Abstraction and Structures in Energy

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Abstract:

Zero postulation and the principles of the Theory of Abstraction are used to study structures of energy inside a black hole, which is incredibly heavy and incredibly small. We chase the questions, how matter (with various structures) is formed from energy and the energy making up matter has to be in what orientation to form the matter that we see. We arrive at the fundamental model and the equations describing the formation of structure in energy.

Introduction:

The Theory of Abstraction and the principle of Zero Postulation describes everything that there 'is' in the universe (including spacetime) as an expression of energy. It explains directivity towards optimized solutions and cluster formation in the universe as inherent properties of the energy itself. It must also be able to describe the formation of various structures in energy. As such, it is of great interest for us to investigate how the matter that we see around us is formed from the various orientations of the structured formed in energy.

A theory that is able to describe the world in totality has to keep the number of basic postulates it depends upon to zero or near zero. Reductionism hits a dead end in this regard. On the other hand, abstraction as the starting point of building up a theory may be seen to be of fitting use. It would be much more than a new way of tackling the problem. Even abstract postulates do away with the shackles that bind our theories into the system and bar them from being total descriptions of the system. The abstraction we

are talking about here may be defined as, "Postulation of non-postulation" or, in other words, "A system of postulation that gives equal weights to all possible solutions inside the system and favors none of such solutions over others."

Abstraction automatically gives rise to optimized solutions within the universal set of all possible solutions, as has been shown in this book. It is these optimized solutions that make up and drive the non-abstract parts of the world, while the non-optimized solutions remain 'hidden' from the material world, inside the abstract world.

Starting from a basis of no postulation, we build our theory. As we go on piling up possibilities, we come to a similar basis for understanding the four non-contact forces of nature known till date. The difference in ranges of these forces is explained from this basis in this book. Zero postulation or abstraction as the basis of theory synthesis allows us to explore even imaginary and chaotic non-favored solutions as possibilities. With no postulation as the fundamental basis, we are thus able to pile up postulated results or favored results, but not the other way round. We keep describing such implications of abstraction in this book. We deal with the abstraction of observable parameters involved in a given system (quantum, relativistic, chaotic, non-chaotic) and formulate a similar basis of understanding them.

Let us consider the example of a three-point isolated system. Let the points be 'A', 'B' and 'C'. Let A and B be material points, whereas, C be situated anywhere on the straight line joining A and B. The material parts of both A and B tends to move in all possible directions. These possible directions include the directions towards each other. Thus, at point C, for obvious reasons, an additional effect will be felt due to the tendency of material to flow from A to B and from B to A, as compared to all other directions.

The points A, B and C being considered parts of an isolated system and all three points being assumed fundamentally similar (with the only difference that A and B contain material, while C is empty), the factors R and S must be equal.

Thus, we have:

$$\frac{F}{T} = c\frac{\lambda}{D}$$

D being considered the 'distance' between A and C and x the distance between A and B (say), the distance between B and C is $D - x$. This distance can be any length of any given dimensions (as determined by the scaling-ratio of observations) between two points in spacetime.

The effect on C due to the material-point A can thus be written as,

$$\frac{F_A}{T_A} = c\frac{\lambda_A}{x}$$

Similarly, the effect on the empty point C due to the material-point B is,

$$\frac{F_B}{T_B} = c\frac{\lambda_B}{D-x};$$

where F_A and F_B are the respective values of flows towards the point C due to A and B, respectively. T_A and T_B are the respective values of time and λ_A and λ_B are the respective values of the differences in concentrations of the concerned entity between A and B.

Substituting x in the above two equations, we have,

$$c\frac{T_A\lambda_A}{F_A} = D - c\frac{T_B\lambda_B}{F_B} \qquad \ldots(1)$$

Considering the points to be having equal factors, i.e., considering $\lambda_A = \lambda_B = \lambda$ (say), $F_A = F_B = F$ (say) and $T_A = T_B = T$ (say), equation (1) reduces to,

$$c\frac{T\lambda}{F} = D - c\frac{T\lambda}{F}$$

i.e.,

$$\frac{F}{T} = 2c\left(\frac{\lambda}{D}\right) \qquad \ldots(2)$$

Equation (2) describes fundamentally the effect (i.e., the flow F in time T) of two material-points having same factorial conditions regarding one or a number of entities. Considering a collection of such points and applying a statistical approach, the logistic equation for $\left(\frac{F}{T}\right)$ can be written as,

$$2c\left(\frac{\lambda}{D}\right)_{t+1} = 2Kc\left(\frac{\lambda}{D}\right)_t\left[1 - 2c\left(\frac{\lambda}{D}\right)_t\right]$$

i.e.,

$$\left(\frac{\lambda}{D}\right)_{t+1} = K\left(\frac{\lambda}{D}\right)_t \left[1 - 2c\left(\frac{\lambda}{D}\right)_t\right] \quad \ldots (3)$$

where K is a constant.

Also, the quadratic map can be written as,

$$2c\left(\frac{\lambda}{D}\right)_{t+1} = K - \left(2c\frac{\lambda}{D}\right)_t^2$$

i.e.,

$$2c\left(\frac{\lambda}{D}\right)_{t+1} = K - 4c^2\left(\frac{\lambda}{D}\right)_t^2 \quad \ldots (4)$$

All trajectories described by the quadratic map become asymptotic to $-\infty$ for

$$K < -0.25 \text{ and } K > 2$$

As we deal with the flow of a given material entity towards one given point or the effects on a given point, the expression for the attractor for each such point can be written as,

$$\left(2c\frac{\lambda}{D}\right)^* = \left(1 - \frac{1}{K}\right) \quad \ldots (5);$$

where $O < K < B$.

$\left(2c\frac{\lambda}{D}\right)^*$ is a point in the desired dimensional plot into which the trajectories seem to crowd. As we do not need to deal with more than one attractor or periodic point, the trajectories will tend to revisit only the attractor point concerned, to the desired level of accuracy of observations and calculations.

For $K \geq 3$, the trajectory behaviour becomes increasingly sensitive to the value of K. There are a few more points to be noted regarding the dependence of the trajectory behavior on the values of K:

1. For $K \leq 1$, the attractor is a fixed point and has a value 0.

2. For $1 < K < 3$, the attractor is a fixed point and its value is > 0 but < 0.667.

3. For $3 \leq K \leq 3.57$, period doubling occurs, with the attractor consisting of $2, 4, 8,$ etc., periodic points as K increases within that range.

