International Journal of Mathematical Engineering and Science
ISSN : 2277-6982 Volume 1 Issue 10 (October 2012)
http://www.ijmes.com/ https://sites.google.com/site/ijmesjournal/

On Somewhat πgb-Continuous and Somewhat πgb-Open Functions

D.Sreeja [*] and C.Janaki [**]
Asst.Professor, Dept. of Mathematics, CMS College of Science and Commerce Coimbatore-6.
[*]E-mail:sreejadamu@gmail.com
Asst.Professor, Dept. of Mathematics, L.R.G Govt.Arts College for Women, Tirupur-4.
[**]E-mail:janakicsekar@yahoo.com

Abstract: In this paper, we have introduced new continuous and open functions called somewhat πgb-continuous and somewhat πgb -open functions by using πgb-open sets . Further somewhat almost πgb-open sets, and somewhat M- πgb-open functions are also discussed. Its various characterizations and properties are established.

Keywords: somewhat πgb-continuous, somewhat πgb- irresolute, somewhat πgb-open,somewhat πgb-dense, somewhat πgb-seperable, somewhat almost πgb-open sets and somewhat M- πgb-open functions.

1.Introduction:

Andrijevic [1] introduced a new class of generalized open sets called b-open sets in a topological space. This type of sets was discussed by Ekici and Caldas [6] under the name of γ-open sets. Levine [10] introduced the concept of generalized closed sets in topological space and a class of topological spaces called $T_{½}$ spaces. The concepts of feebly continuous functions and feebly open functions were first introduced and studied by Zdenek Frolik [15]. Gentry and Hoyle [7] introduced and studied the concept of somewhat open functions which are Frolik functions with some conditions being dropped. These ideas are closely related to weakly equivalent topologies which was first introduced by [14].S.S Benchalli and Priyanka M.Bansali [2] introduced somewhat b-continuous and somewhat b-open functions in topological spaces.

In this paper, by using πgb-open sets new functions called somewhat πgb- continuous, somewhat πgb- irresolute and somewhat πgb -open functions are introduced and discussed. Also somewhat almost πgb-open sets and somewhat M- πgb-open functions are discussed. These findings results in procuring several characterizations and properties of this class.

2 .Preliminaries

Throughout this paper (X, τ) and (Y, τ) represent non-empty topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a space (X,τ) cl(A) and int(A) denote the closure of A and the interior of A respectively. (X, τ) will be replaced by X if there is no chance of confusion.

Definition2.1: A subset A of a space X is said to be:(1) semi open [9] if A ⊆ Cl(Int(A)).

(2) a regular open set if A= int (cl(A)) and a regular closed set if A= cl(int (A));

(3) b-open [1] or sp-open [4], γ –open [6] if A ⊂ cl(int(A))∪int (cl(A)).

Definition 2.2 :A subset A of a space (X, τ) is called

1) a generalized b-closed (briefly gb-closed)[8] if bcl(A) ⊂ U whenever A ⊂ U and U is open.

2) πg-closed [5] if cl(A)⊂U whenever A⊂ U and U is π-open.

3) πgb -closed [12] if bcl(A)⊂U whenever A ⊂U and U is π-open in (X, τ).

By πGBC(τ) we mean the family of all πgb- closed subsets of the space(X, τ).

Definition 2.3: A function f: (X, τ) $\rightarrow$ (Y, σ) is called

1) πgb- continuous [12] if every $f^{-1}(V)$ is πgb- closed in (X, τ) for every closed set V of (Y,σ).

2) πgb- irresolute [12] if $f^{-1}(V)$ is πgb- closed in(X, τ) for every πgb -closed set V in (Y, σ).

Definition 2.4[13]: A map f: X$\rightarrow$Y is said to be πgb-open if for every open set F of X,f(F) is πgb-open in Y.

Definition2.5[13]:A map f: (X, τ) $\rightarrow$(Y, σ) is said to be a M- πgb-open map if the image f(A) is πgb-open in Y for every πgb-open set A in X.

Definition 2.6: A subset D of a topological space X is said to be dense (or everywhere dense) in X if the closure of D is equal to X . Equivalently, D is dense if and only if D intersects every non-empty open set.

Definition 2.7 [7]**:**A function f : X $\rightarrow$ Y is said to be somewhat continuousif for U $\in$ σ and f−1(U) $\neq$ φ there exists an open set V in X such that V $\neq$ φ and V $\subseteq$ $f^{-1}(U)$.

