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On a Basic Analogue of Generalized *H*-function

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Abstract. In this paper, we investigate the basic analogue of a new hypergeometric function, which is a generalization of the basic *I*-function. In this regard, the application of Riemann-Liouville and Weyl fractional q-integral operator with new hypergeometric function has been discussed. Similar result obtained by other authors follows as special cases of our findings.

Keywords: Basic analogues of *H* and *I*-function, Basic hypergeometric function, Fractional q-integral operators.

MSC 2010: 26A33, 33D15, 33D60.

1 Introduction

In the past century, many authors have generalized *H*-function. In a recent paper, Südland et al. [10] have introduced a generalization of Saxena's *I*-function [9], which is also a generalization of Fox's *H*-function. This function is known as Aleph function. In their paper, Saxena and Pogány [7] have studied fractional integration formulae for the Aleph functions.

Südland et al. [11] studied the generalized fractional driftless Fokker-Planck equation with power law coefficient. As a result a special function was found, which is a particular case of the Aleph function. The Aleph was defined by means of Mellin-Barnes type contour integrals as

$$\aleph[z] := \aleph^{m,n}_{u_i,v_i,\tau_i;r}\left[z \left| \begin{matrix} (a_j, A_j)_{1,n}, & [\tau_i(a_{ji}, A_{ji})]_{n+1,u_i;r} \\ (b_j, B_j)_{1,m}, & [\tau_i(b_{ji}, B_{ji})]_{m+1,v_i;r} \end{matrix} \right. \right] = \frac{1}{2\pi\omega}\int_{\mathsf{L}} \Omega^{m,n}_{u_i,v_i,\tau_i;r}(\zeta) z^{\zeta}\, d\zeta \tag{1.1}$$

for all $z \neq 0$, where $\omega = \sqrt{-1}$ and

$$\Omega^{m,n}_{u_i,v_i,\tau_i;r}(\zeta)=\frac{\prod_{j=1}^{m}\Gamma(b_j-B_j\zeta).\prod_{j=1}^{n}\Gamma(1-a_j+A_j\zeta)}{\sum_{i=1}^{r}\left\{\tau_i\prod_{j=n+1}^{u_i}\Gamma(a_{ji}-A_{ji}\zeta).\prod_{j=m+1}^{v_i}\Gamma(1-b_{ji}+B_{ji}\zeta)\right\}}. \quad (1.2)$$

The parameters u_i, v_i are non-negative integers satisfying the inequality $0\le n\le u_i$, $1\le m\le v_i$ and $\tau_i>0; i=1,\cdots$. The parameters A_j, B_j, A_{ji}, B_{ji} are positive real numbers and a_j, b_j, a_{ji}, b_{ji} are complex numbers. The $L=L_{\omega\gamma\infty}$ is a suitable contour of the Mellin-Barnes type in the complex ζ -plane which runs from $\gamma-\omega\infty$ to $\gamma+\omega\infty$ with $\gamma\in$, such that the poles of $\Gamma(b_j-B_j\zeta), j=1,\cdots$ separating from those of $\Gamma(1-a_j+A_j\zeta)$, $j=1,\cdots$. All the poles of the integrand (1.2) are assumed to be simple, and empty products are interpreted as unity. For the existence conditions, we refer to [8].

We define a basic analogue of this $\aleph$-function in term of Mellin-Barnes type contour integrals in the following manner

$$\aleph[z;q]:=\aleph^{m,n}_{u_i,v_i,\tau_i;r}\left[z;q\left|\begin{matrix}(a_j,A_j)_{1,n}, & [\tau_i(a_{ji},A_{ji})]_{n+1,u_i;r}\\ (b_j,B_j)_{1,m}, & [\tau_i(b_{ji},B_{ji})]_{m+1,v_i;r}\end{matrix}\right.\right]$$
$$=\frac{1}{2\pi\omega}\int_{C}\Omega^{m,n}_{u_i,v_i,\tau_i;r}(\zeta;q)\pi z^{\zeta}\,d\zeta \quad (1.3)$$

