2.3_ MEASURING COMPLEXITY

We have gradually approached to a conceptualization of complexity that can be applied to any objet and that can be measured based on the amount of the object's information that is 'meaningful' to an observer. It therefore refers to information which meaning can be 'known in advance'; i.e.: it can be 'learned' as 'knowledge'.

Consequently, we define 'knowledge' as 'meaningful information that can be referred to two or more different objects or events'; i.e.: to a class of objects or events.

This definition coincides with the common use of the term 'regularity' in scientific Complexity conceptualizations [that will be reviewed in the second part of this chapter], and it is noteworthy that it excludes certain type of meaningful information which cannot be applied to a class of objects that can be found in some 'singular' objects as symbolic objects[1].

Based on the above definition, we propose measuring objects' Complexity accounting their information, according to the one that can be converted into knowledge [i.e.: that can be 'known'], while the Complexity Degree will account such percentage in relation to its potential maximum.

An object's Complexity Degree has therefore a very different meaning to its Complexity, they become essentially independent measures.

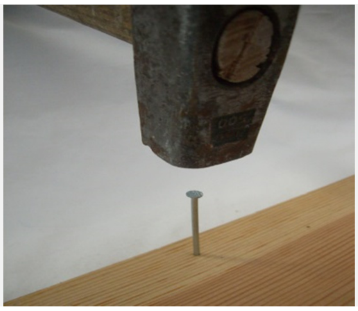

Image 17: The task of hammering a nail *let us see that the Complexity of an 'object' is independent of its 'Complexity Degree'.*

The total knowledge required [Absolute Complexity] to hammer a nail is low, while its predictability is almost complete.

Moreover, the already mentioned difficulty of measuring the Absolute Complexity of real objects requires that we differentiate two types of complexity measures that we may designate as Absolute and Conditional Complexity, which we characterize below.

2.3.1_ THE DIFFERENT MEASURES OF COMPLEXITY

2.3.1.1_ 'ABSOLUTE' COMPLEXITY MEASURES

The absolute measures review all the knowledge that can be drawn from any object and they have a major conceptual importance to *narrow* the proposed *complexity* conceptualization.

These measures are difficult to use when referring to real systems or phenomena as we can hardly ever be *reasonably certain* of having accounted all their meaningful information, but nevertheless they may be operational in *conceptual objects* whose information has been entirely generated with rules.

[1] See 2.4.1.4_COMPLEXITY OF SYMBOLIC OBJECTS; ART, LITERATURE, MUSIC…

ABSOLUTE COMPLEXITY

The **Absolute Complexity** of an object C[I] revises the total amount of *knowledge* that can be enunciated from the object's information and must meet two conditions:

An ***Inclusion* condition** that implies that *if an object J is strictly included in another object I, its Absolute Complexity is necessarily lower*, since I contains J [i.e.: includes its complexity] plus other rules relating to information not included in J[2]:

$$\forall J \subset I: C[J] < C[I] \tag{13}$$

An ***Evolution* condition** that affects only to *Adaptive Systems* and refers to their ever increasing Absolute Complexity.

$$T2 \gg T1 \rightarrow \forall I \in AS: C[I]^{T2} > C[I]^{T1} \tag{14}$$

Therefore, Absolute Complexity C[I] allows to compare the evolution level of any type of systems [even if they belong to very different classes]; **if C [I]> C [J] then I is a more *evolved* system than J.**

However, Absolute Complexity incorporates two issues that reduce its *operativeness*:

- It includes the complexity of all types of objects contained in the revised object, and therefore its measurement usually requires a great effort [except in quite small size objects].

$$C[I] \sim \sum_{\forall J \in I} C[J] \tag{15}$$

- We can never be completely sure of having detected all regularities in an object[3].

And this allows us to state that the *Absolute Complexity is not an operational concept, except perhaps in very low complexity 'conceptual' objects or systems.*

ABSOLUTE COMPLEXITY DEGREE

This measure $C[I]_{\%}$ refers to the total percentage of the object's information that can be generated by 'rules'; so in terms of communication theory equates its 'Redundancy Degree'.

$$\forall I \in \Omega: C[I]_{\%} \sim \frac{R[I]}{H[I]_{max}} \leftrightarrow 1 - \frac{H[I]}{H[I]_{max}} \tag{16}$$

---

[2] The statement can be easily proved, since I includes the knowledge contained in J and at least one more knowledge rule; which defines 'J inclusion or belonging to I'.

[3] 'We do not know what we do not know' and therefore it is not possible to know when we have enunciated all the knowledge rules that can be stated referred to an 'object'.

The Absolute Complexity Degree is a measure of maximum compression rate and predictability of any information source.

2.3.1.2_CONDITIONAL COMPLEXITY MEASURES

'Conditional Complexity measures only account the knowledge enunciable in relation to an object that has certain *meaning* X [or which is coincident with a concept X], which is set as a *condition* for the complexity measuring.

In terms of communication it equates selecting a 'known' message or concept X among those possible in an object [if the concept is not possible in an object, then the measuring would be pointless as it would necessarily be 'zero'] and measuring the extent to what the object's meaning corresponds to such message X':

- If the object totally matches such known message X the information needed to understand it from that perspective will be minimal [as we already know X]
- If the object does not match at all any possible X [any possibly 'known' message], the information needed to understand it will be its full description.

From all knowledge rules enunciable from the objects information only those 'relevant' to 'concept' X shall be accounted. Concept 'X' becomes equivalent to an 'analytical perspective', and Conditional Complexity measures are greatly operational because when reviewing 'objects' we usually want to do it from a certain perspective X[4].

Image 18: Map of the world. *Kolmogórov [1965] proposes that when we look at a map searching for geographic information, then the information regarding the structure of the paper or the ink, are not relevant although it is also information.*

In a measure of the "absolute" complexity of the object they should be accounted, but in a measure of its "conditional" complexity, it will not always be necessary; it will depend on the chosen "condition".

