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A Various Method to Solve the Optimality for the Transportation Problem

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Abstract: In this paper, we have obtain optimal solution Transportation problem. The work is showed is more close with intial basic feasible solution (IBFS) obtained by vogel's approximation method (VAM). The comparison of optimal solution has been shown in this paper, by the Modi and Zero Point (z-p) method. The z-p method is applied for finding an optimal solution for T.P. directly. The proposed method is unique it gives always optimal solution without disturbance of degeneracy condition. The z-p method requires least interations to reach optimality condition as compared to the existing Modi Method. The z-p method avoides the degeneracy in the T.P.. The procedure for the solutions is illustrated with an example. Further, comparative study among the new algorithm and the existing algorithm is established by the means of sample.

Key words: Transportation problem (T.P.), VAM, Modi Method z-p method, Intial basic feasible solution (IBFS), L.P.P.

1 Introduction

The Transportation problem is one of the oldest applications of the L.P.P. The basic Transportation problem was origionally developed by Hitch Cock [5]. Efficient method of solution derived from simplex algorithm, which was developed in 1947, primarily by Dantzing [3] and then by charnes and coopers [2].

The Transportation problem is a special kind of the network optimization problems. It has the special data structure in solution. Characterized as a Transportation group.

The T.P constitutes on important part of the logistics supply chains. The problem basically deals with the determination of a coast plan for transporting a single commodity from a number of sources to a number of destination [6]. Network model of the TP is shown in figure 1. It aims to find the best way to fulfill the demand of n-demand points using the capacities of m-supply points.

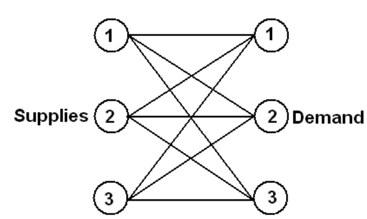

Figure -1: Network model of the Transportation problem.

All the optimal solution algorithm for solving TP needs an IBFS to obtain optimal solution:

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2 Methods of obtaining Intial Basic Feasible Solutions

a. North Corner Method [See 19]
b. Vogel Approximation Method [See 19]
c. Least cost Method [See 19]

a. Intial Basic Feasible Solutions by North West Corner Method

Table-1

	D1	D2	D3	D4	D5	D6	Supply
S1	2 (1)	3 (2)	5	11	4	2	3
S2	4	7	9 (3)	5 (1)	10	4	4
S3	12	25	9	6 (2)	26 (1)	12	3
S4	8	2	9	24	10 (1)	8 (2)	3
Demand	1	2	3	3	2	2	$\sum a_i = \sum b_j$ 13

$X_{11}=1, X_{12}=2, X_{13}=0, X_{23}=3, X_{24}=1, X_{34}=2, X_{45}=1, X_{46}=2$

Total Transportation cost is 104

b. Intial Basic Feasible Solutions by Least Cost Method

Table-2

	D1	D2	D3	D4	D5	D6	Supply
S1	2 (1)	3	5	11	4	2 (2)	3
S2	4	7 (1)	9	5 (3)	10	4	4
S3	12	25	9 (3)	6	26	12	3
S4	8	7 (1)	9	24	10 (2)	8 (2)	3
Demand	1	2	3	3	2	2	$\sum a_i = \sum b_j$ 13

$X_{11}=1, X_{12}=0, X_{16}=2, X_{22}=1, X_{24}=3, X_{33}=3, X_{42}=1, X_{45}=2$

Total Transportation cost is 82

c. Intial Basic Feasible Solutions by VAM

Table-3

	D1	D2	D3	D4	D5	D6	Supply
S1	2	3	5 [1]	11	4 [2]	2	3
S2	4 [1]	7 [1]	9	5	10	4 [2]	4
S3	12	25	9	6 [3]	26	12	3
S4	8	7 [1]	9 [2]	24	10	8	3
Demand	1	2	3	3	2	2	$\sum a_i = \sum b_j$ 13

$X_{13}=1$, $X_{15}=2$, $X_{21}=1$, $X_{22}=1$, $X_{24}=0$, $X_{26}=2$, $X_{34}=3$, $X_{42}=1$, $X_{43}=2$

Total Transportation cost is 75

3 Optimality

a. Taking IBFS due to Vogel's approximation method, because it has been sean that VAM determine on IBFS which is very close to the optimum solution, that is the number of interations required to reach optimality is lesser in this case. Now we proceed for optimality using MODI method.