where $\omega=\sqrt{-1}$,

$$\Omega^{m,n}_{u_i,v_i,\tau_i;r}(\zeta;q)=\frac{\prod_{j=1}^{m}G(q^{b_j-B_j\zeta}).\prod_{j=1}^{n}G(q^{1-a_j+A_j\zeta})}{\sum_{i=1}^{r}\left\{\tau_i\prod_{j=n+1}^{u_i}G(q^{a_{ji}-A_{ji}\zeta}).\prod_{j=m+1}^{v_i}G(q^{1-b_{ji}+B_{ji}\zeta})\right\}} \quad (1.4)$$
$$\times\frac{1}{G(q^{1-\zeta})\sin\pi\zeta},$$

and

$$G(q^{\delta})=\left\{\prod_{j=0}^{\infty}(1-q^{\delta+j})\right\}^{-1}=\frac{1}{(q^{\delta};q)_{\infty}} \quad (1.5)$$

The parameters u_i, v_i are non-negative integers satisfying the inequality $0 \le n \le u_i$, $1 \le m \le v_i$ and $\tau_i > 0;\ i = 1, \cdots$; r is finite and A_j, B_j, A_{ji}, B_{ji} are positive real numbers and a_j, b_j, a_{ji}, b_{ji} are complex numbers. The $\mathsf{C} = \mathsf{C}_{\omega\gamma\infty}$ is a suitable contour of Mellin-Barnes type in the complex ζ -plane, which runs from $\gamma - \omega\infty$ to $\gamma + \omega\infty$ with $\gamma \in \mathbb{R}$, such that the poles of $G(q^{b_j - B_j\zeta}),\ j = 1, \cdots$ separating from those of $G(q^{1-a_j+A_j\zeta}),\ j = 1, \cdots$. All the poles of the integrand (1.4) are assumed to be simple and empty products are interpreted as unity. The integral converges if $\Re[\zeta \log(z) - \log \sin \pi\zeta] < 0,$ for large value of $|\zeta|$ on the contour, that is if $|\arg(z) - \varpi_2 \varpi_1^{-1} \log|z|| < \pi$, where $0 < |q| < 1$, $\log q = -\varpi = -(\varpi_1 + \omega\varpi_2)$, ϖ, ϖ_1, ϖ are definite quantities, ϖ_1 and ϖ_2 being real.

When all $\tau_i = 1$; (1.3) yields the q -analogous of the I -function due to Saxena and Kumar [6].

Again, when $r = 1, u_i = u, v_i = v$ and $c_i = 1$; (1.3) yields the q -analogous of the H -function due to Saxena et al. [7].

2 Preliminary Notes

In this section, we first recall some definitions and fundamental facts of basic analogue of special function and integral operator.

The fractional q -calculus is the q -extension of the ordinary calculus. Agarwal [1] introduced, the q -analogue of the Riemann-Liouville fractional integral operator in the following form:

$$\mathrm{I}_q^{\alpha} f(x) := \frac{1}{\Gamma_q(\alpha)} \int_0^x (x - tq)_{\alpha-1} f(t) d(t;q), \tag{2.1}$$

and Al-Salam [2] introduced, the q -analogue of the Weyl fractional integral operator in the following form:

$$\mathrm{K}_q^{\mu} f(x) := \frac{q^{-\mu(\mu-1)/2}}{\Gamma_q(\mu)} \int_x^{\infty} (t - x)_{\mu-1} f(tq^{1-\mu}) d(t;q), \tag{2.2}$$

where $\Re(\alpha) > 0, Re(\mu) > 0$, $|q| < 1$. So that

$$\mathrm{I}_q^{0} f(x) = f(x) = K_q^0 f(x). \tag{2.3}$$

The theory of q-calculus for a real parameter $q \in □$, a q-real number $[a]_q$ was introduced (see [4]) as:

$$[a]_q := \frac{1-q^a}{1-q}, \quad a \in □ \tag{2.4}$$

The q-analog of the Pochhammer's symbol (q-shift factorial) is defined by:

$$(a;q)_0 = 1; \ (a;q)_n = \prod_{j=0}^{n-1}(1-aq^j) = \frac{(a;q)_\infty}{(a^n;q)_\infty}, \tag{2.5}$$

where $a \in □ \quad □$.

