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Maintenance Activity Single- Machines Scheduling and Due-Date Assignment Simultaneously

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Abstract. Present work deals with analysis maintenance activity single-machine scheduling and due-date assignment simultaneously. The objective is to find the optimal maintenance position as well as the optimal location of the common due-date for minimizing the total of earliness, tardiness and due-date costs. We introduce a polynomial 0 (n^4) time solution for the problem. To solve the scheduling problem addressed in this work, we have to determine the job sequence, the common due-date, and the location of a maintenance activity. We also present two special cases of the problem and show that they can be optimally solved by a lower order algorithm.

Keywords: Scheduling, maintenance activity, due –date assignment, Earliness, tardiness.

1 Introduction

Recently, machine scheduling problems with the effect of learning or aging have received increasing attention. In scheduling with the learning effect, the actual processing time of a job decreases as a function if it is scheduled later in a sequence, while in scheduling with the aging effect the actual processing time of a job increases as a function if it is scheduled later in a sequence. Up to now, numerous papers have been investigated on scheduling problems with the effect of learning or aging to minimize performance measures. For details on this stream of research, , the reader may refer to the comprehensive Surveys by Janiak and Mikhail (1), Biskup (2), and Janiak and Rudek (3).

Applications of the common due-date problem in real-life situations can readily be found. For example: a common due-date might reflect an assembly environment in which the components of a product should all be ready at the same time in order to avoid staging delays, or a shop where several jobs constitute a single customer's order. Panwalkar et al. (4) first introduced a due-date assignment problem in scheduling. They considered that all the jobs have a common due-date. The objective was to find an optimal common due-date and an optimal schedule which minimizes the total earliness, tardiness and due-date costs. Their study provided a $O(n\log n)$ solution for the problem. Additionally, due to the many applications, production scheduling problems with a maintenance activity planning to improve the production efficiency or in preventing the machine from malfunction have been one of the most popular topics among researchers. During the maintenance activity, the machine becomes unavailable for processing jobs. Scheduling under such an environment is known as scheduling with availability constraints. As of now, plentiful research has been conducted on availability constraints under different environments, such as Chen and Yang (5), Yao and Huang (6), Gawiejnowicz (7), Chen and Tsou (8),Yang and Yang (9). Schmidt (10) and Ma et al. (11) provided extensively surveys related to machine scheduling problems with availability constraints.

Recently (12) represented transmission constraints, but did not recognize interconnection failures and Maintenance, (13) recognized the composite generation and transmission reliability but did not consider transmission maintenance. In this note we study a classical due-date assignment problem with the option of scheduling a maintenance activity. Panwalker, Smith and Seidmann [4] addressed the following single machine scheduling and common due-date assignment problem: All jobs have a common (but unknown) due-date. The objective is to find an optimal value of the due-date and optimal sequence which minimizes the total penalty based on the due-date value and the earliness or tardiness of each job. Panwalker et al. consider a set of n jobs available at time zero. The common due-date d is a decision variable. The processing time of job j is denoted by p_j, $j = 1,2,..., n.$ For a given schedule, the completion time of job j is denoted by C_j. The earliness and tardiness of job j are defined as $E_{[j]} = \max\{0, d - C_j\}$ and $T_j = \max\{0, C_j - d\}$, $j = 1,2,..., n.$, respectively. Three cost components are assumed: for earliness, for tardiness and for (delaying the) due-date. The unit penalties for earliness, tardiness and due-date are denoted by α, β and γ, respectively. The objective is to minimize the total cost,

i.e. $z = f(d,\pi) = \sum_{j=1}^{n}(\alpha E_{[j]} + \beta T_{[j]} + fd).$

The objective is to minimize the total earliness, tardiness and due-date costs. We show that there exists a polynomial time solution for the proposed problem. We also discuss two special cases of the problem and show that they can be optimally solved by a lower order algorithm.

Notations

We used the following notation throughout the paper and will introduce additional notation when needed:

n : The total number of jobs;

b : The basic maintenance time;

c : The maintenance factor;