4. For $3.57 < K \leq 4$, we have the region of chaos, where the attractor can be erratic (chaotic with infinitely many points) or stable.

For all calculations, the desired conditions may be placed at the attractor. A trajectory never gets completely and exactly all the way into an attractor though, but only approaches it asymptotically. In the region of chaos, we apply the method of searching for windows or zones of K-values for which iterations from any initial conditions will produce the periodic attractor, instead of a chaotic one. For the logistic equation(3), the most common such zone lies at $K \approx 3.83$ and for the quadratic map(4), at $K \approx 1.76$.

Inside A Black Hole:

A black hole may be considered to be a system, whose packing-density has a certain minimum value. If the black hole's mass is M, which is concentrated within a radius R then the minimum value of the ratio

$$\frac{M}{R} = \frac{c^2}{2G} \approx 10^{27} kg/m$$

Be it the large vastness of the universe or the delicate smallness of the sub-atomic world, by choosing a suitable constant scaling ratio for both, we may obtain their representations. These representations, following a certain constant scaling ratio, will be self-same. In the previous chapters, I have mentioned the chaotic behavior in the quantum world. Choosing suitable scaling ratios, we may turn the universe itself into such a chaotic quantum system, having its own necessary quantum states and trajectory behavior. In that case, the study of the universe reduces to the study of some sort of a quantum chaotic system.

On the other hand, choosing some other necessary scaling ratios, the atomic and the sub-atomic realm may be extended to become the universe itself, complete with its own macroscopic trajectory behavior. Instead of formulating different ways of looking at worlds of different sizes, if we adjust the way of viewing i.e., the scaling ratios in such a fashion that the representations of the worlds merge, we will be looking at representative worlds of study which are practically self-same. The Laws of Physical Transactions formulated in previous chapters may then be applied in order to study such self-same representations of the worlds of various scales. Unification of the ways of studying at different ranges of scaling may thus be achieved by suitable landscaping (adjusting different scales to a suitable scaling ratio, in order to make all the scales of study similar in size).

Further, a similar approach may be applied to study the Bose-Einstein Condensation. A certain critical packing density of the constituents of each world of a certain landscape must ensure a condensation of similar sort. The quantum states (or some similar states) of each such landscape will merge and give spikes for that critical scaling ratio in their respective representations.

The Abstraction Theory is applied in landscaping. A collection of objects may be made to be vast or meager depending upon the scale of observations. This idea may be developed to unite the worlds of the great vastness of the universe and the minuteness of the sub-atomic realm. Keeping constant a scaling ratio for both worlds, these may actually be converted into two self-same representatives with respect to scaling. The Laws of Physical Transactions are made use of to study Bose-Einstein condensation. As the packing density of concerned constituents increase to a certain critical value, there may be evolution of energy from the system.

Looking at a large enough part of the universe, we may draw an analogy to a system of scattered particles, in motion or rest, relative to each other. These particles may or may not be similar to each other, if we look at a given locality. Our idea, however, is that we can always represent even the whole of the universe on a piece of paper of our desired size. We can very well do the same with localities of sub-atomic sizes.

We may represent both the worlds, viz., the microscopic and the macroscopic, within any desired standard size. Theoretically, we are only to diminish the snaps of the universe and magnify the snaps of the microscopic world in order to put both into representations of a definite scaling-size. Looking at such a representation of the macroscopic world (due to the large number of constituents and the large distances separating them involved) we will find it to be a complex mixture of various kinds of particles. On the other hand, looking at such a representation of the microscopic world, (due to the small distances separating the constituents) it will be like the actual universe

itself, with various types of constituent parts involved. Such a representation of the microscopic and the macroscopic worlds will bring out hidden properties and behaviors of both worlds, as well as providing for a similar basis of studying them both.

Let us consider a given representation with fractal dimension D_F. The fractal dimension is purely geometrical, i.e., it only depends on the shape of the representation. A suitable probability measure $d\mu$, according to the particular phenomenon considered is assigned to the given representation. A coarse grained probability density, as the mass of the hypercube Λ_i of size l is defined as,

$$P_i(l) = \int_{\Lambda_i} d\mu\,(x)$$

where $i = 1,2,3, \dots, N(l)$.

The information dimension D_I is such that,

$$\sum_{i=1}^{N(l)} P_i \ln(P_i) \simeq D_I \ln(l) \qquad \dots(6);$$

where $D_I \leq D_F$.

The number of boxes containing the dominant contributions to the total mass and thus relevant part of the information, is,

$$N_R(l) \propto l^{-D_I} \qquad \dots(7).$$

For each box Λ_i, $D_I = D_F$ for a uniform distribution. When $D_I < D_F$, the measure itself may be called fractal since it is singular with respect to the uniform distribution,

$$P^* = \frac{1}{N(l)} \propto l^{D_F}$$

For each box Λ_i.Thus, $\frac{P_i}{P_i^*}$ can diverge in the limit of vanishing l.

Simulations of the mass-moment scaling yields,

$$\langle P_i(l)^q \rangle \equiv \sum_{i=1}^{N(l)} P_i(l)^{q+1} \propto l^{q.d_{q+1}} \quad \ldots (8).$$

The d_q are the Renyi dimensions which generalize the information dimension $D_I = d_1$ as well as the fractal dimension $D_F = d_0$. If the d_q's are not constant, anomalous scaling is to be employed and, as the order q varies, the amount of the difference $D_q - D_F$ gives a first rough measure of the heterogeneity of the probability distribution.

The moment generic observables A computed on scale l is such that,

$$\langle A(l)^q \rangle \propto l^{g(q)} \quad \ldots (9)$$

Anomalous scaling, i.e., a non-linear shape of the function $g(q)$ is the more common situation, where one does not require unnecessarily to consider only a finite number of scaling components. In some cases, one may observe strong time variations in the degree of chaoticity. This intermittency phenomenon involves an anomalous scaling with respect to time-dilations identifying the parameter e^{-t} with the parameter l used in spatial dialations. A measure of the degree of intermittency requires the introduction of infinite sets of exponents which are analogous to the Renyi dimensions and can be related to a multifractal structure given by the dynamical system in the functional trajectory space.