The q-gamma function (cf. Gasper and Rahman [4]) is defined as follows:

$$\Gamma_q(b) = \frac{(q;q)_\infty}{(q^b;q)_\infty (1-q)^{b-1}} = \frac{(q;q)_{b-1}}{(1-q)^{b-1}}, (b \neq 0,-1,-2,\cdots \tag{2.6}$$

and

$$(x-y)_\nu = x^\nu \prod_{n=0}^{\infty}\left[\frac{1-(y/x)q^n}{1-(y/x)q^{\nu+n}}\right]. \tag{2.7}$$

The q-binomial summation theorem is given by

$${}_1\Phi_0[a;-;z] = \sum_{n=0}^{\infty}\frac{(a;q)_n}{(q;q)_n}z^n = \frac{(az;q)_\infty}{(z;q)_\infty}, |z|<1. \tag{2.8}$$

The basic integration (cf. Gasper and Rahman [4]), is defined as:

$$\int_0^x f(t)d(t;q) = x(1-q)\sum_{k=0}^{\infty} q^k f(xq^k), \tag{2.9}$$

$$\int_x^\infty f(t)d(t;q) = x(1-q)\sum_{k=1}^{\infty} q^{-k} f(xq^{-k}). \tag{2.10}$$

For $\Re(\alpha) > 0$, using (2.6), (2.7), (2.9) in the q-analogue of the Riemann-Liouville fractional integral operator (2.1) can be expressed as:

$$\begin{aligned} \mathrm{I}_q^\alpha f(x) &= \frac{x^\alpha (1-q)}{\Gamma_q(\alpha)}\sum_{k=0}^{\infty} q^k (1-q^{k+1})_{\alpha-1} f(xq^k) \\ &= x^\alpha (1-q)^\alpha \sum_{k=0}^{\infty} q^k \frac{(q^\alpha;q)_k}{(q;q)_k} f(xq^k). \end{aligned} \tag{2.11}$$

Similarly, for $\Re(\mu) > 0$ and using (2.6), (2.7), (2.10) in the q-analogue of the Weyl fractional integral operator (2.2) can be expressed as:

$$\begin{aligned}\mathrm{K}_q^{\mu} f(x) &= \frac{x^{\mu}(1-q)q^{-\mu(\mu+1)/2}}{\Gamma_q(\mu)} \sum_{k=0}^{\infty} q^{-k\mu}(1-q^{k+1})_{\mu-1} f(xq^{-\mu-k}) \\ &= x^{\mu}(1-q)^{\mu} q^{-\mu(\mu+1)/2} \sum_{k=0}^{\infty} q^{-k\mu} \frac{(q^{\mu};q)_k}{(q;q)_k} f(xq^{-\mu-k}).\end{aligned} \tag{2.12}$$

For the basic concept of q-calculus we refer the reader to [3].

3 Main Results

In this section, we will establish fractional q-integral formula for the basic analogous of Aleph function.

Theorem 3.1. Let $\Re(\alpha) > 0, |q| < 1$ and I_q^{α} be the q-analogue of the Riemann-Liouville fractional integral operator (2.1), then following result holds:

$$\mathrm{I}_q^{\alpha}\left\{ t^{\rho-1} \aleph_{u_i,v_i,\tau_i;r}^{m,n}\left[\eta t^{\lambda};q \left| \begin{matrix}(a_j,A_j)_{1,n}, & [\tau_i(a_{ji},A_{ji})]_{n+1,u_i;r} \\ (b_j,B_j)_{1,m}, & [\tau_i(b_{ji},B_{ji})]_{m+1,v_i;r}\end{matrix}\right.\right]\right\}$$