Example-

Table-4

	D1	D2	D3	D4	D5	D6	Supply
S1	2	3	5 [1]	11	4 [2]	2	3
S2	4 [1]	7 [1]	9	5	10	4 [2]	4
S3	12	25	9	6 [3]	26	12	3
S4	8	7 [1]	9 [2]	24	10	8	3
Demand	1	2	3	3	2	2	$\sum a_i = \sum b_j$ 13

Here we determine a set Ui and Vj starting $U_2=0$ and using the relation Ui+Vj=Cij for occupied cells as shown below:

Cell (1, 3) $u_1 + v_3 = 5$
Cell (1, 5) $u_2 + v_5 = 4$
Cell (2, 1) $u_2 + v_1 = 4$
Cell (2, 2) $u_2 + v_2 = 7$
Cell (2, 4) $u_2 + v_4 = 5$
Cell (2, 6) $u_3 + v_6 = 4$
Cell (3, 4) $u_4 + v_4 = 6$
Cell (4, 2) $u_4 + v_7 = 7$
Cell (4, 3) $u_4 + v_9 = 9$
Taking $u_2 = 0$ we get the values
$u_1 = -4, u_2 = 0, u_3 = 1, u_4 = 0$
$v_1 = 4, v_2 = 7, v_3 = 9, v_4 = 5, v_6 = 4$

Now we find the net evaluation unoccupied cells by using the relation $u_i+v_j - c_{ij}$
Cell (1, 1) $=u_1+v_1 - c_{11} = -2$
Cell (1, 2) $=u_1+v_2 - c_{12} = 0$
Cell (1, 4) $=u_1+v_4 - c_{14} = -10$
Cell (1, 6) $=u_1+v_6 - c_{16} = -2$
Cell (2, 3) $=u_2+v_3 - c_{23} = -18$
Cell (2, 5) $=u_2+v_5 - c_{25} = -10$
Cell (3, 1) $=u_3+v_1 - c_{31} = -15$
Cell (3, 2) $=u_3+v_2 - c_{32} = -33$
Cell (3, 3) $=u_3+v_3 - c_{33} = -17$
Cell (3, 5) $=u_3+v_5 - c_{35} = -25$
Cell (3, 6) $=u_3+v_6 - c_{36} = -15$
Cell (4, 1) $=u_4+v_1 - c_{41} = -45$
Cell (4, 4) $=u_4+v_4 - c_{44} = -21$
Cell (4, 5) $=u_4+v_5 - c_{45} = -10$
Cell (4, 6) $=u_4+v_6 - c_{46} = -4$
Here, all the Zij - Cij ≤ 0, the current basic feasible solution is an optimum one.
The optimum solution is :
$X_{11}=1, X_{15}=2, X_{21}=1, X_{22}=1, X_{24}=0$ $X_{26}=2, X_{34=}3, X_{42}=1, X_{43}=2$
The Transportation cost associated with optimum schedule z =75.

b. Zero Point Method (Z-P) Method

We now introduce a new method called the zero point method for finding an optimal solution to T.P. the Z-P method are as follows:

Step-1: Construct the Transportation table for the given Transportation problem.

Step-2: Substract each row entries of the Transportation table from the row minimum and then substract each column entries of the resulting Transportation.

Step-3: In the reduced cost matrix there will be at least one zero in each row and column, then find the suffix value of all the zero's in the reduced cost matrix by following simplification, the suffix value is denoted by S, therefore S={Add the costs of nearest adjacent sides of zero's/ No. of cost added}

Step-4: Choose the maximum of S, if it has one maximum value then first apply to that demand corresponding to the cell, if it has more equal vlues then select {ai, bj} and supply to that demand maximum possible.
Step-5: After the above step, the exhausted demands (columns) or supplies (row) are to be trimmed. The resultant matrix much possess at least one zero in each row and coloumn else repeat step-2.
Stip-6: Repeat step-3 to step-5 until the optimal cost is obtained.