i : The position of job preceding the maintenance activity;
d : The common due-date;
a_j : The aging factor of job j, $j = 1, 2, ..., n$;
$a_{[j]}$: The aging factor of a job scheduled in the j th position in a sequence, $j = 1, 2, ..., n$;
p_j : The normal processing time of job j, $j = 1, 2, ..., n$;
$p_{[j]}$: The normal processing time of a job scheduled in the j th position in a sequence, $j = 1, 2, ..., n$;
p_{jr} : The actual processing time of job j scheduled in the r th position in a sequence, $j, r = 1, 2, ..., n$;
C_j : The completion time of job j, $j = 1, 2, ..., n$;
E_j : The earliness of job j, $j = 1, 2, ..., n$, i.e. $E_j = \max\{0, d - C_j\}$;
T_j : The tardiness of job j, $j = 1, 2, ..., n$, i.e. $T_j = \max\{0, C_j - d\}$;
$C_{[j]}$: The completion time of a job scheduled in the j th position in a sequence, $j = 1, 2, ..., n$;
$E_{[j]}$: The earliness of a job scheduled in the j th position in a sequence, $j = 1, 2, ..., n$, i.e. $E_{[j]} = \max\{0, d - C_j\}$;
$T_{[j]}$: The tardiness of a job scheduled in the j th position in a sequence, $j = 1, 2, ..., n$, i.e. $T_j = \max\{0, C_j - d\}$.

2 Formulation

There are n independent jobs to be processed on a single machine. All the jobs are non-resumable and available for processing at time zero. Job j has a normal processing time P_j and a job-dependent aging factor a_j, where $a_j > 0$. Let job j is scheduled in the r^{th} position is a sequence, its actual processing time is $P_{jr} = P_j r^{a_j}$. All the jobs share a common due-date d.
The position and the starting time of the maintenance activity are not known in advance. It can be scheduled immediately after the processing of any one job has been completed. We further assume that: (1) after the maintenance activity, the machine will revert to its initial condition and the aging effect will start a new (2) the machine

maintenance duration is a linear function of its starting time and is represented as $f(t) = b + ct$,where $b > 0$ and $c \geq 0$ are constants, and t is the scheduled in the r^{th} position after the maintains activity in a sequence, the its actual processing time is given by $P_{jr} = P_j(r-i)^{a_j}$, where i denotes the position of a job preceding the maintain activity (i.e. position ($i+a$) is the first position after the maintains activity. The problem under consideration is to find jointly the optimal common due date d . The optimal maintenance position and the optimal job sequence π such that the following cost function is minimized.

$$z = f(d,\pi) = \sum_{j=1}^{n} (\alpha E_{[j]} + \beta T_{[j]} + fd) \quad (1)$$

Where $\alpha > 0$, $\beta > 0$ and $\gamma > 0$ ($\beta > \gamma$) are the earliness tardiness and due-date unit time penalties respectively, i.e. a job is finished on the due date, it will incur the manufacturing penalty cost only. If a job is finished earlier than its due date, it will incur the manufacturing penalty cost and the earliness penalty cost. On the other hand, if a job a finished later then its due-date, if will incur the manufacturing penalty costand the tardiness penalty cost, the problem can be denoted as $1/ma, P_{jr} = P_j r^{a_j} / \sum_{j=1}^{n} (\alpha E_j + \beta T_j + \gamma d)$, where ma in the second field denotes the maintenance activity.

Property 1: for any specified sequence π there exists an optimal due-date d in which coincides with some job completion times.

Property 2: For any specified sequence π , there exists an optimal common due-date equal to $c_{[k]}$ where $k = \lceil n(\beta - \gamma)/(\alpha + \beta) \rceil$.

Property 3: let there be two sequences of numbers x_i and y_i the sum $\sum_{i=1}^{n} x_i y_i$ of products of the corresponding elements is the least (largest) if the sequence are monotonic in the opposite (same) sense.

3 Optimal solution

By property 2.we can determine the optimal position of common due date d. if the maintenance activity is performed prior to the due-date (i.e. $i < k$), then the total cost is given by

$$z = \sum_{j=1}^{n} (\alpha E_{[j]} + \beta T_{[j]} + yd)$$

$$=\alpha\left[\sum_{j=1}^{i}(d-c_{[j]})+\sum_{j=i+1}^{k}(d-c_{[i]})\right]+\beta\sum_{j=k+1}^{n}(c_{[j]}-d)$$
$$+n\gamma\left[\sum_{j=1}^{i}P_{[j]}j^{a_{[j]}}+\sum_{j=i+1}^{k}P_{[j]}(j-i)^{a_{[j]}}+\left(b+c\sum_{j=1}^{i}P_{[j]}J^{a_{[j]}}\right)\right]$$
$$=\alpha\left[\sum_{j=1}^{i}(j-1)\sum_{j=1}^{i}P_{[j]}j^{a_{[j]}}+\sum_{j=i+1}^{k}(j-1)P_{[j]}(j-i)^{a_{[j]}}\right]+i\left(b+c\sum_{j=1}^{i}P_{[j]}J^{a_{[j]}}\right)\Bigg]$$
$$+\beta\sum_{j=i+1}^{k}(n-j+1)P_{[j]}(j-i)^{a_{[j]}}+n\gamma\left[\sum_{j=1}^{i}P_{[j]}j^{a_{[j]}}\right.$$
$$\left.+\sum_{j=i+1}^{k}P_{[j]}(j-i)^{a_{[j]}}+\left(b+c\sum_{j=1}^{i}P_{[j]}J^{a_{[j]}}\right)\right]$$