But also Conditional Complexity measures have great importance because **except for some conceptual objects, Absolute Complexity cannot be measured; Conditional Complexity measures allow us to work with objects information, restraining the part of their information that needs to be reviewed to that relevant for concept X.**

The choice of X concept will therefore be the most important in determining the nature and extent of the Conditional Complexity of objects. Different concepts X provide different complexity measures, and we can interpret them in relation to the previously proposed perspectives as[5]:

[4] While Conditional complexity is equivalent to choosing a particular analytical perspective, Absolute Complexity is somehow equivalent to reviewing objects from 'any possible perspective'. Therefore, Conditional Complexity measures are in general more interesting because usually when we review an object we do it for a specific purpose, not for any possible purpose.

[5] Conditional Complexity measures require choosing a *concept* X, which can be interpreted both as a *meaning*, *emergent property* or *class*. Meaning, organization of a class and emergent property become equivalent terms for measuring Conditional Complexity.

- From a systemic perspective, we measure the amount of…
  - … Structure of a system 'I' that matches the organization of a class X.
  - … A property X that emerges in a system I.
- From an information and meaning perspective, they measure the amount of…
  - … That a meaning X emerges from the source information.
  - … Meaning of a source I that matches a message X chosen by an observer.

And this last statement leads us to the importance of a formulation from Communication Theory that we review next: the Mutual Information between two sources.

MUTUAL INFORMATION BETWEEN TWO SOURCES

The Mutual Information between two sources can be conceptualized as "the average reduction of uncertainty of [a source] due to knowledge of another" [Crutchfield & Feldman, 2001:5] and its formula is:

Mutual Information

$$I[I;X] = H[X] - H_X[I] \qquad (17)$$

H[X] is the maximum Entropy of X; i.e.: the maximum amount of ignorance that we may have about X [which therefore coincides with the maximum amount of information that we can acquire in relation to X] and its value depends on the range of possible symbols in X information/description.

$H_x[I]$ is the Conditional Entropy of I which is a measure of "how uncertain we are on I when we know X" [Shannon 1949:12], and its formula is:

Conditional Entropy

$$H_X[I] = -\sum_{i,j} p(i,j) * log_2 \, p_i(j) \qquad (18)$$

CONDITIONAL COMPLEXITY

Conditional Complexity $C_X[I]$ measures the amount to which a given concept X emerges in a given object I[6] and its calculation requires two issues:

- Calculating the amount of I's information that matches concept X
- Verify the conditions that could cause X to be false referred to I[7].

This requires adapting the formula of the Mutual Information in the first place, in order to provide us a measure of the amount of a concept X that is present in the object I:

$$C_X[I] \sim f\left[I[I,x]\right] = f\left[H[x] - H_X[I]\right] \qquad (19)$$

Where *f* is a function yet to be determined

[6] The approach is largely consistent with *the Relative Complexity* proposed by Kolmogorov [1965:4] as a measure of "the quantity of information conveyed by an individual object x about an individual object I" [see ALGORITHMIC INFORMATION THEORY]

[7] We approach the fact that when we express it as a normalized measure [i.e.: as a measure of Complexity Degree] it will be possible to relate it with Fuzzy Logic [See 2.4_ COMPLEXITY, KNOWLEDGE AND LOGIC]

Given a concept X and an object I; the relevant symbols for X are those that determine the object's *meaning*; i.e.: those to which X 'implicitly' assigns a value and therefore can make X referred to I to be 'true', 'partially true' or 'false'.

And if we consider X as a string with N relevant symbols, there is an upper bound to the maximum amount of information that I can convey regarding X that is reached when all the information of X is present in I, i.e.:

$$C_x[I]_{max} \sim I[I,x]_{max} = N^8 \quad (20)$$

However, information associated with each relevant symbol of X can often be not totally but partially present in I and it becomes necessary to propose a formulation that allows us to determine $C_x[I]$ in those situations, i.e.:

$$C_x[I] \sim f\left[\sum_{i=1}^{N} f_x[i]\right] \quad (21)$$

Being '$f_x[i]$' the mathematical functions that measure the amount of information regarding each relevant symbol of X that is present in I; *f* is the function that transforms all those functions in a global measure of the amount concept X emerges in I.

Additionally, it will be necessary to model the factors that could cause X to be false referred to I, even if part of the information expected in X is in the same I.

Further developing both issues requires that we first review in detail some general concepts, as we will carry out later in the text [See Chapter 3].

CONDITIONAL COMPLEXITY DEGREE

The Conditional Complexity Degree $C_X[I]_{\%}$, measures the degree to which a concept X emerges in an object I. It is a *normalization of Conditional Complexity in relation to its maximum possible value*[9]:

$$\forall I, x \in \Omega: C_x[I]_{\%} \cong \frac{C_x[I]}{C_x[I]_{max}} \quad (22)$$

This implies that measuring *Complexity Degree* requires establishing a maximum value $C_x[I]_{max}$[10], something that is not possible for all X concepts [however, it is for the majority of them].

The *Complexity Degree* is probably the most interesting measure for systems modeling; since X can measure any concept [or property] some of which will be of great interest[11].

[8] The maximum Conditional Complexity of an object is limited by the maximum information content of the message or *concept* in relation to which it is assessed. The greater the number of *relevant symbols* associated with the concept, the greater the total information that the object can convey about the concept, and therefore its *Conditional Complexity*.

[9] For better understanding, from now on we designate is just as *Complexity Degree*.

[10] Being totally fair, we do not need to determine $C_x[I]_{max}$, but a 'relative' maximum that allows us to compare objects.

Conditional Complexity measures are largely based on the *Mutual Information* between two objects I[I;x] formulation, and as a consequence they inherit two interesting qualities of such formula[12]:

- If known X we are more certain on 'I', then 'I' conveys information about X, and therefore I[I,x]>0, which in terms of Complexity we can translate as if known X we are more certain on I then $C_x[I]>0$

$$C_x[I] > 0 \leftrightarrow known\ X\ we\ are\ more\ certain\ about\ I \quad (23)$$

- If X and I are independent; H[X] is equal to zero or H[I] is equal to zero, then I[I,x]=0; that we translate as if X and I are independent, C[X]=0 or C[I]=0, then $C_x[I]=0$

$$C_x[I] = 0 \leftrightarrow [X\ \cap I = 0] \vee [C[X] = 0] \vee [C[I] = 0] \quad (24)$$

2.3.1.3_ COHERENCE OF ABSOLUTE AND RELATIVE MEASURES

The distinction of four *complexity measures* is a tool that allows us to more easily review certain issues, but it also must be noted that all of them can be explained as a unique perspective; they all correspond to a single Complexity conceptualization.