Definition 2.8[11]**:** A function f : X $\rightarrow$ Y is said to be somewhat semi continuous if for U $\in$ σ and $f^{-1}(U) \neq \varphi$ there exists a semi open set V in X such that V $\neq$ φ and V $\subseteq$ $f^{-1}(U)$.

Remark 2.9 [11]**:**Every somewhat continuous function is somewhat semi continuous function.

Definition2.10 [7]**:** A function f : X $\rightarrow$ Y is said to be somewhat open function provided that for U $\in$ τ and U $\neq$ φ, there exists an open set V in Y such that V $\neq$ φ and V $\subseteq$ f(U).

Definition2.11[11]**:** A function f : X $\rightarrow$ Y is said to be somewhat semi open function provided that for U $\in$ τ and U $\neq$ φ, there exists a semi open set V in Y such that V $\neq$ φ and V $\subseteq$ f(U).

Remark 2.12[11]: Every somewhat open function is somewhat semi open function but the converse need not be true in general.

3.Somewhat πgb-continuous functions

Definition 3.1 Let (X, τ) and (Y, σ) be any two topological spaces. A function f : X $\rightarrow$ Y is said to be somewhat πgb-continuous function if for every U $\in$ σ and $f^{-1}(U) \neq \phi$ there exists a πgb-open set V in X such that V $\neq$ ϕ and V $\subseteq$ $f^{-1}(U)$.

Example 3.2 Let X = {a, b, c}, τ = {X, {a},{a,b},ϕ}, σ = {X, ϕ, {a}}. Now define a function f : (X, τ) $\rightarrow$ (X, σ) as follows : f(a) = b, f(b) = a ,f(c)=c.Then clearly f is somewhat πgb-continuous function.

Theorem 3.3 Every somewhat semi continuous function is somewhat πgb-continuous function.

Proof. Let f : X $\rightarrow$ Y be somewhat semi continuous function. Let U be any open set in Y such that $f^{-1}(U) \neq \phi$. Since f is somewhat semi continuous, there exists a semi open set V in X such that V $\neq$ ϕ and V $\subseteq$ $f^{-1}(U)$. Since every semi open set is πgb-open, there exists a πgb-open set V such that V $\neq\phi$ and V $\subseteq$ $f^{-1}(U)$, which implies that f is somewhat b-continuous function.

Remark 3.4 Converse of the above theorem need not be true in general which follows from the following example.

Example 3.5 Let X = {a, b, c}, τ = {X, {a,b},ϕ}, σ = {X, ϕ, {a}}. Define a function f : (X, τ) $\rightarrow$ (X, σ) by f(a) = a, f(b) = b,f(c)=c. Then f is somewhat πgb-continuous function but not somewhat semi continuous function.

Theorem 3.6 Every somewhat continuous function is somewhat πgb-continuous function.

Proof. Follows from Theorem 3.3 and Remark 2.4.

Remark 3.7 Converse of the above theorem need not be true in general which follows from the following example.

Example 3.8 Let X = {a, b, c, d}, τ = {X, {c}, ϕ}, σ = {X, ϕ, {a}, {b}, {a, b}}.Define a function f : (X, τ) → (X, σ) by f(a) = a, f(b) = c, f(c) = d and f(d) = b. Then clearly f is somewhat πgb-continuous function but not a somewhat continuous function.

Theorem 3.9 Let f : (X, τ) → (Y, σ) and g : (Y, σ) → (Z, η) be any two functions. If f is somewhat πgb-continuous function and g is continuous function, then g ∘ f is somewhat πgb-continuous function.

Proof. Let U ∈η. Suppose that $g^{-1}(U) \neq \phi$. Since U ∈ η and g is continuous function $g^{-1}(U) \in \sigma$. Suppose that $f^{-1}g^{-1}(U) \neq \phi$. Since by hypothesis f is somewhat πgb-continuous function, there exists a πgb-open set V in X such that V ≠ ϕ and $V \subseteq f^{-1}(g^{-1}(U))$. But $f^{-1}(g^{-1}(U)) = (g \circ f)^{-1}(U)$, which implies that $V \subseteq (g \circ f)^{-1}(U)$. Therefore g ∘ f is somewhat πgb-continuous function.

Remark 3.10 If f is continuous function and g is somewhat πgb-continuous function, then it is not necessarily true that g ∘ f is somewhat πgb-continuous function.

