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On Completely $g^{\mu}b$ –irresolute Functions in supra topological spaces

M.TRINITA PRICILLA[1] and I.AROCKIARANI [2]
[1]Assistant Professor, Department of Mathematics
Jansons Institute of Technology,
Coimbatore-641659
abishai_kennet@yahoo.in
[2]Associate Professor, Department of Mathematics
NirmalaCollege for Women
Coimbatore-641 018.

Abstract. The focus of this paper is to formulate the notion of completely $g^{\mu}b$-irresolute function which is a stronger form of $g^{\mu}b$-irresolute function in supra topological spaces. Further the class of $g^{\mu}b$-closed sets are utilized to define the applications namely strongly $g^{\mu}b$-normal space, strongly $g^{\mu}b$-regular space, mildly $g^{\mu}b$-regular spaces and some of their characterizations are obtained.

Keywords: Completely $g^{\mu}b$-irresolute function, strongly $g^{\mu}b$-normal space, strongly $g^{\mu}b$-regular space, mildly $g^{\mu}b$-regular space.

1 Introduction

In 1970, Levine [7] introduced the concept of generalized closed sets in topological space and a class of topological spaces called T spaces. Extensive research on generalizing closedness was done in recent years by many Mathematicians [3, 4, 7, 8, and 9]. In 1972, Grossley and Hildebrand[4] introduced the notion of irresoluteness. Further many different forms of irresolute functions have been developed over the years.Andrijevic [2] defined a new class of generalized open sets in a topological space, the so-called b-open sets.

The notion of supra topological spaces, S-S continuous functions and S^* - continuous functions was initiated by A.S.Mashhour et al [9] in 1983. In 2010, O.R.Sayed and Takashi Noiri [11] formulate the concept of supra b - open sets and supra b - continuity on topological spaces. In this paper, we present and characterize the concepts of completely$g^{\mu}b$-irresolute functions. As applications some new classes of spaces namely strongly $g^{\mu}b$-regular space, mildly $g^{\mu}b$-regular spaces are established to derive their properties. Also some related properties of these functions are analyzed.

2. PRELIMINARIES

Definition: 2.1 [7]

A subclass $\tau^* \subset P($ is called a supra topology on X if $X \in \tau^*$ and τ^* is closed under arbitrary union. (X, τ^*) is called a supra topological space (or supra space).The members of τ^* are called supra open sets.

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Definition: 2.2 [7]

The supra closure of a set A is defined as $cl^{\mu}(A) = \cap\{B : B \text{ is supra closed and } A \subseteq B\}$

The supra interior of a set A is defined as $Int^{\mu}(A) = \cup\{B : B \text{ is supra open and } A \supseteq B\}$

Definition: 2.3[3]

Let (X, μ) be a supra topological space. A set A of X is called supra generalized b - closed set (simply g^{μ} b - closed) if $bcl^{\mu}(A) \subseteq U$ whenever $A \subseteq U$ and U is supra open. The complement of supra generalized b - closed set is supra generalized b - open set.

Definition: 2.4[13]

A Subset A of (X, μ) is said to be supra regular open if $A = Int^{\mu}(Cl^{\mu}(A))$ and supra regular closed if $A = cl^{\mu}(Int^{\mu}(A))$.

Definition: 2.5 [11]

A function $f : (X, \tau) \to (Y, \sigma)$ is said to be g^{μ} b –continuous if $f^{-1}(V)$ is g^{μ} b - closed in (X, τ) for every supra closed set V of (Y, σ).

Definition: 2.6 [11]

A function $f : (X, \tau) \to (Y, \sigma)$ is said to be g^{μ} b –irresolute if $f^{-1}(V)$ is g^{μ} b - closed in (X, τ) for every g^{μ} b – closed set V of (Y, σ).

Definition: 2.7[11]

A map $f : (X, \tau) \to (Y, \sigma)$ is said to be M- $g^{\mu}b$ -closed map if the image f(A) is $g^{\mu}b$ -closed in (Y, σ) for every $g^{\mu}b$ -closed set A in (X, τ).

3. Characterizations of completely g^{μ} b-irresolute Functions

Definition: 3.1

A function $f : (X, \tau) \to (Y, \sigma)$ is said to be completely g^{μ}b-irresolute if $f^{-1}(V)$ is regular$^{\mu}$ open in (X, τ) for every g^{μ}b-open set V in (Y, σ).

Definition: 3.2

A function $f : (X, \tau) \to (Y, \sigma)$ is said to be completely$^{\mu}$continuous if $f^{-1}(V)$ is regular$^{\mu}$ open in (X, τ) for every supra -open set V in (Y, σ).