Complexity Degree measurements can be understood as a *normalization* between 0 and 1 of *Conditional Complexity* measures [which does not involve conceptual change].

$$C_x[I]_{\%} = \frac{C_x[I]}{C_x[I]_{max}} \quad (25)$$

And *Absolute measures* can be understood as *Conditional Complexity* measures in relation to two specific X concepts:

- Absolute Complexity can be considered as a particular case of Conditional Complexity when the chosen concept is knowledge.

$$x = "knowledge" \leftrightarrow C[I] = C_x[I] \quad (26)$$

- Absolute Complexity Degree is a particular case of Conditional Complexity Degree when the chosen concept is predictable.

$$x = "predictable" \leftrightarrow C[I]_{\%} = C_x[I]_{\%} \quad (27)$$

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[11] For instance, if X is "optimal organization", $C_X[I]_{\%}$ will be measuring the "Degree to which a system's structure is optimal", which is a way to characterize/measure systems sustainability Degree [Alvira, 2014a].

[12] The formula of the common information has other properties such as symmetry, i.e.: I[I,x]=I[x,I], which is not share by the Conditional Complexity since X is a concept and I is an object [sometimes 'I' can be a concept, but usually it is not]. Therefore, usually they are not interchangeable. In this sense we can interpret Frege's statement [1892b: 183] "An equation is reversible; an object's falling under a concept is an irreversible equation".

We have previously described the Conditional Complexity Measures as special cases of the Absolute Complexity measures, but now we see that we can look it the other way around; Absolute Complexity measures can be considered special cases of Conditional Complexity measures.

There is a recursiveness between the two types of measures that allow us to state that the same *complexity conceptualization* underlies the four of them. And the interest of differentiating four Complexity measures is merely *expository*, as it will allow us to review and understand more clearly some of the issues that we propose/review in this theory.

2.3.2_ REVIEW OF SOME EXISTING CONCEPTUALIZATIONS AND FORMULATIONS

There are a considerably high number of different approaches and proposals for measuring complexity some of them we review next dividing them into two groups:

First, we review several proposals that have emerged in the context of the so-called **Complexity Sciences** which share two common characteristics:

- They propose mathematical formulations that *measure information*.
- They can be conceptualized as *one emergency level complexity* modelings.

Secondly, we review a **non-scientific proposal;** measuring the *complexity of literary texts* that has quite different characteristics to those corresponding to the scientific formulations, whose review allows us to introduce several additional interesting questions:

- Mathematical aspects lose weight against the interaction among a variety of factors that include subjective issues.
- It can be conceptualized as a *several emergency levels complexity* modeling.

Let us review both types of proposals

2.3.2.1_ COMPLEXITY MEASURING IN COMPLEXITY SCIENCES

Formulations proposed from the Complexity Sciences build on information measurement *as a mean for complexity measuring*, and it becomes necessary to review an alternative proposal to Communication Theory for measuring information: *Algorithmic Information Theory.*

When Shannon proposes its Entropy formulation as a **measure of the information conveyed by a message,** it becomes clear that it is a conceptualization of information from a very specific perspective [a communication], and two 'objections' are raised against that proposal:

- Information measuring depends on the context in which communication occurs; it measures the un-likelihood of receiving a message from a set of possible messages, and if this set varies, the amount of information provided by the same message may be different[13].
- It considers that the amount of information conveyed by a message is independent of its value as message, and a question arises... what happens when we receive an *improbable* and unknown message whose content is completely *useless*?[14]

And these two questions can be combined into one: is it possible to propose an invariant information measure for each possible message or object [that is independent of context] and that includes the value of the conveyed message?[15]

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[13] The Entropy formula measures the amount of information transmitted according to our uncertainty related to its reception, which will be greater the greater the number of possible messages is. From this perspective, if we receive a message whose content we know, and we know that we are going to receive it, it will not be "reporting" us anything; while if we receive a message that we do not know, it will be "informing" us of all its content.

[14] Informing someone of something that is useless to him/her... Can be considered to be providing him/her of any information?

[15] We can interpret this concern regarding the 'value of the message' to be underlying Kolmogorov's assertion [1965] "what real meaning is there? ... How much information is contained in War and Peace? [...] It Should be noted that the broader problem of measuring the information connected with creative human endeavor is of the utmost significance"

And in an attempt to answer this question, Algorithmic Information Theory 'emerges'; which will be the origin for many complexity measures later developed.

ALGORITHMIC INFORMATION THEORY

This theory arises from the joint contribution of three authors who, looking for different goals, raised similar/complementary proposals [Solomonoff, Kolmogorov and Chaitín]. This allows us to focus on at least three different interpretations of this theory[16]:

- As a model for inductive inference or ability to infer future or *unknown* events from other past or *known* events [Solomonoff 1964].
- As an alternative to Shannon Entropy for measuring information [Kolmogorov 1965].
- As a measure of algorithmic randomness [non-computability] of information strings [Chaitín 1967].

These three interpretations and the question raised above, which this theory will try to answer with an invariant information measurement for each object that 'captures' its value as message, may be then considered to overlap.

While Shannon entropy measures the information as the number of bits required to distinguish a received message from the other messages that may have been sent/received; Kolmogórov seeks developing a measure of information, independent from the context in which a message is transmitted; i.e.: **when the set of possible messages are all messages** [Li & Vitanyi 1997]

He therefore proposes the **Kolmogórov complexity or $K_U[I]$** whose definition is the length of the shortest program 'l[p]' that can generate an information string 'I' in a universal Turing machine 'U'[17].

Kolmogórov complexity

$$K_U[I] = min\{l[p]: U[p] = I\} \quad (28)$$

Kolmogorov complexity measures the *computability* [or ability to generate information strings using 'rules'] of objects, modeling them as numeric strings. It is therefore a measure of their *Algorithmic Information Content* [AIC][18].

[16] For an interesting abstract refer to Li & Vitanyi [1997] or ZAWADA [2009].