Example 3.11 Let X = {a, b, c}, τ = σ = {X, ϕ, {a},{b},{a,b},} and η = {X, ϕ, {b,c},{c}}.Define f : (X, τ) → (X, σ) by f(a) =a,f(b)=b and f(c)=c and define g : (X, σ) →(X, η) by g(a) = b ,g(b) = a and g(c)=c. Then clearly f is continuous function and g is somewhat πgb-continuous function but g ∘ f is not a somewhat πgb-continuous function.

Definition 3.12 Let A be a subset of a topological space (X, τ). Then A is said to be πgb-dense in X if there is no proper πgb-closed set C in X such that A⊂ C ⊂ X.

Theorem 3.13 Let f : (X, τ) → (Y, σ) be a function. Then the following are equivalent:

(i) f is somewhat πgb-continuous function.

(ii) If C is a closed subset of Y such that $f^{-1}(C) \neq X$, then there is a proper πgb-closed subset D of X such that $D \supset f^{-1}(C)$.

(iii) If M is a πgb-dense subset of X then f(M) is a dense subset of Y .

Proof. (i) ⇒ (ii) : Let C be a closed subset of Y such that $f^{-1}(C) \neq X$.Then Y − C is an open set in Y such that $f^{-1}(Y - C) = X - f^{-1}(C) \neq \phi$.By hypothesis (i) there exists a πgb-open set V in X such that V ≠ ϕ and $V \subset f^{-1}(Y - C) = X - f^{-1}(C)$. This means that $X - V \supset f^{-1}(C)$ and X − V = D is a πgb-closed set in X. This proves (ii).

(ii) ⇒ (i) : Let U ∈ σ and $f^{-1}(U) \neq \phi$. Then Y − U is closed and $f^{-1}(Y - U) = X - f^{-1}(U) \neq \phi$. By hypothesis of (ii) there exists a proper πgb-closed set D such that $f^{-1}(Y - U) \subset D$. This implies that $X - D \subset f^{-1}(U)$ and X –D is πgb-open and X − D ≠ ϕ.

(ii) ⇒ (iii) : Let M be a πgb-dense set in X. We have to show that f(M) is dense in Y . Suppose not, then there exists a proper πgb-closed set C in Y such that f(M) ⊂ C ⊂ Y . Clearly $f^{-1}(C) \neq X$. Hence by (ii) there exists a proper πgb-closed set D such that $M \subset f^{-1}(C) \subset D \subset X$. This contradicts the fact that M is πgb-dense in X.

(iii) ⇒ (ii) : Suppose that (ii) is not true. This means there exists a closed set C in Y such that $f^{-1}(C) \neq X$. But there is no proper πgb-closed set D in X such that $f^{-1}(C) \subset D$. This means that $f^{-1}(C)$ is πgb-dense in X. But by (iii) $f(f^{-1}(C)) = C$ must be dense in Y , which is contradiction to the choice of C.

International Journal of Mathematical Engineering and Science
ISSN : 2277-6982 Volume 1 Issue 10 (October 2012)
http://www.ijmes.com/ https://sites.google.com/site/ijmesjournal/

Theorem 3.14 Let (X, τ) and (Y, σ) be any two topological spaces, A be an open set in X and f : (A, τ/A) → (Y, σ) be somewhat πgb-continuous function such that f(A) is dense in Y . Then any extension F of f is somewhat πgb –continuous function.

Proof. Let U be any open set in (Y, σ) such that $F^{-1}(U) \neq \phi$. Since f(A) ⊂ Y is dense in Y and U ∩ f(A) ≠ ϕ it follows that $F^{-1}(U) \cap A \neq \phi$. That is $f^{-1}(U) \cap A \neq \phi$. Hence by hypothesis on f, there exists a πgb-open set V in A such that V ≠ ϕ and $V \subset f^{-1}(U) \subset F^{-1}(U)$ which implies F is somewhat πgb-continuous function.

Theorem 3.15 Let (X, τ) and (Y, σ) be any two topological spaces, X =A ∪ B where A and B are regular open and πgb-closed subsets of X and f : (X, τ) → (Y, σ) be a function such that f/A and f/B are somewhat πgb-continuous functions. Then f is somewhat πgb-continuous function.

Proof. Let U be any open set in (Y, σ) such that $f^{-1}(U) \neq \phi$. Then $(f/A)^{-1}(U) \neq \phi$ or $(f/B)^{-1}(U) \neq \phi$ or both $(f/A)^{-1}(U) \neq \phi$ and $(f/B^{)-1}(U) \neq \phi$.