Definition: 3.3

A function $f : (X, \tau) \to (Y, \sigma)$ is said to be completely g^{μ}b continuous if $f^{-1}(V)$ is supra open in (X, τ) for every g^{μ}b- open set V in (Y, σ).

Theorem: 3.4

(i) Every completely g^{μ}b-irresolute function is g^{μ}b-irresolute.
(ii) Every completely g^{μ}b-irresolute function is g^{μ}b-continuous.

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(iii) Every completely$^{\mu}$continuous function is g^{μ}b-continuous.
(iv) Every completely$^{\mu}$continuous function is completely g^{μ}b-irresolute.
(v) Every completely g^{μ}b-irresolute function is completely g^{μ}b continuous.

Proof: It is obvious.

Remark: 3.4

The converse of the theorem need not be true as shown by the following example

Example: 3.5

Let X = {a b,c}; $\tau = \{\phi, X, \{a\}, \{c\}, \{a, c\}\}$Define $f:(X,\tau)\to(X,\tau)$ be an identity function. Here f is both g^{μ}b-irresolute andg^{μ} b-continuous functions. But $f^{-1}\{a\}=\{a\}$is not regular$^{\mu}$ -closed. Therefore f is not completely g^{μ} b-irresolute.

Theorem: 3.6

A function $f:(X,\tau)\to(Y,\sigma)$ is completely g^{μ}b-irresolute if the inverse image of each g^{μ}b-closed set is regular$^{\mu}$ closed in (X,τ).

Proof: Let V be g^{μ}b-closed in (Y,σ). Then $Y-V$ is g^{μ}b-open in Y. By hypothesis, $f^{-1}(Y-V)$ is regular$^{\mu}$ open in X implies $X-f^{-1}(V)$ is regular$^{\mu}$ open in X. That is, $f^{-1}(V)$is regular$^{\mu}$ closed in X. Hence f is completely g^{μ}b-irresolute.

Theorem: 3.7

The following are equivalent for a function $f:(X,\tau)\to(Y,\sigma)$.

1. f is completely g^{μ}b-irresolute
2. For each $x \in X$ and each g^{μ} b-open set Vof Y containing $f(x)$, there exists a regular$^{\mu}$-open set U in X containing x such that $f(U) \subset V$.
3. $f^{-1}(V)$is regular$^{\mu}$-open in X for every g^{μ} b-open set Vof Y.
4. $f^{-1}(F)$is regular$^{\mu}$-closed in X for every g^{μ} b-closedset Fof Y.

Proof: It is obvious.

Theorem: 3.8

The following hold for function $f:(X,\tau)\to(Y,\sigma)$ and $g:(Y,\sigma)\to(Z,\gamma)$.

(a) If f is completely g^{μ}b-irresolute and g is g^{μ}b-continuous then $g\circ f:(X,\tau)\to(Z,\gamma)$ is completely$^{\mu}$continuous function.

(b) If f is completely g^{μ}b-irresolute and g is g^{μ}b-irresolute then $g\circ f:(X,\tau)\to(Z,\gamma)$ is completely g^{μ}b-irresolute function.

(c) If f is completely$^{\mu}$ continuous and g is completely g^{μ}b-irresolute then $g\circ f:(X,\tau)\to(Z,\gamma)$ is completely g^{μ}b-irresolute function.

Proof: Straight forward.

Theorem: 3.9

If a mapping $f:(X,\tau)\to(Y,\sigma)$ is M-g$^{\mu}$b-closed then for each subset B of Y and each g$^{\mu}$b-open set U of X containing $f^{-1}(B)$ there exists g$^{\mu}$b-open set V in Y containing B such that $f^{-1}\{V\}\subset U.$

Proof: Let B be a subset of Y and U be g$^{\mu}$b-open set of X such that $f^{-1}\{B\}\subset U.$ Then $Y-f(X-U)=V$ is g$^{\mu}$b-open set of Y containing B such that $f^{-1}\{V\}\subset U.$

4. Applications

Definition: 4.1

A space X is said to be almost $^{\mu}$ -connected (resp.g$^{\mu}$b-connected)if there does not exist disjoint regular $^{\mu}$ open (resp.g$^{\mu}$b-open) sets A and B such that $A\cup B=X.$

Definition: 4.2

A space X is said to be r$^{\mu}$ -disconnected if there exists two regular$^{\mu}$-open sets R and W such that $X=R\cup W \ and \ R\cap W=\phi$ Otherwise X is called r$^{\mu}$ -connected.