[17] In current terms, a Universal Turing Machine is a 'computer' [for a complete description see Turing 1936].

[18] Kolmogorov complexity is currently considered a measure of "algorithmic randomness" of information strings [not of their complexity]. "The works of Shakespeare have a lower AIC than a random gibberish of the same length that would typically be typed by the proverbial roomful of monkeys" [Gell-Mann, 1995b: 17], since $K_u[I]$ reaches its maximum value when a numeric string no 'regularities' and the shortest program to generate it is "to hand the computer a copy of x and say 'print this'" [Feldman & Crutchfield 1998]

For numeric strings of sufficient length, the greater length of the program -the maximum value of '$K_U[I]_{max}$'- is reached when a string I is *algorithmically random* and the shortest program to generate the string is to tell the computer 'print I':

Maximum Complexity

$$K_U[I]_{max} = min\{l[p]: p = "print\ I"\} \tag{29}$$

To assess the 'content' of the message the Relative Complexity between two objects $K_U[I,x]$ is proposed, which is the minimum length l[p] of the program p to obtain I from X[19], i.e.:

Relative Complexity

$$K_U[I,x] = min\{l[p]: U[p,x] = I\} \tag{30}$$

Kolmogorov Complexity review allows us to highlight several important issues:

- It starts a journey aiming to detect and measure objects' *meaningful information* [in contrast with Communication Theory whose main objective was to optimize the design/sizing of the communication channels]
- It suggests that in the majority of objects we only want to measure certain information [that has a certain meaning X] and we can measure it using Relative Complexity formula $K_U[I,x]$.
- both measures are related, since $K_U[I]$ can be characterized as a particular case of $K_U[I,x]$ when X is an empty string[20]

$$x = \emptyset \leftrightarrow K_U[I] = K_U[I,x] \tag{31}$$

It can be therefore considered as the first step towards measuring objects' meaning; ... and **while the transmitted information measured by Entropy maximizes for equiprobability between the variety of symbols that describe an object, their non-equiprobability[21] starts to emerge as a signal of 'meaning';** i.e.: that 'something interesting is happening' in an object.

The presence of patterns or 'regularities in information strings allows us to consider them as *meaningful information*[22].

[19] Kolmogórov [1965] states that we are usually interested in the Relative Complexity between an object I and another X; i.e.: measuring the amount of information an object I possess within a particular type of information X. NOTE: Kolmogórov uses the term *relative* in a different way to Shannon; it does not refer to a *normalized* but to a *conditioned* measure.

[20] Li & Vitanyi [cited in Faloutsos & Megalooikonomou 2007] designate *Kolmogorov Complexity* as *unconditional Complexity* and consider it as a special case of *Conditional Complexity* when X is an empty string, i.e.: it does not pose any *conditions*.

[21] It is equivalent to the presence of patterns or regularities. Solomonoff [1964:8] defines 'regularities' as "deviations of the relative frequencies of various symbols from the average"

[22] Solomonoff [1964] designates as "meaningless" those strings that when provided to a Turing machine do not generate any other string; and "meaningful" strings those which supplied to the Turing machine generate another string as a result. And proposes that "we shall regard an input as 'meaningful' if every symbol of the output takes only a finite number of operations to compute it' [1964: 15], identifying 'meaning' as 'computability' from other strings.

Review of Algorithmic Information Theory allows us to highlight some *key issues* to understand all subsequent complexity formulations and conceptualizations:

First, the importance of 'predictability' or the ability to generate 'objects' information by known rules which results equivalent to 'compress' it. Only information that can be compressed in some way can have a meaning[23].

A **'deterministic' perspective** arise from this question, seeking to measure complexity from the difficulty of generating or describing objects information, and shares a conceptual base with the Kolmogorov complexity combinatorial approach.

Secondly, objects with very high or low $K_U[I]$ values have commonly little interest. They are algorithmically random objects [e.g., some monkeys typing] or trivial objects [e.g., a person pressing the '1' key for an indefinite period].

The "most interesting" objects have intermediate values of $K_u[I]$; and here begins our *problematic* relationship with complexity; the objects that most interest us [e.g., biological systems] have *intermediate* $K_U[I]$ values; but ... what defines that *middle* point and why?[24]

A **'probabilistic' perspective** arises from this second question, which seeks to measure the amount of *organization* [understood as restrictions to possible states of the systems] using the Entropy formula and the probabilistic approach of Communication Theory.

Additionally, Kolmogorov complexity provides another important aspect, **generally we are not interested in measuring all but part of the information of an object, and we can measure by comparison between objects; the most interesting measures are the 'relative' measures.**

The abovementioned issues are another way to approach to our proposal of Conditional Complexity from information and meaning perspective, yet we still need to address two complementary issues:

- Go one step further in abstraction, considering that X can be any conceptual object, i.e.: *any linguistic term or construction with meaning*
- Outline a more flexible computability concept, reinterpreting it as a *sufficiently reliable reconstruction of such particular meaning*[25]

This last issue requires displacing the goal from the measurement of *the amount of X's information that is in I* to the measurement of *the amount of X's meaning that is 'true' referred to I*; displacement

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It equates to identify objects meaning with our ability to *generate* their information from patterns; which in turn comes to be equivalent to "understanding" them.

[23] I.e.: that we can "designate". The same linguistic terms that we use to describe objects meanings are a compression of a much larger amount of information on the referred objects.

[24] I.e.: they locate between strings computable with very simple algorithms and non-computable strings. And this relates to the extent to which is possible compressing the information string. The "interesting" strings are those that allow compression up to an 'intermediate' point, but … is it possible to define that point?. The answer to this question [difficult] will give rise to developments that attempt to measure objects' *Degree of Organization*.

[25] It implies that it will not be necessary to reconstruct with accuracy each object's information, but being able to describe/imagine it accurately enough. It refers to the object's general form; its *content as message* or *identity as system.*

that seems 'acceptable' referred to a reality that not always can be mathematically modeled nor computed.

This allows us to understand why there are many different proposals for modeling complexity; accepting that X can be any concept allows accepting different complexity modeling if concepts X being measured are different. Conditional Complexity may be referring to very different questions depending on the chosen concept X.