$$= \begin{cases} t^{\alpha+\rho-1}(1-q)^{\alpha} \aleph_{u_i+1,v_i+1,\tau_i;r}^{m,n+1}\left[\eta t^{\lambda};q \left| \begin{matrix}(1-\rho,\lambda), (a_j,A_j)_{1,n}, [\tau_i(a_{ji},A_{ji})]_{n+1,u_i;r} \\ (b_j,B_j)_{1,m}, [\tau_i(b_{ji},B_{ji})]_{m+1,v_i;r}, (1-\rho-\alpha,\lambda)\end{matrix}\right.\right], & \text{for } \lambda \geq 0 \\ t^{\alpha+\rho-1}(1-q)^{\alpha} \aleph_{u_i+1,v_i+1,\tau_i;r}^{m+1,n}\left[\eta t^{\lambda};q \left| \begin{matrix}(a_j,A_j)_{1,n}, [\tau_i(a_{ji},A_{ji})]_{n+1,u_i;r}, (\rho+\alpha,-\lambda) \\ (\rho,-\lambda), (b_j,B_j)_{1,m}, [\tau_i(b_{ji},B_{ji})]_{m+1,v_i;r}\end{matrix}\right.\right], & \text{for } \lambda < 0 \end{cases} \tag{3.1}$$

where $\Re[\zeta \log \eta - \log \sin \pi\zeta] < 0$ and $\rho > 0$.

Proof. For the sake of convenience, we denote the left side of (3.1) by S and applying the equation (2.11) and (1.3), we obtain

$$\mathsf{S} := t^{\alpha}(1-q)^{\alpha}\sum_{k=0}^{\infty} q^{k}\frac{(q^{\alpha};q)_k}{(q;q)_k}(tq^{k})^{\rho-1}\frac{1}{2\pi\omega}\int_{\mathsf{C}}\Omega_{u_i,v_i,\tau_i;r}^{m,n}(\zeta;q)\pi(\eta t^{\lambda}q^{\lambda k})^{\zeta}\,d\zeta \tag{3.2}$$

Under the conditions stated above, we can interchange the order of summation and integration. Applying q-binomial theorem (2.8) expression (3.2) reduces to

$$\begin{aligned}\mathsf{S} &= \frac{t^{\alpha+\rho-1}(1-q)^{\alpha}}{2\pi\omega}\int_{\mathsf{C}}\Omega_{u_i,v_i,\tau_i;r}^{m,n}(\zeta;q)\pi(\eta t^{\lambda})^{\zeta}\,{}_1\Phi_0[q^{\alpha};-;q^{\rho+\lambda\zeta}]d\zeta \\ &= \frac{t^{\alpha+\rho-1}(1-q)^{\alpha}}{2\pi\omega}\int_{\mathsf{C}}\Omega_{u_i,v_i,\tau_i;r}^{m,n}(\zeta;q)\pi(\eta t^{\lambda})^{\zeta}\frac{G(q^{\rho+\lambda\zeta})}{G(q^{\rho+\alpha+\lambda\zeta})}d\zeta.\end{aligned} \tag{3.3}$$

Depending on the sign of λ, (3.3) can be expressd as the right hand side of (3.1). □

Theorem 3.2. Let $\Re(\mu)>0, |q|<1$ and K_q^{α} be the q-analogue of the Weyl fractional integral operator (2.2), then following result holds:

$$\mathrm{K}_q^{\mu}\left\{t^{\rho-1}\aleph_{u_i,v_i,\tau_i;r}^{m,n}\left[\eta t^{\lambda};q\left|\begin{matrix}(a_j,A_j)_{1,n}, & [\tau_i(a_{ji},A_{ji})]_{n+1,u_i;r}\\ (b_j,B_j)_{1,m}, & [\tau_i(b_{ji},B_{ji})]_{m+1,v_i;r}\end{matrix}\right.\right]\right\}$$