Case 1. Suppose $(f/A)^{-1}(U) \neq \phi$. Since f/A is somewhat πgb-continuous, there exists a πgb-open set V in A such that V ≠ ϕ and $V \subset (f/A)^{-1}(U) \subseteq f^{-1}(U)$. Since X-V is πgb-closed in A and A is
regular open and πgb-closed in X, X-V is πgb-closed in X. Thus f is somewhat πgb-continuous function.

Case 2. Suppose $(f/B)^{-1}(U) \neq \phi$.Since f/B is somewhat πgb-continuous function, there exists a πgb-open set V in B such that V ≠ ϕ and $V \subset (f/B)^{-1}(U) \subseteq f^{-1}(U)$. Since X-V is πgb-closed in B and B is regular open and πgb-closed in X, X-V is πgb-closed in X.. Thus f is somewhat πgb-continuous function.

Case 3. Suppose $(f/A)^{-1}(U) \neq \phi$ and $(f/B)^{-1}(U) \neq \phi$.This follows from both the cases 1 and 2. Thus f is somewhat πgb-continuous function.

Definition 3.16 A topological space X is said to be πgb-separable if there exists a countable subset B of X which is πgb-dense in X.

Theorem 3.17 If f is somewhat πgb-continuous function from X onto Y and if X is πgb-separable, then Y is separable.

Proof. Let f : X → Y be somewhat πgb-continuous function such that X is πgb-separable. Then by definition there exists a countable subset B of X which is πgb-dense in X. Then by Theorem 3.13, f(B) is dense in Y . Since B is countable f(B) is also countable which is dense in Y , which indicates that Y is separable.

4. Somewhat πgb-irresolute function

Definition 4.1: A function f is said to be somewhat πgb-irresolute if for U∈πGBO(σ) and $f^{-1}(U) \neq \phi$, there exists a non-empty πgb-open set V in X such that $V \subset f^{-1}(U)$.

Example4.2: Let X = {a, b, c}, τ = {ϕ, {a}, {b, c}, X} and σ = {ϕ, {a}, {a, b}, X}. The function f:(X, τ)→ (X, σ) defined by f(a) = c, f(b) = a and f(c) = b is somewhat πgb-irresolute but not somewhat-irresolute.

Example 4.3: Let X = {a, b, c}, τ = {ϕ, {a}, {b}, {a, b}, X} and σ = {ϕ, {a}, {b,c}, X}. The function f :(X,τ)→ (X, σ) defined by f(a) = c, f(b) = a and f(c) = b is not somewhat πgb-irresolute and somewhat-irresolute.

Theorem 4.4: If f is somewhat πgb-irresolute and g is πgb-irresolute, then g•f is somewhat πgb-irresolute.

Proof. Let U ∈ πGBO (η). Suppose that $g^{-1}(U) \neq \phi$. Since U ∈ πGBO(η) and g is πgb -irresolute function $g^{-1}(U) \in$ πGBO (σ). Suppose that $f^{-1}g^{-1}(U) \neq \phi$. Since by hypothesis f is somewhat πgb- irresolute function, there exists a πgb-open set V in X such that V ≠ ϕ and $V \subseteq f^{-1}(g^{-1}(U))$. But $f^{-1}(g^{-1}(U)) = (g \circ f)^{-1}(U)$, which implies that $V \subseteq (g \circ f)^{-1}(U)$. Therefore g ∘ f is somewhat πgb-continuous function.

Theorem 4.5 Let f : (X, τ) → (Y, σ) be a function. Then the following are equivalent:

(i) f is somewhat πgb-irresolute function.

(ii) If C is a πgb-closed subset of Y such that $f^{-1}(C) \neq X$, then there is a proper πgb-closed subset D of X such that $D \supset f^{-1}(C)$.

(iii) If M is a πgb-dense subset of X then f(M) is a πgb-dense subset of Y .

Proof. (i) ⇒ (ii) : Let C be a πgb-closed subset of Y such that $f^{-1}(C) \neq X$.Then Y − C is an πgb-open set in Y such that $f^{-1}(Y - C) = X - f^{-1}(C) \neq \phi$.By hypothesis (i) there exists a πgb-open set V in X such that V ≠ ϕ and $V \subset f^{-1}(Y - C) = X - f^{-1}(C)$. This means that $X - V \supset f^{-1}(C)$ and X − V = D is a proper πgb-closed set in X. This proves (ii).