It is equivalent to transforming Complexity Theory from its widespread acceptance today as focused on the study of Organization [and to a lesser extent on Emergent phenomena] into a Theory that gives equal importance to Organization, Emergency, Meaning and Logic.

We assign 'organization' to any object in which any non-linear 'property', 'meaning' or 'concept' X emerges, and the extent to which that concept emerges equals its grade of fuzzy membership to the 'class' X[26].

Algorithmic Information Theory provides us many interesting issues, but it is important to indicate that *Kolmogorov complexity* cannot actually be considered a complexity measure:

- **It does not comply with the fundamental statement of Systems and Complexity Theories,** as it maximizes its value precisely when objects are *algorithmically random* and describing them requires describing each of their parts. And if to describe the object [the whole] we need to list all its information [each of its parts], the whole is equal to the sum of its parts and therefore it is not a complex object.
- **Complexity always involves nonlinearity** but algorithmically random objects Kolmogorov Complexity is a linear measure[27].

Another relevant issue about Kolmogorov complexity is that **the 'algorithmic computability' criterion gives equal weight to any regularity present in an object.** From the emergency perspective it is equivalent to considering one emergency level, and this issue will be repeated by almost all the proposals in the *sciences of complexity*.

The question of 'what the correct complexity measure is' becomes then 'pointless', because when measuring one emergency level, many different formulations are possible depending on the concept or complexity manifestation X measured [subsequently, we propose some 'minimum conditions' that all formulations should meet].

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[26] Additionally, Organizational rules that involve emergency, find certain parallelism with the syntactic rules governing the language [from which 'meaning' emerges'], as well as the rules of logic, which establish the validity of logical inference [from which 'truth values' emerge]

[27] This information non-linear transformation into meaning implies an upper bound to the maximum value of complexity, which shall always be a 'finite' amount. However, if the string is incomputable and information is infinite, $K_U[I]$ is infinite, and if we accept that "most of the numeric strings are algorithmically random" [Li & Vitanyi 1997] then in most situations $K_u[I]$ is a 'linear' information measurement.

Let us review other interesting complexity conceptualizations/formulations that we shall divide into three groups according to their main approach to objects' complexity:

- Their algorithmic information content.
- The possible systems states.
- The systems graphic analysis, which may be related to either of the above two approaches.

And the main interest of the review is to relate each of them to our proposal; showing coincidences with ideas already reviewed or collecting ideas that we incorporate in the present Theory, and checking if they can be conceptualized as special cases of this theory.

APPROACHES TO COMPLEXITY FROM OBJECTS' AIC

We herein review three proposals that use Turing machines and propose different 'developments' based on Algorithmic Information Theory, aiming to transform the amount of 'information' into a measure of 'meaningful information'[28]:

Depth [Bennett 1988] adopts a *dynamic approach*; proposing that the time needed by a universal Turing machine to generate an object from its minimum program is a measure of its *evolution*; the higher the logical depth the longer its evolution time[29]:

Depth $$D[I] = \{t[min[p]]: U[p] = I\} \quad (32)$$

Where t[p] is the runtime of program 'p'

In addition, the author proposes the *Relative Depth* of a string I relative to X at a significance level s $D_S[I/x]$ as "the least time required to compute I from X by a program that is s-incompressible relative to X" [Bennett, 1998:17]:

Relative Depth $$D_S[I/x] = min\{t[p,x]: [l[p] - l[(p/x)^*] < s] \wedge [U[p,x] = I]\} \quad (33)$$

Being 'p' a program that can generate 'I' from 'x'; 'p*' the minimum program; and 's' a limit that 'p' cannot be compressed in relation to 'x' , i.e. [(p/x)*]

Sophistication [Atlan & Koppel 1987] proposes that the size of the most concise description of the structure or regularities of an object is a measure of the *amount of planning* necessary to create the object:

Sophistication $$SOPH_s[I] = min\{l[p]: [\exists D: l[p] + l[D] \leq AIC[I] + s]\} \quad (34)$$

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[28] We can interpret this way Koppel's statement [1987:1087] that we generally are interested in objects' meaningful complexity, that can be low even if its Kolmogórov complexity is large

[29] However, a challenge to this approach ['construction' time as a measure of evolution] will be objects that include iterative statements [e.g. fractals, math series ….], which can be difficult to model from this perspective [something suggested by Gell-Mann, 1995b]

Being ‘I’ a string of information; 'l[p]' the length of a program ‘p’; 'D' a dataset; AIC[I] the Algorithmic Information Content of ‘I’; and 'p,D' a minimal description of ‘I’ in s, i.e.:

$$I[p, D] = l[p] + l[D] \leq AIC[I] + s \quad (35)$$

Effective Complexity [Gell-Mann & Lloyd, 2003] can be considered a development of *Sophistication* and is defined as “the length of a highly compressed description of the regularities” [of an object] and is calculated as:

Effective Complexity

$$K[E] = AIC[E] \quad (36)$$

Where E is the set of N strings 'r' describing each of the regularities in object I, and AIC the Algorithmic Information Content of E.

The authors of *Effective Complexity* refer to several issues that we have reviewed in our proposal:

- They suggest that Complexity involves some subjectivity; i.e.: it depends partly on the observer's point of view.
- They propose that its usefulness is mainly for object’s comparison.
- They refer to a *regularity* concept very similar to the one that we herein propose.

These three proposals raise several similarities due to their common origin in Algorithmic Information Theory, which leads them to propose measures similar to K_U[I] and K_U[I,x], allowing us to establish also certain parallelism with our *Absolute/Conditional Complexity* proposals:

- *Depth* and *Sophistication* allow a comparison on both levels, although its complexity metrics are different
- *Effective Complexity* may be located in an intermediate position between the two, seeking to measure all regularities but in a certain description *level of detail.*

They build on a *computational* approach using Turing machines to achieve *objectivity*, but in turn introduce some limitations:

- They only allow to model one emergency level complexity.
- They inherit from K_U[I] the impossibility of being totally certain in many situations of having accounted every *regularity* in an object; *indeterminacy* that will also incorporate our *Absolute Complexity* proposal.
- It is not possible to set a maximum value and therefore these measures cannot be expressed as Complexity Degree [an issue incorporated by all proposals based on AIC].