The Grassberger-Procaccia correlation dimension ν is defined by considering the scaling of the correlation integral,

$$C(l) = \lim_{M \to \infty} \frac{1}{M^2} \sum_i \sum_{j \neq i} \theta \left(l - \left| x_i - x_j \right| \right);$$

where θ is the Heaviside step function and $C(l)$ is the percentage of pairs (x_i, x_j) with distance $\left| x_i - x_j \right| \leq l$.

In the limit $l \to 0$,

$$C(l) \propto l^{\nu}.$$

In general,

$$\nu \leq D_F.$$

ν is a more relevant scaling index than D_F since it is related to the point probability distribution on the attractor, while D_F cannot take into account an eventual homogeneity in the visit frequencies.

Let us define the number of points in an F-dimensional spherical representation of the world, with radius l and centre at x_i as,

$$n_i(l) = \lim_{M\to\infty} \frac{1}{M-1} \sum_{j\neq i} \theta\,(l - |x_i - x_j|) \qquad \ldots(10).$$

We must introduce a whole set of generalized scaling exponents

$$\langle n(l)^q \rangle = \lim_{M\to\infty} \frac{1}{M} \sum_{i=1}^{M} n_i(l)^q \propto l^{\emptyset(q)}$$

where $\emptyset(1) = \nu$.

Considering a uniform partition of phase space into boxes of size l it is convenient to introduce the probability $P_K(l)$ that a point x_i falls into the K^{th} box. In this case, the moments of P_K can be estimated by summing up the boxes,

$$\langle p(l)^q \rangle = \sum_{K=1}^{N(l)} P_K(l)^{q+1} \propto l^{q.d_{q+1}} \qquad \ldots(11)$$

A moment of reflection shows

$$\emptyset\,(q)/q = d_{q+1}$$

because of the ergodicity $n_i(l) \sim P_K(l)$, if x_i belongs to the K^{th} box and since one can use either an 'ensemble' average (weighted sum over the boxes) or a 'temporal' average (sum of the time evolution $x(l)$).

The fractal dimension for $q = -1$ is,

$$D_F = d_0 = -\emptyset(-1)$$

while the correlation dimension is,

$$\nu = d_2 = \emptyset(1)$$

According to the Theory of Physical Abstraction, each point x should have the same singularity structure,

$$\Delta V_x(r) \propto r^h, h = \frac{1}{3} \quad \ldots (12)$$

In other words $\varepsilon(x)$ tends to be smoothly distributed in a region of R^3. The eddy turn-over time and the kinetic energy per unit mass at scale r are defined as,

$$t(r) \sim \frac{r}{\Delta V(r)} \quad \ldots (13)$$

and

$$E(r) \sim \Delta V(r)^2 \quad \ldots (14)$$

The transfer rate of energy per unit mass from the eddy at scale r to smaller eddies is then given by

$$\tilde{\varepsilon}(r) = \frac{E(r)}{t(r)} \sim \frac{\Delta V(r)^3}{r} \quad \ldots (15)$$

Since

$$\varepsilon_x(r) = \left(\frac{1}{r^3}\right) \int_{\Lambda_x(r)} \varepsilon(y) d^3y,$$

[$\Lambda_x(r)$ is a cube of edge r around x we have,

$$\int_{\Lambda_x(r)} \varepsilon(y) d^3y \sim r^3 \qquad \ldots (16)$$

$r \to 0$ means r in the initial range and the regions containing a large part of $\varepsilon(x)$ are a physical approximation of a fractal structure. In this β −model approach,

$$\int_{\Lambda_x(r)} \varepsilon(y) d^3y \propto \begin{cases} r^{D_F} & if\ x \in S \\ 0 & if\ x \notin S \end{cases}$$

in an equivalent way

$$\Delta V_x(r) \propto \begin{cases} r^h & if\ x \in S \\ 0 & if\ x \notin S; \end{cases}$$

where $h = (D_F - 2)/3$

At scale r, there is only a fraction,

$$r^{3-D_F} \propto \frac{r^{-D_F}}{r^{-3}}$$

occupied by active eddies.

The transfer energy from the eddy at scale l_n (active eddy) to the scale l_{n+1} is $\varepsilon_n \propto \frac{v_n^3}{l_n}$.

Since, the energy transfer rate is constant in the cascade process, for $\beta = 2^{D_F-3}$, we have,

$$\varepsilon_n = \beta \varepsilon_{n+1}, \frac{v_n^3}{l_n} = \beta \frac{v_{n+1}^3}{l_{n+1}} \qquad \ldots (17)$$

Iterating, we have,

$$v_n \propto l_n^{1/3}(l_n/l_0)^{\frac{D_F-3}{3}}$$

Each eddy at scale l_n is divided into eddies of scale l_{n+1} in such a way that the energy transfer for a fraction β of eddies increases by a factor $\frac{1}{\beta}$, while it becomes zero for the other ones.

In order to generalize the β-model, we have at scale l_n, N_n active eddies. Each eddy $l_n(k)$ generates active eddies covering a fraction of volume $\beta_{n+1}(k)$. *k* labels the mother-eddy and $k = 1, \dots, N_n$.

Since the rate of energy transfer is constant among mother-eddies and their effects, we have,

$$\frac{v_n(k)^3}{l_n} = \beta_{n+1}(k)\frac{v_{n+1}(k)^3}{l_{n+1}} \quad \dots (18)$$

The iteration of v_n gives an eddy generated by a particular history of fragmentations $[\beta_1, \dots, \beta_n]$, such that,

$$v_n \propto l_n^{1/3}\left(\prod_{i=1}^{n}\beta_i\right)^{-1/3} \quad \dots (19)$$

The fraction of volume occupied by an eddy generated by $[\beta_1, \dots, \beta_n]$ is $\prod_{i=1}^{n}\beta_i$, such that,

$$\langle|\Delta V(l_n)|^P\rangle \propto l_n^{P/3}\int\prod_{i=1}^{n}d\beta_i\ \beta_i^{(1-P/3)}P(\beta_1, \dots, \beta_n)$$

With no correlation among different steps of the fragmentation, i.e., with $P(\beta_1, \dots, \beta_n) = \prod_{i=1}^{n}P(\beta_i)$, the exponent concerned,

$$\zeta_P = \frac{P}{3} - \ln_2\{\beta^{(1-P/3)}\} \quad \ldots(20)$$

Let us now consider a given representation of the universe. Let the packing density of the constituents be $\emptyset$. This packing density function $\emptyset$ will affect any given constituent point inside it in accordance with the Laws of Physical Transactions. The given constituent point concerned will in turn affect $\emptyset$ while interacting. For a given critical state of study of the total effects, we therefore are going to have a shear stress $\emptyset$ and a mean effective stress f. The critical state line is the loci of critical state conditions in the $\varepsilon - f - \emptyset$ space. Its projection on the $f - \emptyset$ space defines a strength parameter,

$$M = \frac{\emptyset}{f} = \frac{6\sin\emptyset}{3 - \sin\emptyset} \quad \ldots(21).$$

The second equality applies to axis-symmetric, axial compression and it is a function of the constant volume critical state packing density function $\emptyset$.