$$=\begin{cases} t^{\mu+\rho-1}(1-q)^{\alpha}q^{-\mu(\rho-1)-\mu(\mu+1)/2}\aleph_{u_i+1,v_i+1,\tau_i;r}^{m+1,n} \\ \qquad \left[\eta t^{\lambda}q^{-\lambda\mu};q\left|\begin{matrix}(a_j,A_j)_{1,n},[\tau_i(a_{ji},A_{ji})]_{n+1,u_i;r},(1-\rho,\lambda),\\ (1-\rho-\mu,\lambda),(b_j,B_j)_{1,m},[\tau_i(b_{ji},B_{ji})]_{m+1,v_i;r}\end{matrix}\right.\right], \text{for}\,\lambda\geq 0 \\ t^{\mu+\rho-1}(1-q)^{\alpha}q^{-\mu(\rho-1)-\mu(\mu+1)/2}\aleph_{u_i+1,v_i+1,\tau_i;r}^{m,n+1} \\ \qquad \left[\eta t^{\lambda}q^{-\lambda\mu};q\left|\begin{matrix}(\rho+\mu,-\lambda),(a_j,A_j)_{1,n},[\tau_i(a_{ji},A_{ji})]_{n+1,u_i;r}\\ (b_j,B_j)_{1,m},[\tau_i(b_{ji},B_{ji})]_{m+1,v_i;r},(\rho,-\lambda)\end{matrix}\right.\right], \text{for}\,\lambda<0\end{cases} \tag{3.4}$$

where $\Re[\zeta\log\eta-\log\sin\pi\zeta]<0$ and $\rho>0$.

Proof. For the sake of convenience, we denote the left hand side of (3.4) by U. Applying the equation (2.12) and (1.3), we have

$$\begin{aligned}\mathsf{U} &:= t^{\mu}(1-q)^{\mu}q^{-\mu(\mu+1)/2}\sum_{k=0}^{\infty}q^{-\mu k}\frac{(q^{\mu};q)_k}{(q;q)_k}(tq^{-\mu-k})^{\rho-1} \\ &\times\frac{1}{2\pi\omega}\int_{\mathsf{C}}\Omega_{u_i,v_i,\tau_i;r}^{m,n}(\zeta;q)\pi(\eta t^{\lambda}q^{-\lambda\mu-\lambda k})^{\zeta}\,d\zeta\end{aligned} \tag{3.5}$$

Under the conditions stated above, we can interchange the order of summation and integration. Applying q-binomial theorem (2.8) the expression (3.5) reduces to

$$\begin{aligned} \mathsf{U} &= \frac{t^{\mu+\rho-1}(1-q)^{\mu} q^{-\mu(\rho-1)-\mu(\mu+1)/2}}{2\pi\omega} \int_{\mathsf{C}} \Omega^{m,n}_{u_i,v_i,\tau_i;r}(\zeta;q)\pi(\eta t^{\lambda} q^{-\lambda\mu})^{\zeta} \\ &\quad \times {}_1\Phi_0[q^{\mu};-;q^{1-\rho-\mu-\lambda\zeta}]d\zeta \\ &= \frac{t^{\mu+\rho-1}(1-q)^{\mu} q^{-\mu(\rho-1)-\mu(\mu+1)/2}}{2\pi\omega} \int_{\mathsf{C}} \Omega^{m,n}_{u_i,v_i,\tau_i;r}(\zeta;q)\pi(\eta t^{\lambda} q^{-\lambda\mu})^{\zeta} \\ &\quad \times \frac{G(q^{1-\rho-\mu-\lambda\zeta})}{G(q^{1-\rho-\lambda\zeta})} d\zeta, \end{aligned} \tag{3.6}$$

Depending on the sign of λ, the expression (3.6) can be reduced to the right hand side of (3.4). □

4 Special Cases

The q-extension of Aleph function defined by (1.3) in terms of Mellin-Barnes type of basic integrals is most general in nature, which includes a number of basic analogues of special functions. In this section we discuss only the case involving $I_q(\cdot)$-function and $H_q(\cdot)$-function.

4.1 Special Case of Theorem 3.1

If we set $\tau_i = 1,\ i = 1,\cdots$ in Theorem 3.1 we obtain the following formula for $I_q(\cdot)$-function.