Furthermore, the Turing machines measures objects’ information based on *regularities’ descriptions lengths*, but many objects [both real and conceptual] do not allow modelling as numeric strings all their meanings and the description length of their regularities will not necessarily keep a relationship with their complexity[30].

And our proposal will mean a fundamental change in relation to the last issue, suggesting that the amount of complexity provided by each *regularity* or *rule* depends on its significance level, and within the same significance level all rules will provide the same *amount of complexity* to any object.

APPROACHES TO COMPLEXITY FROM THE POSSIBLE STATES OF SYSTEMS

These approaches are especially targeted to Adaptive Systems analysis [although there are also purely *thermodynamic* interpretations], understanding their complexity as a measure of their *amount of organization*, which is revised based on their potential states on their phase spaces, using Entropy formula.

The justification lies in *two complementary interpretations of the concept of organization*:

First, **organization is considered to be built by the relationships between elements of the system and their corresponding constraints as to per their possible arrangements [current and/or future][31].** A relationship is established between *organization* and *certainty*, as the latter emerges as a consequence of the existence of organization.

As a system reduces its possible configurations [increases its organization] our certainty about its state at a given time increases, which may materialize as:

- Certainty about its future state; once we know the system's current state not all future states are equally probable and some are not possible. There are *attractors or ergodic behaviors*[32].
- Certainty about its microscopic state; known its global state [compressed information] we can approximately reproduce its microscopic state [detailed or uncompressed information][33].

Both types of certainty can be assessed from the *possible states* of the system on its phase space and the amount of *constraints/possibilities* in such space can be considered a measure of its *order or organization*.

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[30] Both issues are further developed later [see COMPLEXITY OF LITERARY TEXTS]

[31] Von Foerster 1960. The author proposes 'Self-Organizing Systems' [i.e.: Adaptive Systems] as those which order [number of constraints] increases over time. This can be related to the herein proposed concepts of Absolute Complexity and Evolution; considering each constraint as a 'knowledge rule'.

[32] I.e.: not all points of system's phase space have equal probability. This relates to system's predictability.

[33] It relates to the concept of Entropy in Thermodynamics: "The Entropy of a thermodynamic system is a measure of the degree of ignorance of a person whose sole knowledge about its microstate consists of the values of the macroscopic quantities x_i which define its thermodynamic state" [Jaynes 1978:28]

And secondly, **systems' organization is considered as *that* which opposes to the second law of thermodynamics,** which endurance requires negative entropy, and therefore can be measured using a variety of entropy formulations[34].

Thus we arrive to a relationship between Organization and Entropy based on two interpretations of the last; [*thermodynamic* and *informational* interpretations]:

- From the information perspective, it provides a measure of uncertainty [hence certainty] about the system[35]
- From the thermodynamic perspective, it provides a measure of system's distance to thermal equilibrium.

Let us review two different proposals that we may include in this type of approach:

Foerster [1960] proposes **measuring the 'order' of a system** as the redundancy on the information of its description in its phase space, expressed in relative terms[36]:

Redundancy $$R[I] = H[I]_{max} - H[I] \quad (37)$$

Relative Redundancy $$R[I]_{\%} = 1 - \frac{H}{H_{max}} \quad (38)$$

The author suggests that *Self Organizing Systems* can increase their amount of *order* reducing their amount of *disorder* or increasing their number of different elements and proposes to measure the *maximum order* of a system as:

Maximum Entropy $$H[I] = Z * ln[e * n] \quad (39)$$

Where 'n' is the average number of elements of the system; Z the number of 'cells' in which the phase space is divided [each containing a sufficient number of elements 'n' of the system].

And for a system which elements can connect 'two to two', the maximum potential order results:

Maximum Order $$R[I]_{\%} = 1 - \frac{Z * ln\left[\frac{e * n}{2}\right]}{Z * ln[e * n]} \quad (40)$$

[34] We can interpret in this sense Lovelock's statement [1979:31-32] "wherever we find a highly improbable molecular assembly -a distribution which is sufficiently different from the background to be recognizable as an entity [or that] would require the expenditure of energy for its assembly from the background of molecules at equilibrium- it is probably life [an Adaptive System] or one of its products, […] and the extent to which it is different or improbable is a measure of the entropy reduction".

[35] According to Gibbs [cited in Jaynes 1978:18] Entropy refers to "the probability that the phase of a system falls within certain limits at a certain time". And Jaynes continues: "Gibbs recognized that in fact we are only describing our imperfect knowledge about a single system"

[36] Foerster [1960:7] considers redundancy as a measure of system's relative order, and proposes that "order has a relative connotation rather than an absolute one; namely, with respect to the maximum disorder the elements of the set may be able to display".

Although Foerster does not use the word *complexity*, his proposal for measuring *order* can be conceptualized as a measure of *Complexity Degree* [in certain way, a *pioneer* one] in which the concept measured is 'certainty about the microscopic state of the system'[37]

Subsequent approaches to the concept of organization consider that systems' structure [organization] 'maximizes' at an intermediate position between perfectly ordered [trivial] and disordered [random] states and therefore propose formulations that provide a zero value in both limit states, while maximizing for intermediate situations[38].

Another proposal for complexity measuring is **LCM Complexity** which combines Entropy [as a measure of the information content of the system] with a measure of system's *disequilibrium* D[I] understood as distance to equiprobability situation[39]:

LCM Complexity

$$C_{LMC}[I] = H[I] * D[I] \tag{41}$$

Disequilibrium

$$D[I] = \sum_{i=1}^{n} [p_i - \overline{p_i}]^2 \tag{42}$$

And the Entropy for a system 'I' with 'n' possible states it can be calculated as:

Entropy

$$H[I] = -k \sum_{i=1}^{n} p_i * log_2 \, p_i \tag{43}$$

Where 'k' is a real positive constant and p_i the probabilities[40] associated with each 'n' possible system state.

The maximum value is reached in the 'balance' situation, when:

$$H[I]_{max} = k * log_2 \, n \tag{44}$$

Thus we see that both proposals can be conceptualized within the proposed framework; the first as a measure of *Complexity Degree* conditioned to the concept of *certainty* while the second to the concept of *organization*.