Theorem: 4.3

If X is r$^{\mu}$ -connected space and $f:(X,\tau)\to(Y,\sigma)$ is completely g$^{\mu}$b-irresolute surjection, then Y is g$^{\mu}$b-connected.

Proof: Suppose Y is not g$^{\mu}$b-connected then there exist non-empty g$^{\mu}$b -open sets $H_1 \ and \ H_2$ in Y such that $H_1\cap H_2=\phi \ and \ Y=H_1\cup H_2$. Since f is completely g$^{\mu}$b-irresolute function, we have $f^{-1}(H_1)\cap f^{-1}(H_2)=\phi \ and$ $X=f^{-1}(H_1)\cup f^{-1}(H_2)$ .Since f is surjection $f^{-1}(Hj)=\phi$ and $f^{-1}(Hj)\in R^{\mu}o(X)$ for j=1,2. This implies X is not r$^{\mu}$ -connected which is a contradiction.

Definition: 4.4

A supra topological space X is said to be g$^{\mu}$b -regular($almost^{\mu} regular$) if for each supra closed(resp. regular $^{\mu}$ closed) set F of X and each $x\notin F$,there exist disjoint g$^{\mu}$b -open(resp.supra open) sets U and V such that $x\in U \ and \ F\subset V.$

Definition: 4.5

A space X is called strongly g$^{\mu}$b-regular if for each g$^{\mu}$b-closed subsets F and each point $x\notin F$, there exists disjoint g$^{\mu}$b-open sets U and V in X such that $x\in U \ and \ F\subset V.$

Definition: 4.6

A space X is called mildly g^{μ}b-regular if for each regular$^{\mu}$-closed subset F and every point $x \notin F$, there exists disjoint g^{μ}b-open sets U and V in X such that $x \in U$ *and* $F \subset V$.

Theorem: 4.7

(i) If f is completely g^{μ} b-irresolute, g^{μ} b-open from an $g^{\mu}b$-regular space X onto a space Y, then Y is strongly g^{μ} b-regular.

(ii) If f is completely g^{μ} b-irresolute, g^{μ} b-open from an almost $^{\mu}$ regular space X onto a space Y, then Y is strongly g^{μ} b-regular.

Proof: (i)Let F be g^{μ}b-closed set of Y and let $y \notin F$. Take $y = f(x)$. Since f is completely g^{μ}b-irresolute $f^{-1}(F)$ is regular$^{\mu}$-closed and so supra closed in X and $x \notin f^{-1}(F)$. By almost regularity of X, there exists disjoint g^{μ}b-open sets U and V such that $x \in U$ *and* $f^{-1}(F) \subset V$. We obtain that $y = f(x) \in f(U)$ and $F \subset f(V)$ such that $f(U)$ and $f(V)$ are disjoint g^{μ}b-open sets. Thus, Y is strongly g^{μ}b-regular.

(ii)It is similar to (i)

Theorem: 4.8

If $f:(X,\tau) \to (Y,\sigma)$ is completely g^{μ}b-irresolute, M-g^{μ}b-closed injection of a mildly g^{μ}b-regular space onto a space Y, then Y is strongly g^{μ}b-regular space.

Proof: Let F be g^{μ}b-closed subset of Y and let $y \notin F$. Then $f^{-1}(F)$ is regular$^{\mu}$-closed subset of X such that $f^{-1}(y) = x \notin f^{-1}(F)$. Since X is mildly g^{μ}b-regular space, there exists disjoint g^{μ}b-open sets U and V in X such that $f^{-1}(y) \in U$ *and* $f^{-1}(F) \subset V$.. By theorem 3.15,there exists g^{μ}b-open sets $G = Y - f(X - U)$ such that $f^{-1}(G) \subset U$, $y \in G$ *and* $H = Y - f(X - V)$ *such that* $f^{-1}(H) \subset V, F \subset H$.

Clearly G and H are disjoint g^{μ}b-open subsets of Y. Hence Y is strongly g^{μ}b-regular.

Definition: 4.9

A space X is said to be strongly g^{μ}b-normal (resp. mildly g^{μ}b-normal) if for each pair of distinct g^{μ}b-closed (resp.regular$^{\mu}$-closed)sets A and B of X, there exist distinct g^{μ}b-open sets U and V such that $A \subset U$ *and* $B \subset V$.

Definition: 4.10

A space X is said to be almost$^{\mu}$ -normal if for each regular$^{\mu}$-closed sets A and B such that $A \cap B = \phi$, there exist supra open sets U and V such that $A \subset U$ *and* $B \subset V$.