Corollary 4.1 Let $\Re(\alpha) > 0, |q| < 1$, then following result holds:

$$\mathrm{I}_q^{\alpha}\left\{t^{\rho-1} I_{u_i,v_i;r}^{m,n}\left[\eta t^{\lambda};q\left|\begin{matrix}(a_j,A_j)_{1,n}, & (a_{ji},A_{ji})_{n+1,u_i;r}\\(b_j,B_j)_{1,m}, & (b_{ji},B_{ji})_{m+1,v_i;r}\end{matrix}\right.\right]\right\}$$

$$=\begin{cases} t^{\alpha+\rho-1}(1-q)^{\alpha} I_{u_i+1,v_i+1;r}^{m,n+1}\left[\eta t^{\lambda};q\left|\begin{matrix}(1-\rho,\lambda),(a_j,A_j)_{1,n}, & (a_{ji},A_{ji})_{n+1,u_i;r}\\(b_j,B_j)_{1,m},(b_{ji},B_{ji})_{m+1,v_i;r}, & (1-\rho-\alpha,\lambda)\end{matrix}\right.\right], \text{ for } \lambda\geq 0, \\ t^{\alpha+\rho-1}(1-q)^{\alpha} I_{u_i+1,v_i+1;r}^{m+1,n}\left[\eta t^{\lambda};q\left|\begin{matrix}(a_j,A_j)_{1,n},(a_{ji},A_{ji})_{n+1,u_i;r}, & (\rho+\alpha,-\lambda)\\(\rho,-\lambda),(b_j,B_j)_{1,m}, & (b_{ji},B_{ji})_{m+1,v_i;r}\end{matrix}\right.\right], \text{ for } \lambda<0\end{cases} \tag{4.1}$$

where $\Re[\zeta\log\eta-\log\sin\pi\zeta]<0$ and $\rho>0$.

If we set $r=1,\tau_1=1,u_1=u,v_1=v,a_{j1}=a_j,b_{j1}=b_j,A_{j1}=A_j,B_{j1}=B_j$ in Theorem 3.1 we obtain the following formula for $H_q(\cdot)$-function.

Corollary 4.2 Let $\Re(\alpha)>0,|q|<1$, then following result holds:

$$\mathrm{I}_q^{\alpha}\left\{t^{\rho-1} H_{u,v}^{m,n}\left[\eta t^{\lambda};q\left|\begin{matrix}(a_j,A_j)_{1,u}\\(b_j,B_j)_{1,v}\end{matrix}\right.\right]\right\}$$

$$=\begin{cases} t^{\alpha+\rho-1}(1-q)^{\alpha} H_{u+1,v+1}^{m,n+1}\left[\eta t^{\lambda};q\left|\begin{matrix}(1-\rho,\lambda),(a_j,A_j)_{1,u}\\(b_j,B_j)_{1,v},(1-\rho-\alpha,\lambda)\end{matrix}\right.\right], \text{ for } \lambda\geq 0 \\ t^{\alpha+\rho-1}(1-q)^{\alpha} H_{u+1,v+1}^{m+1,n}\left[\eta t^{\lambda};q\left|\begin{matrix}(a_j,A_j)_{1,u},(\rho+\alpha,-\lambda)\\(\rho,-\lambda),(b_j,B_j)_{1,v},\end{matrix}\right.\right], \text{ for } \lambda<0\end{cases} \tag{4.2}$$

where $\Re[\zeta\log\eta-\log\sin\pi\zeta]<0$ and $\rho>0$.

When $\rho=1$ the Corollary 4.2 reduces to the main result due to Kalla et al. [5].

4.2 Special Case of Theorem 3.2

If we set $\tau_i = 1,\ i = 1, \cdots$ in Theorem 3.2 we obtain the following formula for $I_q(\cdot)$-function.