The small-strain stiffness of a given representation is measured by imposing a smaller strain than the elastic threshold strain concerned. In this range, deformations localize at inter-point contacts and the granular skeleton deforms at constant fabric of spacetime. The nonlinear load-deformation response determines the stress-dependent shear wave velocity,

$$V_S \propto \left(\frac{f - \emptyset}{\varepsilon}\right)^{\beta} \quad \ldots(22)$$

Inside a given cluster, we may have various growth-patterns. The growth may occur mainly at an active zone on the surface of the cluster. For a one-dimensional interface, a fluctuation-dissipation theorem exists, leading to an exact dynamic exponent $z = \frac{3}{2}$. This is in excellent agreement with numerical simulations of ballistic aggregation and Eden clusters. For two-dimensional interfaces, $z \sim 1.5$.

The interface profile is described by a height $h(x,t)$. The simplest nonlinear Langevin equation for a local growth of the profile is,

$$\frac{\partial h}{\partial t} = m\Delta^2 h + \frac{\lambda}{2}(\nabla h)^2 + \eta(x,t) \qquad \ldots(23)$$

The first term on the right-hand side describes relaxation of the interface by a tension term m. The second term is the lowest -order nonlinear term that can appear in the interface growth equation. Higher-order terms may also be present, but they are irrelevant and will not modify the scaling properties concerned. The noise $\eta(x,t)$ has a Gaussian distribution with $\langle \eta(x,t) \rangle = 0$ and $\langle \eta(x,t)\eta(x',t') \rangle = 2D\delta^d(x-x')\delta(t-t')$.

There is also a velocity term, but it is removed by choice of an appropriate moving coordinate system. Equation (23) is invariant under translation $h \to h + \text{constant}$, and obeys the infinitesimal reparametrization,

$$h \to h + b.x, x \to x + \lambda bt,$$

which describes the tilting of the interface by a small angle. When a given constituent point is added, the increment projected along the h-axis is,

$$\delta h = m\sqrt{[1 + (\nabla h)^2]} \simeq m + (m/2)(\nabla h)^2 + \cdots$$

Following the transformation $W(x,t) = e^{[(\lambda/2m)h(x,t)]}$, we have,

$$\frac{\partial W}{\partial t} = m\nabla^2 + \left(\frac{\lambda}{2m}\right)\eta(x,t)W \qquad \ldots(24)$$

which is a diffusion equation in a time-dependent random potential. $W(x,t)$ is the sum of Boltzmann weights for all static configurations of a flow in a $(d+1)$-dimensional space from $(0,0)$ to (x,t). The noise term describes a quenched random potential $(\lambda/2m)\eta(x,t)$exerted by the environment. The second transformation, $m = -\nabla h$ results in

$$\frac{\partial m}{\partial t} + \lambda m.\nabla = m\nabla^2 m - \nabla\eta(x,t)$$

which is the Burger's equation for a vorticity-free velocity field for $\lambda = 1$. In the Burger's equation, further evolution of the pattern proceeds through the larger parabolas growing at the expense of the smaller ones, and parallels the evolution of shock waves.

If the initial profile is $h(x,0) = h_0(x)$, its evolution is given by,

$$h(x,t) = \frac{2m}{\lambda} \ln \left\{ \int_{-\infty}^{\infty} \frac{d^d \xi}{(4\pi m t)^{\frac{d}{2}}} e^{\left[-\frac{(x-\xi)^2}{2mt} + \frac{\lambda}{2m} h_0(\xi) \right]} \right\}$$

Let a given representation have bonds within itself, occupied by a resistance generated inside it due to its packing density $\emptyset$, with probability p. Let it have a support towards the concerned flow with probability $1 - p$. In such a representation, we have,

$$\langle \sum_k \varepsilon_k^n \rangle_{\xi, L} \propto L^{-x_n}, L \to \infty \qquad \ldots (25);$$

where $\langle \sum_k \varepsilon_k^n \rangle$ refers to the average over the sample realizations, L is the system-size $L \lesssim \xi$ and $\xi \propto (p - p_c)$ is the correlation length. ε_k is the energy dissipated in the branch k.

For a finite size scaling behaviour,

$$p\left(\sum_k \varepsilon_k^0, \sum_k \varepsilon_k^1, \ldots, \xi, L \right) = \lambda^{x_0} \lambda^{x_1} \ldots p\left(\sum_k \varepsilon_k^0 / \lambda^{-x_0}, \frac{\sum_k \varepsilon_k^1}{\lambda^{-x_1}}, \ldots, \xi/\lambda, \frac{L}{\lambda} \right) \qquad \ldots (26),$$

(λ is the rescaling parameter) equation (25) implies,

$$\langle \left(\sum_k \varepsilon_k^n \right)^m \rangle_{\xi,L} \propto L^{-m x_n} \qquad \ldots (27)$$

In disordered representations, the fluctuations of the free energy among different replicas may be regarded as the analogue of the temporal intermittency in a chaotic signal. Considering a spin-model of the D-dimensions, the Hamiltonian,

$$H[\{J_{ij}\}] = -\sum_{(i,j)} J_{ij}\,\sigma_i \sigma_j ,$$

where $\sigma_i = \pm 1$ is the of the spin on the site i and the coupling J_{ij} is an independent random variable distributed according to a probability distribution $p(J_{ij})$. Given a coupling realization $\{J_{ij}\}$, the partition function of an N spin system is the trace of the Boltzmann weight $e^{(-\beta\ H_N)}$,

$$Z_N(\beta, \{J_{ij}\}) = \sum_{\{\sigma_i\}} e^{\{-\beta\ H_N[\{J_{ij}\}]\}}$$

The free energy per spin in the limit $N \to \infty$ is,

$$F(\beta) = \lim_{N\to\infty} -\frac{1}{N\beta} \langle \ln Z_n \rangle$$
$$= \lim_{N\to\infty} -\frac{1}{N\beta} \int p(J_{ij}) dJ_{ij}\ \ln Z_N(\beta, \{J_{ij}\})$$