(ii) ⇒ (i) : Let U ∈ πGBO(σ) and $f^{-1}(U) \neq \phi$. Then Y − U is πgb-closed and $f^{-1}(Y - U) = X - f^{-1}(U) \neq \phi$. By hypothesis of (ii) there exists a proper πgb-closed set D such that $f^{-1}(Y - U) \subset D$. This implies that $X - D \subset f^{-1}(U)$ and X –D is πgb-open and X − D ≠ ϕ.

(ii) ⇒ (iii) : Let M be a πgb-dense set in X. We have to show that f(M) is πgb-dense in Y . Suppose not, then there exists a proper πgb-closed set C in Y such that f(M) ⊂ C ⊂ Y . Clearly $f^{-1}(C) \neq X$. Hence by (ii) there exists a proper πgb-closed set D in X such that $M \subset f^{-1}(C) \subset D \subset X$. This contradicts the fact that M is πgb-dense in X.

(iii) ⇒ (ii) : Suppose that (ii) is not true. This means there exists a πgb-closed set C in Y such that $f^{-1}(C) \neq X$. But there is no proper πgb-closed set D in X such that $f^{-1}(C) \subset D$. This means that $f^{-1}(C)$ is πgb-dense in X. But by (iii) $f(f^{-1}(C)) = C$ must be πgb-dense in Y , which is contradiction to the choice of C.

Theorem 4.6: Let (X, τ) and (Y, σ) be any two topological spaces, X =A ∪ B where A and B are regular open and πgb-closed sets of X and f : (X, τ) → (Y, σ) be a function such that f/A and f/B are somewhat πgb-irresolute functions. Then f is somewhat πgb-irresolute function.

Proof. Let U be any πgb-open set in (Y, σ) such that $f^{-1}(U) \neq \phi$. Then $(f/A)^{-1}(U) \neq \phi$ or $(f/B)^{-1}(U) \neq \phi$ or both $(f/A)^{-1}(U) \neq \phi$ and $(f/B)^{-1}(U) \neq \phi$.

Case 1. Suppose $(f/A)^{-1}(U) \neq \phi$.Since f/A is somewhat πgb- irresolute, there exists a πgb-open set V in A such that V ≠ ϕ and $V \subset (f/A)^{-1}(U) \subseteq f^{-1}(U)$. Since X-V is πgb-closed in A and A is regular open and πgb-closed sets in X, X-V is πgb-closed in X. Thus f is somewhat πgb- irresolute function.

The proof is similar for other two cases.

Definition 4.7: If X is a set and τ and σ are topologies for X, then τ is said to be equivalent to σ [3] provided if U ∈ τ and U ≠ ϕ, then there is an open set V in (X, σ) such that V ≠ ϕ and V ⊂ U and if U ∈ σ and U ≠ ϕ, then there is an open set V in (X, τ) such that V ≠ ϕ and V ⊂ U.

Definition 4.8: If X is a set and τ and σ are topologies for X, then τ is said to be πgb- equivalent to σ provided if U ∈ τ and U ≠ ϕ, then there is a πgb-open set V in (X, σ) such that V ≠ ϕ and V ⊂ U and if U ∈ σ and U ≠ ϕ then there is a πgb-open set V in (X, τ) such that V ≠ ϕ and V ⊂ U.

Theorem 4.9 Let f : (X, τ) → (Y, σ) be somewhat continuous function and let τ^* be a topology for X, which is πgb-equivalent to τ then the function f : (X, τ^*) → (Y, σ) is somewhat πgb-continuous function.

Proof. Let U be any open set in (Y, σ) such that $f^{-1}(U)$ ≠ ϕ. Since by hypothesis f : (X, τ) → (Y, σ) is somewhat continuous by definition there exists an open set O in (X, τ) such that O ≠ ϕ and O ⊆ $f^{-1}(U)$. Since O is an open set in (X, τ) such that O ≠ ϕ and since by hypothesis τ is πgb- equivalent to τ^* by definition there exists a πgb-open set V in (X, τ^*) such that V ≠ ϕ and V ⊂ O ⊂ $f^{-1}(U)$. Hence O ⊂ $f^{-1}(U)$ Thus for any open set U in (Y, σ) such that $f^{-1}(U)$ ≠ ϕ there exist a πgb-open set V in (X, τ^*) such that V ⊂ $f^{-1}(U)$. So f : (X, τ^*) → (Y, σ) is somewhat πgb-continuous function.