$$+\sum_{j=k+1}^{n}\beta(n-j+1)P_{[j]}(j-i)^{a_{[j]}} \tag{2}$$

Let x_{jr} be a 0/1 variable such that $x_{jr}=1$ if job j is scheduled in the rth position to be processed on the machines and $x_{jr}=0$ otherwise. Then for given $i<k$, the problem can be formulated as the following assignment problem

Minimize $z=\sum_{j=1}^{n}(\alpha E_{[j]}+\beta T_{[j]}+\gamma d)$

$$=b(n\gamma+\alpha i)+\sum_{j=1}^{n}\left\{\sum_{r=1}^{n}\left[n\gamma(1+c)+\alpha(r-1+(i))\right]P_j r^{a_j}x_{jr}\right\}$$
$$+\sum_{r=i+1}^{k}\left[n\gamma+\alpha(r-1)\right]P_j(r-i)^{a_j}x_{jr}$$
$$+\sum_{r=k+1}^{n}\left[\left(\beta(n-r-1)\right)\right]P_j(r-i)^{a_j}x_{jr} \tag{3}$$

Subject to

$$\sum_{r=1}^{n}x_{jr}=1, i=1,2,\ldots.,n \tag{4}$$

$$\sum_{j=1}^{n}x_{jr}=1, r=1,2,\ldots.,n \tag{5}$$

$$x_{jr} = 1 \text{ Or } 0, i = 1,2,...., n, r = 1,2,...., n \tag{6}$$

On the other hand, if themaintenance activity is performed after the due-date (i.e. $i \geq k$), then the total cost is given by

$$z = \sum_{j=1}^{n} (\alpha E_{[j]} + \beta T_{[j]} + \gamma d)$$
$$= \alpha \sum_{j=1}^{i} (d - c_{[j]}) + \beta \left[\sum_{j=k+1}^{i} (c_{[i]} - d) + \sum_{j=i+1}^{k} (c_{[j]} - d) \right] + n\gamma \sum_{j=1}^{k} P_{[j]} j^{a_{[j]}}$$
$$= \alpha \sum_{j=1}^{k} (j-1) P_{[j]} j^{a_{[j]}} + \beta \left[\sum_{j=k+1}^{i} (n-j+1) P_{[j]} j^{a_{[j]}} + \sum_{j=i+1}^{k} (n-j+1) P_{[j]} (j-i)^{a_{[j]}} \right]$$
$$+ \beta(n-i) \left(b + c \sum_{j=1}^{i} P_{[j]} j^{a_{[j]}} + n\gamma \sum_{j=1}^{k} P_{[j]} j^{a_{[j]}} \right)$$
$$= \beta b(n-i) + \sum_{j=1}^{k} \left[n\gamma + \alpha(j-1) + \beta c(n-i) \right] P_{[j]} j^{a_{[j]}}$$
$$+ \sum_{j=k+1}^{i} \left[\beta(n-j+1) + \beta c(n-i) \right] P_{[j]} j^{a_{[j]}}$$
$$+ \sum_{j=i+1}^{n} \beta(n-j+1) P_{[j]} (j-1)^{a_{[j]}} \tag{7}$$

Then, for given $i \geq k$, the problem can be formulated as the following assignment problem

$$\text{Minimize } z = \sum_{j=1}^{n} (\alpha E_{[j]} + \beta T_{[j]} + \gamma d)$$
$$= \beta b(n-i) + \sum_{j=1}^{n} \left\{ \sum_{r=1}^{k} \left[n\gamma + \alpha(r-1) + \beta c(n-i) \right] P_{[j]} j^{a_{[j]}} x_{jr} \right.$$
$$+ \sum_{r=k+1}^{i} \left[\beta(n-r+1) + \beta c(n-i) \right] P_{[j]} j^{a_{[j]}} x_{jr}$$
$$\left. + \sum_{r=i+1}^{n} \beta(n-r+1) P_{[j]} (r-1)^{a_{[j]}} x_{jr} \right\} \tag{8}$$

Subject to

$$\sum_{r=1}^{n} x_{jr} = 1, j = 1,2,....,n \tag{9}$$

$$\sum_{j=1}^{n} x_{jr} = 1, r = 1,2,....,n \tag{10}$$

$$x_{jr} = 1 \text{ Or } 0, j = 1,2,...., n, r = 1,2,...., n \tag{11}$$

Once the position of the maintenance activity is determined. Solving the associated assignment problem requires an effort of $0(n^3)$ time. Since the maintenance activity can be scheduled immediately after any hob, n different position must be evaluated to obtain the global optimal schedule.

