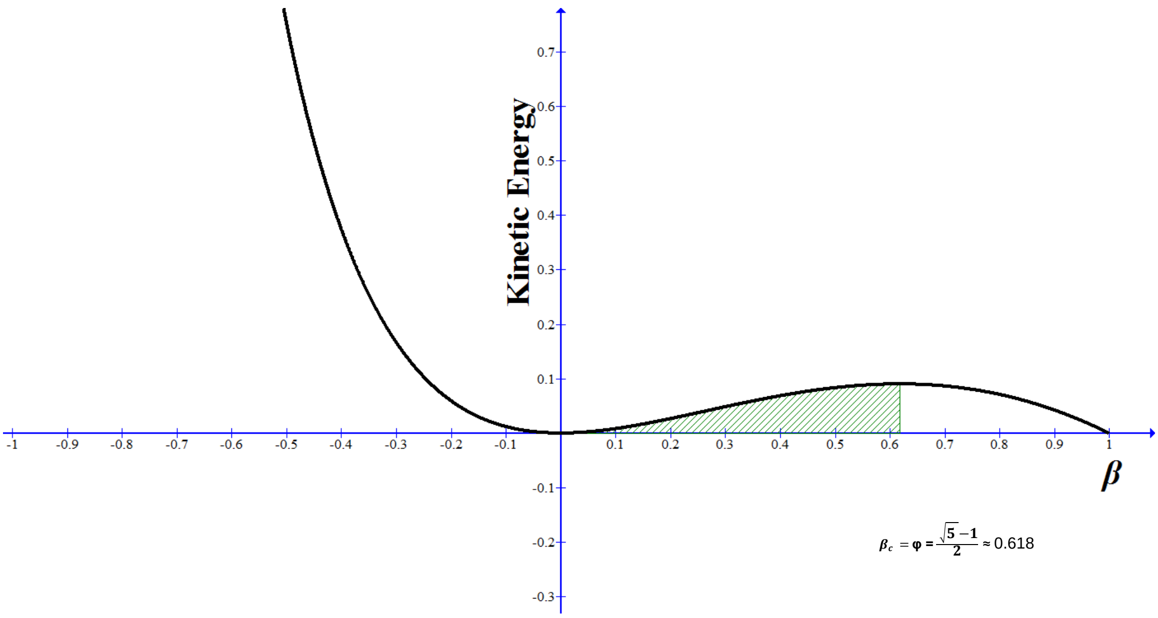

Figure a6. Kinetic energy as a function of velocity

As shown by the Fig.2, the density of *departing* bodies relative to an observer in *F* is predicted to *decrease* with *β*, approaching zero as *β* ⟶ 1, while the density in F for *approaching* bodies is predicted to increase with *β*, up to infinitely higher values as *β* ⟶ -1.

Strikingly, for departing bodies the kinetic energy displays a non-monotonic behavior. It increases with β up to a maximum at velocity β = β_{cr} , and then decreases to zero at β = 1. Calculating β_{cr} is obtained by deriving Eq. a16 and equating the result to zero:

$$\frac{d}{d\beta}\left(\beta^2 \frac{(1-\beta)}{(1+\beta)}\right) = 2\,\beta\,\frac{(1-\beta)}{(1+\beta)} + \beta^2\frac{[(1+\beta)(-1)-(1-\beta)(1)]}{(1+\beta)^2} = 2\,\beta\frac{(1-\beta^2-\beta)}{(1+\beta)^2} = 0 \qquad \text{.... (a17)}$$

for β ≠ 0 and we get:

$$\beta^2 + \beta - 1 = 0 \qquad \text{...(a18)}$$

Which solve for positive β at:

$$\boldsymbol{\beta_{cr}} = \frac{\sqrt{5}-1}{2} = \Phi \approx 0.618 \qquad \text{....(a19)}$$

Where Φ is the Golden Ratio [A1-A2]. This is a striking result given the properties of this phenomenal number, due to its importance, together with the Fibonacci numbers, in mathematics, aesthetics, art, music, and more [e.g., A3-A5] and its key role in nature, including in biology and life sciences[A6-A8], physics [e.g., A9-A10], chemistry [e.g., A11], neuroscience [e.g., A12-A13], and more.

Substituting $\boldsymbol{\beta_{cr}}$ in the energy expression (Eq. a16) yields:

$$E_{max} = ½\, E_0 \quad \Phi^2\,\frac{1-\Phi}{1+\Phi} \qquad \text{.... (a20)}$$

From Eq. a18 we can write: $\Phi^2 + \Phi - 1 = 0$, which implies $1 - \Phi = \Phi^2$ and $1+\Phi = \frac{1}{\Phi}$.

Substitution in Eq. a20 gives:

$$E_{max} = ½\, \Phi^5\, E_0 \;\approx \frac{0.09016994}{2}\; E_0 \approx 0.04508497\, E_0 \qquad \text{.... (a21)}$$

Interestingly, the ratio $\frac{E_{max}}{E_0} = \frac{E_{max}}{m_0\, c^2} \approx 0.04508497$, which is precisely half of L. Hardy's probability of entanglement (0.09016994) [A14-A15]. This result confirms with a recent experimental finding [16], which demonstrated that applying a magnetic field at right angles to an aligned chain of cobalt niobate atoms, makes the cobalt enter a quantum critical state, in which the ratio between the frequencies of the first two notes of the resonance equals the Golden Ratio; the highest-order *E8* symmetry group discovered in mathematics [A17].

B. Derivation of a relativistic Newton's Second Law

For relativistic velocities, Newton's second law is given by:

$$F = \frac{\partial P}{\partial t} = \frac{\partial (mv)}{\partial t} = m\,\frac{\partial (v)}{\partial t} + v\,\frac{\partial (m)}{\partial t}$$

$$= m\,a + v\,\frac{\partial (m)}{\partial v}\,\frac{\partial (v)}{\partial t} = m\,a + v\,a\,\frac{\partial (m)}{\partial v} \qquad \ldots(b1)$$

Or:

$$\mathrm{F} = \left(m + v\,\frac{\partial (m)}{\partial v}\right) a \qquad \ldots.(b2)$$

Substitution the term for *m* from Table 1, and deriving m with respect to *v* yields:

$$\mathrm{F} = \frac{1-2\beta-\beta^2}{(1+\beta)^2}\;m_0\;\mathrm{a} \qquad \ldots.(b3)$$

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