It should be noted that while approaches to Complexity based on AIC bring us closer to *Absolute* and *Conditional Complexity* measures, approaches based on Entropy bring us closer to *Complexity Degree* formulations [normalized measures].

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[37] "If the elements of the system are arranged such that, given one element, the position of all other elements are determined, the entropy –or degree of uncertainty- vanishes and redundancy becomes unity, indicating perfect order" [Von Foerster 1960:7].

[38] See Crutchfield & Feldman [1997]. Moreover, there is an obvious parallel between this issue and the aforementioned Kolmogorov complexity which minimum and maximum values we identify respectively with 'simple' [trivial] or 'unstructured' structures [random] objects.

[39] Lopez Et Al 2010. The designation as 'LCM Complexity' is an acronym of the initials of their last names: Lopez, Colbet and Mancini.

[40] For Jaynes [1978:25] probabilities do not refer to 'stable frequencies' [Objective Probability] but to 'knowledge states' [Subjective Probability], what he considers to be coincident with the original Laplace's definition. This issue matches with our approach in A-V.4.3_ INTERPRETATION AS PROBABILITY SYSTEM OR ASSIGNMENT

In fact, if they are not *normalized* they imply an *undecidability* issue; in the majority of cases it is not possible to *decide* the number of possible states of a system in its phase space.

APPROACHES TO COMPLEXITY BASED ON OBJECTS REPRESENTATION

There are numerous proposals designed for measuring objects' complexity from their graphical representation, which we review following the two previously explained approaches[41]:

A number of proposals relate to the ***deterministic* approach of Algorithmic Information Theory**, and use Graph Theory and combinatorial analysis, measuring the number of vertex and parts and relationships among vertex or adjacencies[42].

These formulations consider that complexity *emerges* as a result of the difficulty of generating the objects, which is equivalent to the difficulty of understanding them.

This approach has many applications in *Systems Architecture*, which aim is optimizing the design of computer architectures; considering that "the simpler a system is, the easier to design, implement and maintain" or in terms of money, the lower its cost in any of these phases [Kinnunen, 2006:15][43].

McCabe [1976] states that software complexity depends on its decision-making structure; the number of basic 'linearly independent' paths that, when taken in combination, can generate every possible path in a program, and proposes **Cyclomatic Complexity or V [I]:**

Cyclomatic Complexity

$$V[I] = e - n + p \qquad (45)$$

Being 'e' the number of sides, 'n' the number of vertex, and 'p' the number of connected elements

In many cases the abovementioned formula can be simplified by directly measuring the number of program 'conditions', namely:

Cyclomatic Complexity

$$V[I] = e - n + p \qquad (46)$$

Being π the number of 'conditions' in the program I[44]

And suggests that programs should be limited not by its physical size but setting a maximum value to their Cyclomatic Complexity.

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[41] The division into two groups coincides with 'deterministic' and 'probabilistic' perspectives suggested by Mowshowitz & Dehmer [2012], having included some of the issues identified by them.

[42] To review an analysis of different 'network complexity measures' from the combinatorial perspective refer to Bonchev & Buck [2005] or Raghuraj & Lakshminarayanan [2006].

[43] Complex architectures are not only difficult to create, but also errors increase during implementation because they are also difficult to understand. It is noteworthy highlighting the meaning of *simple* in this sentence, which differs from its etymological meaning. In the etymological sense, the terms *simple* and *system* are incompatible.

[44] McCABE [1976:314]. The author indicates that sometimes a "statement" can summarize several formulas, and therefore it is necessary counting the number of 'conditions'. A computer instruction like "IF C1 AND C2" should be counted as two different 'conditions'; generating greater complexity than if they were a single condition [McCABE 1976: 315]

Complexity measures of Systems Architecture bring something very interested compared to the previously described approaches; they **displace Complexity measurement from *measuring the regularities' description length* to *counting the number of regularities or rules*[45],** coincident with the criteria that we herein propose.

And another group of proposals relate to the ***probabilistic* approach of Communication Theory**, using the Entropy's formula to measure the *information stored in a network.*

These proposals consider that complexity emerges as a result of systems' information content [which in turn is considered to be their *potential*], and some propose measuring the extent to which a structure is 'good' by comparing it with a *type* organization using the mutual information formula.

Approaches to complexity from object's graphical representation reiterate issues discussed above; *deterministic measures* may be characterized as *Conditional Complexity* measures, designed to measure the *difficulty of generating object's information*, and *probabilistic measures* may be characterized as *Complexity Degree* measures, designed to measure both *description difficulty* and *degree of organization*.

And they bring two issues that we incorporate in this proposal:

- *Deterministic measures* displace the focus from measuring *amount of information* to counting the *number of different rules*.
- *Probabilistic measures* bring us to the possibility of measuring *the degree to which the organization of a structure is optimal by comparison with an organization considered as 'optimal'*.

In addition, their review allows us to state that different objects or purposes require different complexity measures and that the utility of complexity measures will most often be comparative [what in turns will require the 'compared' objects to be similar].

2.3.2.2_ MEASURING COMPLEXITY IN 'NON-SCIENTIFIC' FIELDS

At the beginning of the text we raised an issue that we left unanswered: When we say that a system or a task is *complex* are we talking about the same or different things? ... And although it would be accepted that they are different issues; if we find that we are talking about the same it will be much more interesting for our aim of defining *unified theory*.

Therefore, we review in depth a 'non-scientific' example; the *Complexity of literary texts'*[46], whose analysis is going to bring us some interesting issues.

[45] Kazman & Burth [1998] propose that the complexity of the architecture of a system can be measured by the number of patterns required to cover this architecture. The lower the number of patterns required, the lower the complexity of the program.

[46] We do not mean that complexity of literary text is not dealt with in a scientific manner, but more that it is done in a way that greatly differs from the *Sciences of Complexity*, partly motivated by a different definition of complexity [see A0.0_THE DEFINITION OF COMPLEX AND COMPLEXITY]

COMPLEXITY OF LITERARY TEXTS

The *Complexity of a text* usually refers to its *difficulty to be read and understood by a reader*. Therefore it may be interpreted as a measure of the complexity of performing a task, since what is being assessed is the *difficulty of [the task] of reading and understanding a text*[47].