Theorem: 4.11

If $f:(X,\tau)\to(Y,\sigma)$ is completely g^{μ}b-irresolute,M-g^{μ}b-closed function from a mildly g^{μ}b-normal space X onto a space Y, then Y is strongly g^{μ}b-normal.

Proof: Let A and B be two disjoint g^{μ}b-closed subsets of Y. Then $f^{-1}(A)$ *and* $f^{-1}(B)$ are disjoint regular$^{\mu}$-closed subsets of X. Since X is mildly g^{μ}b-normal space, there exists disjoint g^{μ}b-open set U and V in X such that $f^{-1}(A)\subset U$ *and* $f^{-1}(B)\subset V$. Then by theorem 3.15, $G=Y-f(X-U)$ and $H=Y-f(X-V)$ such that $A\subset G, f^{-1}(G)\subset U\ \ B\subset H, f^{-1}(H)\subset V.$
Clearly G and H are disjoint g^{μ}b-open subsets of Y. Hence Y is strongly g^{μ}b-normal.

Theorem: 4.12

If f is completely g^{μ}b-irresolute, g^{μ}b-open from an almost$^{\mu}$normal space X onto a space Y, then Y is strongly g^{μ}b-normal.

Proof: Let A and B be two disjoint g^{μ}b-closed subsets in Y. Since f is completely g^{μ}b-irresolute $f^{-1}(A)$ *and* $f^{-1}(B)$ are disjoint regular$^{\mu}$-closed and so supra closed in X . By almost$^{\mu}$ normality of X, there exists disjoint supra open sets U and V such that $f^{-1}(A)\subset U$ *and* $f^{-1}(B)\subset V$. We obtain that $A\subset f(U)$ and $B\subset f(V)$ such that $f(U)$ *and* $f(V)$ are disjoint g^{μ}b-open sets. Thus, Y is strongly g^{μ}b-normal.

Definition: 4.13

A space (X,τ) is said to be g^{μ}b-T_0 (resp.r$^{\mu}$ -T_0) if for each pair of distinct points x *and* y *of* X there exists g^{μ}b-open(resp.regular$^{\mu}$-open)set U such that either $x\in U, y\in X\setminus U$ *or* $y\in U, x\in X\setminus U.$

Definition: 4.14

A space (X,τ) is said to be g^{μ}b-T_1 (resp.r$^{\mu}$ -T_1) if for each pair of distinct points x *and* y *of* X there exists g^{μ}b-open(resp.regular$^{\mu}$-open)sets U_1 *and* U_2 such that $x\in U_1, y\in U_2, x\notin U_2$ *and* $y\notin U_1$.

Definition: 4.15

A space X is said to be g^{μ}b-Hausdorff (resp.r$^{\mu}$ -T_2) if for any $x,y\in X, x\neq y,$ there exist g^{μ} b-open(resp.regular$^{\mu}$-open) sets G and H such that $x\in G, y\in H$ *and* $G\cap H=\phi.$

Theorem: 4.16

Let $f:(X,\tau)\to(Y,\sigma)$ be injective and completely g^{μ}b-irresolute function. If Y is g^{μ}b-Hausdorff space, then X is r$^{\mu}$ - T_2.

Proof: Let x *and* y be any two distinct points of X. Since f is injective, $f(x)\neq f(y)$. Since Y is g^{μ}b-Hausdorff space there exists disjoint g^{μ} b-open sets G and H such that $f(x) \in G$ *and* $f(y) \in H$. Since f is completely g^{μ}b-irresolute function it follows that $f^{-1}(G), f^{-1}(H)$ are disjoint regular$^{\mu}$-open sets containing x *and* y respectively. Hence X is r^{μ} - T_2.

Theorem: 4.17

If $f:(X,\tau)\to(Y,\sigma)$ is completely g^{μ}b-irresolute injective function and Y is g^{μ}b- T_1,then X is r^{μ} - T_1.

Proof: Let x, y be distinct points of X. Since Y is g^{μ}b- T_1,there exists g^{μ}b-open sets F_1 and F_2 of Y such that $f(x) \in F_1, f(y) \in F_2; f(x) \notin F_2, f(y) \notin F_1$. Since f is injective completely g^{μ}b-irresolute function we have $x \in f^{-1}(F_1), y \in f^{-1}(F_2)$, $x \notin f^{-1}(F_2), y \notin f^{-1}(F_1)$. Hence X is r^{μ} - T_1.