The free energy per spin of a coupling realization $\{J_{ij}\}$ of a N spin system is,

$$\Xi_N = -\frac{1}{N\beta} \ln Z_N(\beta, \{J_{ij}\}) \qquad \ldots (28)$$

The self-averaging of Ξ_N is

$$F = \lim_{N\to\infty} \Xi_N$$

For a unidimensional system with first neighbor interactions and uniform field h, we can write the partition function as the trace over 2×2 random transfer matrices product. The Hamiltonian is now $H = -\sum_i (J_i \sigma_i \sigma_{i+1} + h\sigma_i)$, such that,

$$Z_N = Tr \prod_{i=1}^{N} M_i, M_i = \begin{pmatrix} e^{\beta J_i + \beta h} & e^{-\beta J_i + \beta h} \\ e^{-\beta J_i - \beta h} & e^{\beta J_i - \beta h} \end{pmatrix} \quad \ldots (29)$$

The moments of the partition function can be estimated as an integral over the spectrum of the possible free energies $[\Xi_{\min}, \Xi_{\max}]$,

$$\langle Z_N(\beta)^q \rangle \propto \int \prod (\Xi)\, d\Xi\ e^{(-\beta \Xi q N)} \quad \ldots (30)$$

The Kolmogorov entropy is related to the sum of the positive Lyapunov exponents which measure the divergence rate along the expanding directions, in accordance with the Theory of Physical Abstraction. For an ergodic measure with a compact support (as proved by Pesin) is,

$$K_1 \leq \sum_{i=1}^{P} \lambda_i ;$$

where P is the number of exponents, $\lambda_i > 0$. In Hamiltonian systems,

$$K_1 = \sum_{i=1}^{P} \lambda_1 = \left. \frac{dL^{(P)}}{dq} \right|_{q=0}$$

A record of measures of a signal $x(t)$ at uniform spacing τ is

$$x_i = x(i\tau); i = 1,2, \ldots, M \gg 1 \quad \ldots (31)$$

Clustering:

Since the number of eddies at scale l with singularity h is proportional to $l^{-d(h)}$, the number of grid points that have to be considered for resolving the set $S(h)$ is

$$N_h(R_e) \sim (L/\eta(h))^{d(h)} \propto R_e^{d(h)/(1+h)};$$

where $R_e = \frac{(\varepsilon L^4)^{1/3}}{\nu}$ and η is the dissipative Kolmogorov length.

Integrating over h, the total number of degrees of freedom is,

$$N(R_e) = \int d\rho(h) N_h(R_e) \propto R_e^{\delta};$$

where $\delta = \max_h [d(h)/1 + h]$.

The estimate $l_{min} = \eta(h_{min})$ assures that all the sets $S(h)$ are taken into account. The number of equations which allows us to get such a fully accurate description is thus;

$$N_T^* \sim \left(\frac{L}{l_{min}}\right)^3 \propto R_e^{3/(1+h_{min})};$$

which may be obtained by considering flows in the required number of directions or dimensions.

Directivity:

The directivity inside a given system, which is equal to the tendency of energy transport concerned, shows up as the 'direction' part of the energy quanta that carry out or tend to carry out the transaction. A particle in an isolated box will tend to move in all possible directions. A bias towards any given direction indicates an imbalance of support towards its movement in that given direction and resistance against it. Considering the movement of an energy quantum in a particular direction, this difference between the concerned support and the concerned resistance must be at least the same of the given energy quantum, in accordance to the Theory of Abstraction. For a given quantum-state ($h\upsilon$),

$$S \sim R = h\upsilon \qquad \ldots(32)$$

where, S and R represent the support and resistance respectively.

This means that at least one half of a total energy-quantum gives it its direction while the other part gives it its magnitude. The direction part remains 'hidden' while only the magnitude part shows up as the value of the given quantum state. Considering the direction part however may reduce quantum-transport to classical transport as we shall see here.

For a given transport of energy-quantum, between an initial and a final point, let the trajectory of the initial point $x_o = x(o)$ be denoted by,

$$x(t) = f^t(x_o)$$

Expanding $f^t(x_o + \delta x_o)$ to linear order, the evolution of the distance to a neighbouring trajectory $x_i(t) + \delta x_i(t)$ is given by the Jacobian matrix J,

$$\delta x_i(t) = \sum_{j=1}^{d} J^t(x_o)_{ij} \ \delta x_{oj},$$

$$J^t(x_o)_{ij} = \frac{\delta x_i(t)}{\delta x_{oj}} \qquad \ldots(33)$$

A trajectory of an energy-quantum as moving on a flat surface is specified by two position coordinates and the direction of motion. The Jacobian matrix describes the deformation of an infinitesimal neighborhood of $x(t)$along the transport. Its eigenvectors and eigenvalues give the directions and the corresponding rates of expansion or contraction. The trajectories that start out in an infinitesimal neighborhood separate along the unstable directions (those whose eigenvalues are greater than unity in magnitude), approach each other along the stable directions (those whose eigenvalues are less than unity in magnitude), and maintain their distance along the marginal directions (those whose eigenvalues equal unity in magnitude).

Holding the hyperbolicity assumption (i.e., for large n the prefactors a_i, reflecting the overall size of the system, are overwhelmed by the exponential growth of the unstable eigenvalues Λ_i, and may thus be neglected), to be justified, we may replace the magnitude of the area of the *i*th strip $|B_i|$ by $\frac{1}{|\Lambda_i|}$ and consider the sum,

$$\lceil n = \sum_{i}^{n} \frac{1}{|\Lambda_i|};$$

where the sum goes over all periodic points of period n. We now define a generating function for sums over all periodic orbits of all lengths,

$$\lceil z = \sum_{n=1}^{\infty} \lceil n \, z^n \qquad \ldots(34)$$

For large n, the nth level sum tends to the limit $\lceil n \to e^{-n\gamma}$, so the escape rate γ is determined by the smallest $z = e^{\gamma}$ for which equation (34) diverges,

$$\lceil z \approx \sum_{n=1}^{\infty} (ze^{-\gamma})^n = \frac{ze^{-\gamma}}{1 - ze^{-\gamma}} \qquad \ldots(35)$$

Making an analogy to the Riemann zeta-function, for periodic orbit cycles,

$$\lceil z = -z \frac{d}{dx} \sum_{p} \ln(1 - t_p);$$

$\lceil(z)$ is a logarithmic derivative of the infinite product

$$\frac{1}{\zeta(z)} = \prod_{p} (1 - t_p), t_p = \frac{z^{n_p}}{|\Lambda_p|} \qquad \ldots(36)$$

This represents the dynamical zeta function for the escape rate of the trajectories of quantum-transport.