4 Special cases

For the case where the aging factor $a_j = a, j = 1,2,.........,n$ i.e. the model with a job independent aging effect for given $i < k$, the scheduling problem above can be formulated as follows.

$$z = \sum_{j=1}^{n} (\alpha E_{[j]} + \beta T_{[j]} + \gamma d)$$

$$= \alpha \left[\sum_{j=1}^{i} (d - c_{[j]}) + \sum_{j=i+1}^{k} (d - c_{[i]}) \right] + \beta \sum_{j=k+1}^{k} (c_{[j]} - d) \Bigg]$$

$$+ n\gamma \left[\sum_{j=1}^{k} P_{[j]} j^a + \sum_{j=i+1}^{k} P_{[j]} (j-i)^a + \left(b + c \sum_{j=1}^{i} P_{[j]} J^a \right) \right]$$

$$= \alpha \left[\sum_{j=1}^{i} (j-1) P_{[j]} J^a + \sum_{j=i+1}^{k} (j-1) P_{[j]} (j-i)^a + i \left(b + c \sum_{j=1}^{i} P_{[j]} J^a \right) \right]$$

$$+ \beta \sum_{j=k+1}^{k} (n-j+1) P_{[j]} (j-i)^a + n\gamma \left[\sum_{j=1}^{i} P_{[j]} j^a + \sum_{j=i+1}^{k} P_{[j]} (j-i)^a + \left(b + c \sum_{j=1}^{i} P_{[j]} J^a \right) \right]$$

$$= b(n\gamma + \alpha i) + \sum_{j=1}^{i} \left[n\gamma (1+c) + \alpha (j - 1 + ci) \right] P_{[j]} J^a$$

$$+ \sum_{j=i+1}^{k} \left[n\gamma + \alpha (j-1) \right] P_{[j]} (j-i)^a + \sum_{j=k+1}^{n} \left[\beta (n-j+1) \right] P_{[j]} (j-i)^a \tag{12}$$

If the maintenance activity is performed after the due-date (i.e. $i \geq k$) then the total cost is given by

$$z = \sum_{j=1}^{n} (\alpha E_{[j]} + \beta T_{[j]} + \gamma d)$$

$$= \alpha \sum_{j=1}^{k} (d - c_{[j]}) + \left[\beta \sum_{j=k+1}^{i} (c_{[i]} - d) + \sum_{j=i+1}^{k} (c_{[j]} - d) \right] + n\gamma \sum_{j=1}^{k} P_{[j]} j^{a}$$

$$= \alpha \sum_{j=1}^{k} (j-1) P_{[j]} j^{a} + \left[\beta \sum_{j=k+1}^{i} (n-j+1) P_{[j]} j^{a} + \sum_{j=i+1}^{k} (n-j+1) P_{[j]} (j-i)^{a} \right]$$

$$+ \beta \sum_{j=i+1}^{k} (n-i) \left(b + c \sum_{j=1}^{i} P_{[j]} J^{a} \right) + n\gamma \sum_{j=1}^{k} P_{[j]} j^{a}$$

$$= \beta b(n-i) + \sum_{j=1}^{k} \left[n\gamma + \alpha(j-1) + \beta c(n-i) \right] P_{[j]} j^{a}$$

$$+ \sum_{j=k+1}^{i} \left[\beta(n-j+1) + \beta c(n-i) \right] P_{[j]} j^{a} x_{jr}$$

$$+ \sum_{j=i+1}^{n} \beta(n-j+1) P_{[j]} (j-1)^{a} \quad (13)$$

Let $M = \begin{cases} b(ny + i\alpha) & i < k \\ \beta b(n-i) & i \geq k \end{cases}$ (14)

Then $z = \sum_{j=1}^{n} (\alpha E_{[j]} + \beta T_{[j]} + \gamma d) = \sum_{j=1}^{n} w_j P_{[j]} + M$ (15)

