GODEL'S INCOMPLETENESS THEOREM AND LOGIC

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Abstract

This article raises some important points about logic, e.g., mathematical logic.

How should we think or act in a logical manner? We should exercise pain, caution and care when making statements, e.g., do our research first and get our facts or premises right, make a careful choice of words, terms or expressions to be used, aim at clarity and at being understood, listen to, consider and accept or adopt others' ideas or points of view if they are relevant, win the support of or acceptance by others for our logical propositions or ideas, et al.. A logical statement could be simple and short or it could be complex, detailed and lengthy. The facts or premises contained in the statement should be true facts, facts whose truthfulness could be verified. It is important that the statement could be verified or proved to be true, e.g., confirmed by an experiment or experiments, or, some other kind of test. For those statements whose truths are not certain or verifiable, we could make them with some qualification or caveat - we could make such statements with a probabilistic, but reasoned, approach, e.g., we could state that something is probably true, most probably true, unlikely to be true, has little likelihood of being true, in all probability true (or untrue or false), true under certain circumstances, or, false under certain circumstances, et al., depending on our intuition and how strongly we felt about the probability of its being true, or, false, though we were not certain of or able to verify its truth or falseness. It is of course important to have clarity and alertness of mind while making statements.

The following is a listing of the kinds of statement we may encounter from our fellow-beings or counterparts:-

1) The statement is entirely true. (The truth is verifiable.)
2) The statement is entirely false. (The falseness is verifiable.)
3) The statement is partially true and partially false. (The partial truth and partial falseness are verifiable.)
4) The statement is partially true and partially false, while the rest of the statement is not verifiable as true, or, false but may be considered probably, or, most probably true, or, false. (The partial truth and partial falseness are verifiable.)
5) The statement is partially true, while the rest of the statement is not verifiable as true, or, false but may be considered probably, or, most probably true, or, false. (The partial truth is verifiable.)
6) The statement is partially false, while the rest of the statement is not verifiable as true, or, false but may be considered probably, or, most probably true, or, false. (The partial falseness is verifiable.)
7) The statement is neither true nor false, but may be considered probably, or, most probably true, or, false. (The statement could not be verified to be true, or, false.)

8) The statement is entirely true under certain circumstances. (This is verifiable.)
9) The statement is entirely false under certain circumstances. (This is verifiable.)
10) The statement is partially true and partially false under certain circumstances. (These partial truth and partial falseness are verifiable.)
11) The statement is partially true and partially false under certain circumstances, while the rest of the statement is not verifiable as true, or, false but may be considered probably, or, most probably true, or, false. (These partial truth and partial falseness are verifiable.)
12) The statement is partially true under certain circumstances, while the rest of the statement is not verifiable as true, or, false but may be considered probably, or, most probably true, or, false. (This partial truth is verifiable.)
13) The statement is partially false under certain circumstances, while the rest of the statement is not verifiable as true, or, false but may be considered probably, or, most probably true, or, false. (This partial falseness is verifiable.)
14) The statement is neither true nor false under certain circumstances, but may be considered probably, or, most probably true, or, false. (This statement could not be verified to be true, or, false.)
15) The statement is neither true nor false, and there is little or hardly any probability that it is true, or, false. (This statement could not be verified to be true, or, false.)
16) The statement is none of the above, i.e., it is neither true nor false under any circumstances, nor probably, or, most probably true, or, false - we cannot make anything or form any conclusion about the statement at all. (This statement could not be verified to be true, or, false.)

The above listing of 16 kinds of statement which we may encounter may not be an exhaustive listing. The listing may be further classified, e.g., there may be a statement which is three-quarter true and one-quarter false, a statement which is two-third true and one-third probably, or, most probably true, or, false, a statement which is one-third true, one-third false and one-third probably, or, most probably true, or, false, a statement which is one-fifth true, two-fifth false and two-fifth probably, or, most probably true, or, false, et al.. There could be lots of fine, subtle distinctions in the logical statements or propositions, which we should be keenly aware of. We should be alert to all the possible implications.

Recall that by Godel's incompleteness theorem there are true statements whose truth is not provable and false statements whose falseness is not provable. The question is if we could not prove or verify the truth or falseness of a statement how could we be certain or know that the statement is true, or, false? (Is it just a hunch, feeling or intuition?) Wouldn't it be a contradiction (of mathematical reasoning wherein rigourous or solid proof is demanded) or absurd to state thus "This statement is true (or, false) but its trueness (or, falseness) cannot be proved"? The most we could say about such a statement is that it is probably, or, most probably true, or, false. Godel appears to have had erred in this respect, for it is practically not possible to know whether a statement or proposition is true, or, false, if its truth, or, falseness is not verifiable or provable, i.e., a statement could only be true, or, false, if it could be proved to be so - otherwise, it would be just a conjecture, e.g., a mathematical

statement which has not been proved is a conjecture while a mathematical statement which has been proved is a theorem. In mathematics, a statement is true only if it is proved to be so. Could an "undecidable" or "unprovable" mathematical statement be true as per Godel's incompleteness theorem (which is actually a contradiction of the important mathematical principle of the need for proofs)? In other words, could we say "By Godel's incompleteness theorem, this mathematical statement is true but its proof is an impossibility", or, "I know this mathematical statement is true though its proof is an impossibility"? Which could invite a counter-argument "If you cannot prove that this mathematical statement is true, how do you know that this mathematical statement is true?", which would make it all look rather absurd.

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