Structure of Matter:

Abstraction says that there is no existence of anything without energy. It also says that points inside energy cluster to form matter of a given property, at the desired scaling-ratio. Let us consider one such system of energy, inside which its constituent points have the tendency to form clusters. In such transactions, the family of evolution-maps

f^t form a group. The evolution rule f^t is a family of mappings of strips of transport B, that we may consider, such that,

1) $f^0(x) = x$
2) $f^t[f^{t'}(x)] = f^{t+t'}(x)$
3) $(x, t) \to f^t(x)$ from $B \times R$ into B is continuous;

where t represents a time interval and $t \in R$.

For infinitesimal times, we may write the trajectory of a given transaction as,

$$x(t+\tau) = f^{t+\tau}(x_0)$$
$$= f[f(x_0, t), \tau] \quad \dots (37)$$

The time derivative of this trajectory at point $x(t)$ is,

$$\left.\frac{dx}{d\tau}\right|_{\tau=0} = \partial_\tau f[f(x_0, t), \tau]|_{\tau=0} = \dot{x}(t) \quad \dots (38)$$

The vector field is a generalized velocity field,

$$\dot{x}(t) = v(x)$$

If x_q represents an equilibrium point, the trajectory remains stuck at x_q forever. Otherwise, the trajectory passing through x_0 at time $t = 0$ may be obtained by,

$$x(t) = f^t(x_0) = x_0 + \int_0^t d\tau\, v[x(\tau)], x(0)$$
$$= x_0 \quad \dots (39)$$

The Euler integrator, which advances the trajectory by $\delta\tau \times \text{velocity}$ at each time step is,

$$x_i = x_i + v_i(x)\delta\tau.$$

This may be used to integrate the equations of the dynamics concerned.

Hamiltonian Chaotic Dynamics:

For a Hamiltonian $H(q,p)$ and equations of motion

$$\dot{q}_i = \frac{\partial H}{\partial p_i}, \dot{p}_i = \frac{\partial H}{\partial q_i}$$

With D degrees of freedom,

$$x = (q,p),$$

$$q = (q_1, q_2, q_3, \dots, q_D),$$

$$p = (p_1, p_2, p_3, \dots, p_D).$$

The value of the Hamiltonian function at the state space point $x = (q,p)$ is constant along the trajectory $x(t)$. Thus the energy along the trajectory $x(t)$ is constant,

$$\frac{d}{dt} H[q(t), p(t)] = \frac{\partial H}{\partial q_i}\dot{q}_i(t) + \frac{\partial H}{\partial p_i}\dot{p}_i(t) = \frac{\partial H}{\partial q_i}\frac{\partial H}{\partial p_i} - \frac{\partial H}{\partial p_i}\frac{\partial H}{\partial q_i}$$
$$= 0$$

The trajectories therefore lie on surfaces of constant energy or level sets of the Hamiltonian $[(q,p): H(q,p) = E]$.

Given a smooth function $g(x)$, the standard map is,

$$x_{n+1} = x_n + y_{n+1}$$

$$y_{n+1} = y_n + g(x_n).$$

This is an area-preserving map. The corresponding n^{th} iterate Jacobian matrix is,

$$M^n(x_0, y_0) = \prod_{K=n}^{1} \begin{pmatrix} 1 + g'(x_K) & 1 \\ g'(x_K) & 1 \end{pmatrix} \quad \dots (40)$$

Let $M = 1$ as the map preserves areas and also B is symplectic, which in turn is because during cluster formation, the support and resistance between the points tend to attain equillibrium.

The standard map corresponds to the choice $g(x) = k/(2\pi \sin(2\pi x))$. When $k = 0, y_{n+1} = y_n = y_0$, so that angular momentum is conserved, and the angle *x* rotates with uniform velocity,

$$x_{n+1} = x_n + y_0 = x_0 + (n+1)y_0$$

The standard map provides a stroboscopic view of the flow generated by a time dependent Hamiltonian,

$$H(x, y, t) = \frac{1}{2}y^2 + G(x)\delta_1(t) \qquad \ldots(41);$$

where δ_1 denotes the periodic delta function,

$$\delta_1(t) = \sum_{m=-\infty}^{\infty} \delta(t - m)$$

and

$$G'(x) = -g(x)$$

A complete description of the dynamics for arbitrary values of the nonlinear parameter k is fairly complex. When K is sufficiently large, single trajectories wander erratically on a large fraction of the phase space.

The component of the dynamics along the continuous symmetry directions of the trajectory behavior or 'drift' may be induced by the symmetries themselves. In presence of a continuous symmetry, an orbit explores the manifold swept by combined actions of the dynamics and the symmetry induced drifts. A group member can be parameterized by angle θ, with the group multiplication law $g(\theta')\, g(\theta) = g(\theta' + \theta)$ and its action on smooth periodic functions $u(\theta + 2\Pi) = u(\theta)$ generated by,

$$g(\theta') = e^{\theta' T}, T = \frac{d}{d\theta} \qquad \ldots(42)$$

The differential operator T is reminiscent of the generator of spatial translations. The constant velocity field $v(x) = v = C.T$ acts on x_j by replacing it by the velocity vector C_j.

Let, G be a group and $gB \to B$ a group action on the state space B. The $[d\ x\ d]$ matrices g acting on vectors in the d-dimensional state space B from a linear representation of the group G. If the action of every element g of a group G commutes with the flow,

$$g\ v(x) = v(gx), gf^t(x) = f^t(gx) \qquad \ldots(43)$$

G is a symmetry of the dynamics and is invariant under G or G-equivalent. For any $x \in B$, the group orbit B_x of x is the set of all group actions,

$$B_x = [g\ x \mid g \in G]$$

The time evolution and the continuous symmetries can be considered on the same Lie group footing. An element of a compact Lie group continuously connected to identity can be written as,

$$g(\theta) = e^{\theta T}, \theta.T = \sum \theta_a T_a\ , a = 1,2,3, \ldots, N;$$

where $\theta.T$ is a Lie element and θ_a are the parameters of the transformation.