Where

$$w_j = \begin{cases} \left[n\gamma(1+c) + \alpha(i-1+ci) \right] . j^{a} & j = 1, 2, ..., i \\ \left[n\gamma + \alpha(j-1) \right] . (j-i)^{a} & j = i+1, i+2, ..., k \\ \left[\beta(n-j+1) \right] . (j-i)^{a} & j = k+1, k+2, ..., n \end{cases} \quad (16)$$

For $i < k$ and

$$w_j = \begin{cases} [n\gamma + \alpha(j-1) + \beta c(n-i)].j^a & j = 1,2,....,i \\ [\beta(n-j+1) + \beta c(n-i)].j^a & j = k+1, k+2,....,k \\ [\beta(n-j+1)].(j-i)^a & j = i+1, i+2,....,n \end{cases} \quad (17)$$

for $i \geq k$

Based on the above analysis and lemmas 2 and 3, the following corollary holds.

Corollary 2. The $1/ma, P_{jr} = P_j r^a / \sum_{j=1}^{n}(\alpha E_{[j]} + \beta T_{[j]} + yd)$ problem can be solved in $0(n^2 \log n)$ time.

Proof. ByProperty 2, we obtain the optimal solution of the common due date k. Once the position of the maintenance activity has been determined, the term M in equation (17) is a constant. Then,we can obtain the local optimal solution via the following steps:

Step 1. For each position j $(j = 1,2,...,n)$, calculate the positional weights w_j mentioned above.

Step 2. Renumber the jobs in a non-increasing order with respect to their normal processing time. By Property 3, assign the job with the largest normal processing time to the position with the smallest value of positional weight w_j,the job with the next largest normal processing time to the position with the next smallest value of positional weight $w_j 0(1)$, etc.

Step 3. By equation (17), calculate the total cost Z.

The time complexity of Step 1 and 3 is $0(1)$ and the time complexity of Step 2 is $0(n \log n)$. Since the maintenance activity can be scheduled immediately after any job, n different positions must be evaluated to obtain the global optimal schedule.

Example

Assume $n = 7$ job and the common aging factor is $a = 0.3$. The processing time is $p_1 = 35,\ p_2 = 20,\ p_3 = 35,\ p_4 = 32, p_5 = 38, p_6 = 42, and\ p_7 = 25$. The earliness, tardiness and due-date unit cost are: $\alpha = 2, \beta = 14, \gamma = 4$. The basic time of maintenance activity is $b = 20$, the parameter of c is 0.05.

Solution. First we calculate the position of the due-date using property 2: $k = [n(\beta - \gamma)/(\alpha + \beta)]$=5

We explain the $c = 0.05$ and $a = 0.3$ present as follow.

Step 1. Calculate w_j, for $j = 1,2,...,7$.

$w_1 = \left[n\gamma(1+c)+\alpha ci)\right].1^a$ =29.7

$w_2 = \left[n\gamma(1+c)+\alpha(1+ci)\right].2^a$ =32.93

$w_3 = \left[n\gamma(1+c)+\alpha(3+ci)\right].3^a$ =33.09

$w_4 = (n\gamma + i\alpha).1^a$ =34

$w_5 = \left[n\gamma(i+1)\alpha\right].2^a$ =35023

$w_6 = \beta(n-r+1).3^a$ =41.71

$w_7 = \beta(n-r+1).4^a$ =22.74

Step 2. The optimal job sequence is (3,6,2,4,7,1,5).
Step 3. The total cost is Z= 11820.86

Table 1
Optimal job sequence and total cost for all the positions of the maintenance activity under job-independent aging effect ($a = 0.3$).

Position of MA	Job Sequence	Total cost
Prior to job 1	(3,4,1,7,2,6,5)	12231.05
Prior to job 2	(3,6,4,7,2,1,5)	12201.48
Prior to job 3	**(3,6,2,4,7,1,5)**	**11820.86**
Prior to job 4	(4,6,7,2,1,3,5)	11956.25
Prior to job 5	(4,1,6,7,2,3,5)	11975.56
Prior to job 6	(3,4,6,7,2,1,5)	12248.48
Not scheduled	(3,1,6,7,2,1,5)	122485.52

5 Conclusion

In this paper, we considered a maintenance activity single –machine scheduling problem and due-date assignment simultaneous. The objective was to find jointly the optimal common due-date, the optimal location of the maintenance activity, and the optimal job sequence for minimizing the total of earliness, tardiness and due-date costs. We showed that the problem can be optimally solved in polynomial time solution. We also discussed two special cases of the proposed problem and showed that they can be optimally solved by a lower order algorithm.

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