Example-

Table-5

	D1	D2	D3	D4	D5	D6	Supply
S1	2	3 [1]	5	11	4 [2]	2	3
S2	4 [1]	7	9 [1]	5	10	4 [2]	4
S3	12	25	9	6 [3]	26	12	3
S4	8	7 [1]	9 [2]	24	10	8	3
Demand	1	2	3	3	2	2	$\sum a_i = \sum b_j$ 13

Applying the Z-P method, which gives the following allocation:
$X_{12}=1$, $X_{15}=2$, $X_{21}=1$, $X_{23}=1$, $X_{26}=2$ $X_{34}=3$, $X_{42}=1$, $X_{43}=2$
Total Transportation cost is 75

Here we see in the table-6 that the optimality solution obtained by MODI method & Z-P method is equal to 75. However we also see that the optimality solution is directly obtained in z-p method without disturbing the degeneracy, however MODI method various iterations are required.

Table -6

S. NO.	Objectives	VAM	MODI	Z-P Method
1	Minimize cost	Nearer to optimal or equal	Equal	Equal
2	Allocation nodes	Equal	Equal	Equal
3	IBFS	As considered	Required	Not required

It will be more clearly shown in Histogram draw below.

Method 1 – NWC Rule
Method 2 – L-C Method
Method 3 – VAM

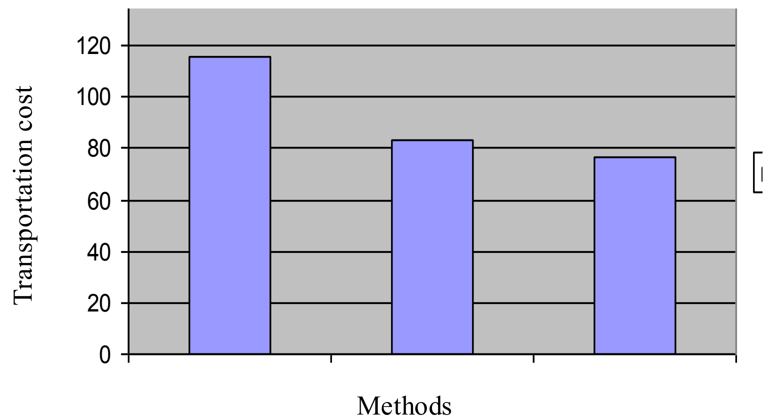

Figure – 2 Comparison of Transportation cost for IBFS

Method 1 – MODI Method
Method 2 – Z-P Method

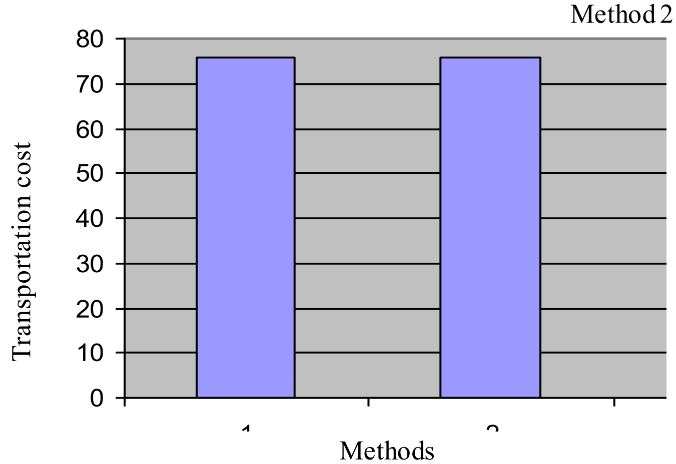

Figure – 3 Comparison of Optimal Values

The Figure-2 shows that the value of VAM is nearer to the optimal value and is darker. Also figure-3 shows that the optimal value obtain by both the methods are equal i.e. 75 with also minimizing the cost. This is shown in the tabulated form in table-7.

Table - 7

Problem	MODI	Z-P Method	Optimum
01	75	75	75

Conclusion

The present paper suggest an optimal heuristics approach for the Transportation Problem. From the investigation and the results given in table 7 and Figure-2 and Figure-3 both the MODI and Z-P Method gains the same optimal value. Here we see that the optimality solution is directly obtained in Z-P method, however in MODI, method various iterations are required. Also, the Z-P method avoides the degeneracy in the Transportation problem.

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