Any representation of a compact Lie group G is fully reducible, and invariant tensors constructed by contractions of T_a are useful for identifying irreducible representations. The simplest such invariant is,

$$T^T.T = \sum_{\propto} C_2^{(\alpha)}\ \|^{(\alpha)} \qquad \ldots(44)$$

equilibria satisfy $f^t(x) - x = 0$ and relative equilibria satisfy $f^t(x) - g(t)x = 0$ for any t.

A relative periodic orbit is periodic in its mean velocity, $C_p = \theta_p/T_p$ comoving frame, but in the stationary frame its trajectory is quasiperiodic. A relative periodic orbit

may be pre-periodic if it is equivariant under a discrete symmetry. Translational symmetry allows for relative equilibria characterized by a fixed profile Eulerian velocity $\mu_{TW}\ (x)$ moving with constant velocity C, i.e.,

$$\mu\ (x,t) = \mu_{TW}\ (x - Ct) \qquad \ldots(45)$$

Let, $f\ (\lambda,\ \mathrm{D})$ be a transaction function for a system, where λ is the difference in concentrations of a given observable quantity between two given points of transaction and D the distance between the points. Let, $f_1, f_2, f_3, \ldots, f_n$ be the complete orthonormal set of eigenfunctions for an operator $\hat{\mathrm{O}}$ corresponding to some observable quantity in the system. f can be expanded such that,

$$f\ =\ j_1 f_1\ +\ j_2 f_2\ +\ j_3 f_3 + \ldots + j_n f_n \qquad \ldots(46)$$

where $j_1, j_2, j_3, \ldots, j_n$ are constants.

Operated with $\hat{\mathrm{O}}$ from the left on both sides yield,

$$\hat{\mathrm{O}} f\ =\ j_1 \hat{\mathrm{O}} f_1\ +\ j_2 \hat{\mathrm{O}} f_2\ +\ j_3 \hat{\mathrm{O}} f_3 + \ldots + j_n \hat{\mathrm{O}} f_n$$

$f_1, f_2, f_3, \ldots, f_n$ being eigenfunctions of $\hat{\mathrm{O}}$, we can write,

$$\hat{\mathrm{O}} f\ =\ j_1 \mathrm{k}_1 f_1\ +\ j_2 \mathrm{k}_2 f_2\ +\ j_3 \mathrm{k}_3 f_3 + \ldots + j_n \mathrm{k}_\mathrm{n} f_n$$

where $k_1, k_2, k_3, \ldots, k_n$ are the eigenvalues corresponding to the eigenfunctions.

Considering the complex conjugate of the transaction-function f in equation (46), we have,

$$f^*\ =\ j_1{}^* f_1{}^*\ +\ j_2{}^* f_2{}^*\ +\ j_3^* f_3{}^* + \ldots + j_n{}^* f_n{}^* \qquad \ldots(47)$$

Using these equations, after integrating over all co-ordinate space, we get,

$$\begin{aligned}&\int f^* \hat{\mathrm{O}} f\, dt\\ &= j_1{}^* j_1 \mathrm{k}_1 \int f_1{}^*\, f_1\, dt\\ &+ j_2{}^* j_2 \mathrm{k}_2 \int f_2{}^*\, f_2\, dt + \ldots + j_n{}^* j_n \mathrm{k}_\mathrm{n} \int f_n{}^*\, f_n dt \qquad \ldots(48)\end{aligned}$$

In this equation, we have got rid of all the terms of the type $j_a^* j_b \hat{O}_b \int f_a^* f_b \, dt$, as these are all zero because of the orthogonality of the eigenfunctions. Only when $a = b$, are all the terms non-zero and these are the ones we have retained. The integrals on the right side of equation (48) are each equal to one because of the normality condition. Therefore, we write,

$$\int f^* \hat{O} f dt = j_1{}^* j_1 \mathrm{k}_1 + j_2{}^* j_2 \mathrm{k}_2 + \ldots + j_n{}^* j_n \mathrm{k}_\mathrm{n}$$

When the system is in the state f, the average value $(\bar{a})$ of the observable k is given by the right-hand side of the previous equation, such that,

$$\bar{a} = \int f^* \hat{O} f dt \qquad \ldots (49)$$

Relation of group theory to physical transactions in symmetrical systems:

Say a given dynamic system has a given set of symmetries or stability points. For all points having similar intrinsic properties within such a system, the probability densities of occurrence are equal and must remain unaltered, being all in a similar environment. Thus the energy and Hamiltonian for the system must not change. If E_i is the energy corresponding to the eigenfunction f_i , we may write,

$$\hat{H} f_i = E_i f_i$$

If a symmetry operation ($\hat{x}$) is performed on the system, we have,

$$\hat{x}\, \hat{H} f_i = \hat{x}\, E_i f_i$$

But since $\hat{x}$ does not affect $\hat{H}$ or E, we may write,

$$\hat{H}\,(\hat{x} f_i) = E_i\,(\hat{x} f_i) \qquad \ldots (50).$$

The function $\hat{x}\, f_i$ is therefore an eigenfunction of $\hat{H}$ with the same eigenvalues as f_i . We can therefore conclude, if the state is non-degenerate, for normalized functions,

$$\hat{x} f_i = \pm f_i$$

Also, as

$$\frac{F}{T} = 2c\left(\frac{\lambda}{D}\right)$$

Using Lyapunov exponents for a transport as described by this equation, and replacing $2c\left(\frac{\lambda}{D}\right)$ by a quantity τ, we have,

$$\frac{d}{d\tau} f^n(\tau) = \frac{\delta n}{\delta o}$$

i.e.,

$$\frac{\delta n}{\delta o} = \prod_{i=1}^{n} f'(\tau_i) \qquad \ldots (51)$$

$$b = \frac{1}{n} \log_e \left(\frac{\delta n}{\delta o}\right)$$

i.e.,

$$b = \frac{1}{n} \sum_{i=1}^{n-1} \log_e \left| f'(\tau_i) \right| \qquad \ldots (52);$$

where b is a constant (the local slope of all possible routes), and

$$\Psi = \lim_{n \to \infty} \frac{1}{n} \sum_{i=0}^{n-1} \log_e \left| f'(\tau_i) \right| \qquad \ldots (53);$$

where Ψ is a constant.