Corollary 4.3 Let $\Re(\mu) > 0, |q| < 1$, then following result holds:

$$\mathrm{K}_q^{\mu}\left\{t^{\rho-1} I_{u_i,v_i;r}^{m,n}\left[\eta t^{\lambda}; q \left| \begin{matrix} (a_j, A_j)_{1,n}, & (a_{ji}, A_{ji})_{n+1,u_i;r} \\ (b_j, B_j)_{1,m}, & (b_{ji}, B_{ji})_{m+1,v_i;r} \end{matrix} \right.\right]\right\}$$

$$= \begin{cases} t^{\mu+\rho-1}(1-q)^{\alpha} q^{-\mu(\rho-1)-\mu(\mu+1)/2} I_{u_i+1,v_i+1,\tau_i;r}^{m+1,n} \\ \left[\eta t^{\lambda} q^{-\lambda\mu}; q \left| \begin{matrix} (a_j, A_j)_{1,n}, (a_{ji}, A_{ji})_{n+1,u_i;r}, (1-\rho, \lambda), \\ (1-\rho-\mu, \lambda), (b_j, B_j)_{1,m}, (b_{ji}, B_{ji})_{m+1,v_i;r} \end{matrix} \right.\right], \text{for} \lambda \geq 0 \\ t^{\mu+\rho-1}(1-q)^{\alpha} q^{-\mu(\rho-1)-\mu(\mu+1)/2} I_{u_i+1,v_i+1,\tau_i;r}^{m,n+1} \\ \left[\eta t^{\lambda} q^{-\lambda\mu}; q \left| \begin{matrix} (\rho+\mu, -\lambda), (a_j, A_j)_{1,n}, (a_{ji}, A_{ji})_{n+1,u_i;r} \\ (b_j, B_j)_{1,m}, (b_{ji}, B_{ji})_{m+1,v_i;r}, (\rho, -\lambda) \end{matrix} \right.\right], \text{for } \lambda < 0 \end{cases} \quad (4.3)$$

where $\Re[\zeta \log \eta - \log \sin \pi\zeta] < 0$ and $\rho > 0$.

If we set $r = 1, \tau_1 = 1, u_1 = u, v_1 = v, a_{j1} = a_j, b_{j1} = b_j, A_{j1} = A_j, B_{j1} = B_j$ in Theorem 3.2 we obtain the following formula for $H_q(\cdot)$-function.

Corollary 4.4 Let $\Re(\mu) > 0, |q| < 1$, then following result holds:

$$\mathrm{K}_q^{\mu}\left\{t^{\rho-1} H_{u,v}^{m,n}\left[\eta t^{\lambda}; q \left| \begin{matrix} (a_j, A_j)_{1,u} \\ (b_j, B_j)_{1,v} \end{matrix} \right.\right]\right\}$$

$$= \begin{cases} t^{\mu+\rho-1}(1-q)^{\alpha} q^{-\mu(\rho-1)-\mu(\mu+1)/2} \\ \times H_{u+1,v+1}^{m+1,n}\left[\eta t^{\lambda} q^{-\lambda\mu}; q \left| \begin{matrix} (a_j, A_j)_{1,u}, (1-\rho, \lambda), \\ (1-\rho-\mu, \lambda), (b_j, B_j)_{1,v} \end{matrix} \right.\right], \text{ for } \lambda \geq 0, \\ t^{\mu+\rho-1}(1-q)^{\alpha} q^{-\mu(\rho-1)-\mu(\mu+1)/2} \\ \times H_{u+1,v+1,\tau_i;r}^{m,n+1}\left[\eta t^{\lambda} q^{-\lambda\mu}; q \left| \begin{matrix} (\rho+\mu, -\lambda), (a_j, A_j)_{1,u} \\ (b_j, B_j)_{1,v}, (\rho, -\lambda) \end{matrix} \right.\right], \text{ for } \lambda < 0 \end{cases} \quad (4.4)$$

where $\Re[\zeta \log \eta - \log \sin \pi\zeta] < 0$ and $\rho > 0$.

When $\rho = 1,\ \lambda = 1$ the Corollary 4.4 reduces to the result due to Yadav and Purohit [12].

5 Conclusion

Since most of the special function can be expressed in term of the q-extension of Aleph function defined by (1.3). It is useful to make tables for the q-extension of Riemann-Liouville and Weyl fractional integral operators. In this regard we refer to Kalla et al. in [5] [eq. no. 3.1-3.11, table 1, p. 320], Yadav and Purohit in [12] [eq. no. 1-40, table 1, p. 241].

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