It can be understood as a type of Conditional Complexity that must be revised *on several emergency levels*. The *difficulty emerges from the interaction of several aspects that are -at least in large part- emergent properties hierarchically organized*[48], and that we group in three categories:

- Quantitative aspects of text [total length, length of sentences, ...]
- Qualitative aspects of text [design, significance levels, ...]
- Aspects of the reader [level of subject knowledge and vocabulary used, ...]

Let us review some issues related to each of the three above categories:

Regarding the **quantitative aspects,** two texts may have similar level of *comprehension difficulty* even if their lengths are different [or have different 'comprehension difficulty' being their lengths equal], and this allows us to state the absence of unequivocal relationship between difficulty and length of texts.

$$\left[l[A] \neq l[B] \nLeftrightarrow C_x[A] \neq C_x[B]\right] \leftrightarrow C_x[A] \neq f\left[l[A]\right] \qquad (47)$$

Being l[A] the length of a text A and *f* any possible function

As for the **qualitative aspects,** its importance is that they include many issues 'no explicit' in the symbols that influence the comprehension difficulty [e.g. the use of 'double meanings'; metaphors; contradictory statements; reading 'between the lines'; …]; and therefore **neither the entropy nor the AIC of the descriptions or linguistic constructions can deliver any information us.**

$$C_x[A] \neq f\left[H[A]\right] \wedge C_x[A] \neq f\left[AIC[A]\right] \qquad (48)$$

Being *f* any possible function and 'A' a text or meaningful linguistic expression.

Sometimes the *comprehension difficulty of a text* may relate to its length, entropy or AIC; but it will not be possible to establish a clear link between them.

The example allows us to clearly review the **difference between the concepts of Absolute and Conditional Complexity:**

- The Absolute Complexity of a text does not relate to the length of the descriptions[49] but with the amount of rules or units of information with different meanings and will always increase its value; the higher amount of different statements incorporated the greater it is.

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[47] HESS and Biggam [2004] conceptualize the complexity of a text as its "degree of challenge [to a reader]".

[48] For example, Hess & Biggam [2004:2] proposal includes: difficulty of words and language structure [vocabulary, sentence types, ..]; text structure [description, sequence, timing, ...]; style of discourse [satire, humor, ...]; gender and text features; knowledge and familiarity with the content required to the reader [historical, geographical, literary, etc. ..]; required level of reasoning [sophistication of themes and ideas, use of abstract metaphors, ...]; format and text design [organization and text design, graphics, ..] extension [length] of the text.

- The Conditional Complexity to the concept 'comprehension difficulty' measures the degree to which the 'difficult to comprehend' meaning can be applied to the text, and it will not necessary relate neither to its length nor to its amount of statements.

There is a *logical independence* between the two measures, which may even show 'inverse correlations'. There may be situations in which a reduction of a text's Absolute Complexity is accompanied by an increase in its *comprehension difficulty*.

Image 19: An Encyclopedia *includes a great amount of knowledge [Absolute Complexity] while having a moderate 'comprehension difficulty' [Conditional Complexity to concept 'difficulty']*

Generating exactly every symbol of a text is evidently not a measure of its complexity as 'comprehension difficulty', and this supports our earlier statement that *measuring Conditional Complexity does not require to fully reconstruct an object's information but its meaning with sufficient approximation.*

And a question arises in relation to measuring issues not explicit in the symbols of a text. If from the statements in the text it is possible to infer other statements ... shall they be accounted in its *complexity*?

The answer in relation to the Conditional Complexity seems obviously positive; if a text incorporates non-explicit rules that the reader needs to detect to understand it [e.g. a text with many metaphors] its 'comprehension difficulty' increases. However, an affirmative answer in relation to the Absolute Complexity [something which is not so evident] would turn it into non-determinable in most situations[50].

And in relation to aspects of the reader we can also draw interesting issues. Comprehension difficulty of a text relates also to the *knowledge of the reader*; or in other words, it requires the existence of a reader and relates to its knowledge.

The *complexity of texts* necessarily shall be assessed in relation to its *potential readers*, and this allows us to understand that non-strictly systemic objects can be complex; they acquire the status of a

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[49] For example, the expression "we do not know what we do not know" has 23 characters while the expression "we cannot see what we cannot see" has 19 characters, but both express a unit or knowledge rule' [which in this case happens to be also coincident] and from the perspective of 'complexity' their knowledge content is the same.

[50] This issue will be further detailed in 5.2.4_ CONTRADICTION AND COMPLEXITY

system when a reader/observer appears, and in many cases it correspond to complexity with several emergency levels[51].

But the intervention of *readers* may lead to contradictory situations/statements. A text expressed in two different languages does not change its *meaning*, yet it has different *comprehension difficulty* for any reader who does not comprehend both languages with equal ease. **The *comprehension difficulty* depends partly on the *observer***

$$A \equiv B \nleftrightarrow C_x[A] = C_x[B] \qquad (49)$$

Where A and B are the same literary text expressed in two different languages[52].

Also, if the knowledge of two readers 1 and 2 is different, we could reach contradictory statements related to the complexity of the same text:

$$C[1] \neq C[2] \rightarrow C_x[A]_1 \neq C_x[A]_2 \qquad (50)$$

Being 1 and 2 two readers with different knowledge C, and A the same text.

The importance of this is that it raises the issue that **it may be impossible to establish complexity measures that are invariant for each object; since the complexity of an object may be different for each different observer.**

Reviewing this issue is going to require us to review the issues that allow us to assess the objectivity of statements that incorporate subjective aspects, leading us towards a review of Epistemological aspects.

Let us therefore review the relation between Complexity and knowledge.

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[51] The complexity of certain objects can only be understood in relation to a receiver or observer [eg symbolic systems as art, literature and music] and often correspond to Complexity with emergency levels because receivers often break-down or decompose hierarchically their information; equivalent to establishing emergency levels.

[52] An example can be a statement written both in natural and logical language, which can pose different comprehension difficulty for a reader despite the "statement" has not changed. Another example may be a proof of a geometry law as Thales Theorem *drawn* or *narrated with words.*

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