Using equation $(53)$the probabilistic plots of all possible routes of transport, i.e., all possible orientations of the points inside the given system of energy (and also the tendencies of these points towards reaching such possible orientations) are to be found

for a given scenario. These plots, in turn, yield the description of the structures inside the given system of energy and as such, the description of the structures of matter inside it.

The free-energy change, ΔF_V is obtained from the winding-number W distribution,

$$e^{-\beta\,\Delta F_v} = \frac{\int \rho_v\,(\Psi,\Psi';\beta)d\Psi}{\int \rho_{v=0}\,(\Psi,\Psi';\beta)d\Psi} = \langle e^{i\left(\frac{m}{\tau_i}\right)v.WL}\rangle \qquad \ldots(54).$$

This free-energy change is the Fourier transform of the winding-number distribution and is periodic under

$$v \to v + \frac{h}{mL}$$

Between two open paths m_1 and $m_1 + m$ a single point momentum distribution n_m is the Fourier transform of the off-diagonal density matrix

$$n(m_j) = \frac{\int \rho_b(m_1,m_2,m_3,\ldots m_j,,m_1+m,\ldots,m_j;\beta)dm_1\ldots dm_j}{\int \rho_b(m_1,m_2,m_3,\ldots m_j,,m_1,\ldots,m_j;\beta)dm_1\ldots dm_j} \qquad \ldots(55).$$

All statistical-mechanical properties of the structures in energy, which in turn are many-point systems themselves, may be determined from the density matrix. The summation over the energy eigenstates of the system;

$$\rho(\Psi,\Psi';\tau_i) = \langle\Psi|e^{\beta H}|\Psi'\rangle = \sum_j e^{-\beta E_j}\Psi_j(\Psi)\Psi_j(\Psi') \qquad \ldots(56).$$

The identity for the discrete path-integral computations of the density matrix is,

$$\rho\left(\Psi,\Psi';\tau_i\right) = \int(\Psi,\Psi_1;\tau)\rho(\Psi_1,\Psi_1;\tau)\dots\rho\left(\Psi_{\mathrm{j}-1},\Psi';\tau\right) \times d\Psi_1\dots\mathrm{d}\Psi_{\mathrm{j}-1} \quad \dots(57).$$

The amount of information required to describe the trajectory of a concerned plot to within an accuracy or length of measuring tool (ε) be (I_ε), say. We have,

$$I_\varepsilon = \sum_{i=1}^{N} P_i \log_2\left(\frac{1}{P_i}\right);$$

where P_i represents the concerned relative frequencies or probabilities of individual observations. Writing logarithmically (arbitrarily), we have,

$$I_\varepsilon = \mathrm{a} + D_i \log_2\left(\frac{1}{\varepsilon}\right);$$

where a is constant, and

$$D_i = \lim_{\varepsilon\to 0}\left[\frac{I_\varepsilon}{\log_2(u/\varepsilon)} - \frac{\mathrm{a}}{\log_2(u/\varepsilon)}\right]$$

i.e.,

$$D_i = \lim_{\varepsilon\to 0}\frac{I_\varepsilon}{\log_2(u/\varepsilon)};$$

$\frac{\mathrm{a}}{\log_2(u/\varepsilon)}$ being sufficently small to be neglected (u represents the unit length of the original).The necessity of I_ε to fall within the desired value is absolute, barring which safety of predictions using the plot concerned is hampered. Ruler-length decreasing to 0, we have,

$$D_e = \lim_{\varepsilon\to 0}\frac{\log N}{\log_2(1/\varepsilon)};$$

where D_e is the concerned dimension of measurements.

Starting measurements relating to two regions, one initial and the other final, each such region may be further considered to be a collection of some other regions. The scaling operation is performed such that:

1. There is a finite number of sub-divisions.
2. Step 1 is repeated on each new facsimilie.

Thus, we have the dimension,

$$D = \frac{\log N}{\log_2\left(\frac{1}{r}\right)};$$

where N is the number of facsimilies and r represents the scaling-ratio (i.e., 1/number of sub-divisions).

Also, for a unit length (u), $u = r^D N$

For irregular forms, however, the estimated unit length,

$$L = \varepsilon N$$

Inserting a constant of proportionality (a) in $N = \left(\frac{1}{r^D}\right)$, for going from unit length to a measured length, we get,

$$N = \mathrm{a}\left(\frac{1}{r^D}\right)$$

Further, inserting new scaling length (ε) in place of (r), we have,

$$N = \mathrm{a}\left(\frac{1}{\varepsilon^D}\right)$$

Thus, from the equations $(L = \varepsilon N)$ and $(N = \mathrm{a}/\varepsilon^D)$, we have,

$$L\varepsilon = \mathrm{a}\varepsilon^{1-D}$$

When the distance between the concerned initial and final points of transport become sufficiently large, however, than the 'size' of the points, the trajectories tend to become chaotic, thereby increasing the uncertainty of predictions. This uncertainty, in turn, depends upon the Lyapunov exponents (ν). Moreover, there is a stretching or

shrinking of a given direction according to the factor $e^{\nu t}$, according as ν being positive or negative in that direction.

Let us suppose a system is characterized by a positive ν, i.e., ν_+ and its initial state is defined within a size ε. Then, in time T, the uncertainty in the co-ordinates concerned will have expanded to the size L of the attractor,

$$L \sim \varepsilon e^{\nu_+ T}$$

or,

$$L \sim \varepsilon e^{KT}$$

Either of these relations may be solved for the prediction-time,

$$T \sim \left(\frac{1}{\nu_+}\right) \log_e \left(\frac{L}{\varepsilon}\right)$$

or,

$$T \sim \left(\frac{1}{K}\right) \log_e \left(\frac{L}{\varepsilon}\right) \qquad \ldots (58).$$

Conclusion:

We investigate the blueprint for the structures in energy. This helps us to understand the formation of structures of matter as we know them. Abstraction takes into account the scaling-ratio concerned for a given accuracy of observations. At different scaling-ratios, these structures can show completely different emergent phenomena. Again, these emergent phenomena depend upon the given set of dimensions that we consider for a given set of observations. The points inside a given system of energy organize themselves at different scaling-ratios to form different orientations or patterns. These different orientations or patterns may give rise to different orientations of matter.

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