Theorem: 4.18

If $f:(X,\tau)\to(Y,\sigma)$ is completely g^{μ} b-irresolute injective function and Y is g^{μ} b-hausdorff,then X is r^{μ} - T_2.

Proof: Let x, y be distinct points of X. Then $f(x) \neq f(y) \in Y$. Since Y is g^{μ}b-hausdorff there exists g^{μ}b-open sets U *and* V such that $f(x) \in U, f(y) \in V$.Since f is completely g^{μ}b-irresolute, $f^{-1}(U), f^{-1}(V)$ are regular$^{\mu}$-open such that $x \in f^{-1}(U)$ *and* $y \in f^{-1}(V)$, $f^{-1}(U) \cap f^{-1}(V) = \phi$. Hence X is r^{μ} - T_2. Proof for (ii) and (iii) is similar to (i)

Theorem: 4.19

(i) If $f:(X,\tau)\to(Y,\sigma)$ is completely g^{μ} b-irresolute injective function and Y is g^{μ} b-hausdorff, then X is g^{μ} b -T_2.

(ii) If $f:(X,\tau)\to(Y,\sigma)$ is completely g^{μ} b-irresolute injective function and Y is g^{μ} b-hausdorff, then X is supra -T_2.

Proof: Similar to theorem 4.18.

Definiton: 4.20

A supra topological space X is said to be

(i) nearly$^{\mu}$ compact if every regular$^{\mu}$ open cover of x has a finite subcover;

(ii) nearly countably$^{\mu}$ compact if every countable cover by regular$^{\mu}$ open sets has a finite subcover ;

(iii) nearly$^{\mu}$ Lindelof if every cover of X by regular$^{\mu}$ open sets has a countable subcover ;

(iv) g^{μ}b-compact if every g^{μ}b-open cover of X has a finite subcover;

(v) countablyg^{μ}b-compact if every g^{μ}b-open countable cover of X has a finite

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subcover;
(vi) $g^{\mu}b$-Lindelof if every cover of X by $g^{\mu}b$-open sets has a countable subcover.

Theorem: 4.21
Let $f:(X,\tau)\to(Y,\sigma)$ be a completely $g^{\mu}b$-irresolute surjective function. Then the following statements hold:

(i) If X is nearly $^{\mu}$ compact, then Y is $g^{\mu}b$-compact.
(ii) If X is nearly $^{\mu}$ Lindelof, then Y is $g^{\mu}b$-Lindelof.

Proof: (i) Let $f:(X,\tau)\to(Y,\sigma)$ be a completely $g^{\mu}b$-irresolute function of nearly $^{\mu}$ compact space X onto a space Y. Let $\{U_\alpha : \alpha\in\Delta\}$ be any $g^{\mu}b$-open cover of Y. Then, $\{f^{-1}(U_\alpha) : \alpha\in\Delta\}$ is a regular $^{\mu}$ open cover of X. Since X is nearly $^{\mu}$ compact, there exists a finite subfamily, $\{f^{-1}(U_{\alpha_i}); i=1,2...n\}$ of $\{f^{-1}(U_\alpha) : \alpha\in\Delta\}$ which cover X. It follows then that $\{U_{\varepsilon_i}; i=1,2...n\}$ is a finite subfamily of $\{U_\alpha : \alpha\in\Delta\}$ which cover Y. Hence, the space Y is $g^{\mu}b$-compact space.

Definition: 4.22
A supra topological space X is said to be

(i) S^{μ}-closed (resp.$g^{\mu}b$-closed compact) if every regular $^{\mu}$ closed (resp.$g^{\mu}b$-closed) cover of X has a finite subcover;
(ii) Countably S^{μ}-closed-compact(resp. countably$g^{\mu}b$-closed compact) if every countable cover of X by regular $^{\mu}$ closed (resp.$g^{\mu}b$-closed) sets has a finite subcover;
(iii) S^{μ}-Lindelof (resp.$g^{\mu}b$-closed Lindelof) if every cover of X by regular closed (resp.$g^{\mu}b$-closed) sets has a countable subcover.

Theorem: 4.23
Let $f:(X,\tau)\to(Y,$ be a completely $g^{\mu}b$-irresolute surjective function. Then the following statements hold:

(i) If X is S^{μ}-closed, then Y is $g^{\mu}b$-closed compact.
(ii) If X is S^{μ}-Lindelof, then Y is $g^{\mu}b$-closed Lindelof.
(iii) If X is Countably S^{μ}-closed, then Y is countably$g^{\mu}b$-closed compact.

Proof: Similar to theorem 4.19.

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