Theorem 4.10: Let f : (X, τ) → (Y, σ) be somewhat πgb-continuous function and let σ^* be a topology for Y which is equivalent to σ. Then f : (X, τ) → (Y, σ^*) is somewhat πgb-continuous function.

Proof. Let U be an open set in (Y, σ^*) such that $f^{-1}(U)$ ≠ ϕ which implies U ≠ ϕ. Since σ and σ^* are equivalent there exists an open set W in (Y, σ) such that W ≠ϕ and W ⊂ U. Now, W is an open set such that W ≠ϕ, which implies $f^{-1}(W)$ ≠ ϕ. Now by hypothesis f : (X, τ) → (Y, σ) is somewhat πgb-continuous function. Therefore there exists a πgb-open set V in X, such that V ⊂ $f^{-1}(W)$. Now W ⊂U implies $f^{-1}(W)$ ⊂ $f^{-1}(U)$.This implies V ⊂ $f^{-1}(W)$ ⊂ $f^{-1}(U)$. So we have V ⊂ $f^{-1}(U)$, which implies that f : (X, τ) → (Y, σ^*) is somewhat πgb-continuous function.

Theorem 4.11: Let f : (X, τ) → (Y, σ) be somewhat πgb- irresolute surjection and let τ^* be a topology for X, which is πgb-equivalent to τ then the function f : (X, τ^*) → (Y, σ) is somewhat πgb- irresolute function.

Proof. Let U be any πgb-open set in (Y, σ) such that $f^{-1}(U)$ ≠ ϕ. Since by hypothesis f : (X, τ) → (Y, σ) is somewhat πgb-irresolute, by definition there exists an πgb-open set O in (X, τ) such that O ≠ ϕ and O ⊆ $f^{-1}(U)$. Since O is an πgb-open set in (X, τ) such that O ≠ ϕ and since by hypothesis τ is πgb- equivalent to τ^* by definition there exists a πgb-open set V in (X, τ^*) such that V ≠ ϕ and V ⊂ O ⊂ $f^{-1}(U)$. Hence O ⊂ $f^{-1}(U)$ Thus for any open set U in (Y, σ) such that $f^{-1}(U)$ ≠ ϕ there exist a πgb-open set V in (X, τ^*) such that V ⊂ $f^{-1}(U)$. So f : (X, τ^*) → (Y, σ) is somewhat πgb- irresolute function.

Theorem 4.12: Let f : (X, τ) → (Y, σ) be somewhat πgb- irresolute surjection function and let σ^* be a topology for Y which is equivalent to σ. Then f : (X, τ) → (Y, σ^*) is somewhat πgb- irresolute function.

Proof. Let U be an open set in (Y, σ^*) such that $f^{-1}(U)$ ≠ ϕ which implies U ≠ ϕ. Since σ and σ^* are equivalent there exists an open set W in (Y, σ) such that W ≠ϕ and W ⊂ U.Now, W is an open set such that W ≠ϕ, which implies $f^{-1}(W)$ ≠ ϕ. Now by hypothesis f : (X, τ) → (Y, σ) is somewhat πgb- irresolute function. Therefore there exists a πgb-open set V in X, such that V ⊂ $f^{-1}(W)$. Now W ⊂U implies $f^{-1}(W)$ ⊂ $f^{-1}(U)$.This implies V ⊂ $f^{-1}(W)$ ⊂ $f^{-1}(U)$. So we have V ⊂ $f^{-1}(U)$, which implies that f : (X, τ) → (Y, σ^*) is somewhat πgb- irresolute function.

5.Somewhat πgb-open functions

Definition 5.1: A function f : (X, τ) → (Y, σ) is said to be somewhat πgb-open function provided that for U ∈ τ and U ≠ ϕ there exists a πgb-open set V in Y such that V ≠ ϕ and V ⊆ f(U).

Example 5.2: Let X = {a, b, c}, τ = {X, ϕ, {a},{a, b},{a,c},} and σ = {X, ϕ,{a}}.Define a function f : (X, τ) → (X, σ) by f(a) = b, f(b) = c, f(c) = a.Then clearly f is somewhat πgb-open function.

Theorem 5.3: Every somewhat semi-open function is somewhat πgb-open function.

Proof. Let f : (X, τ) → (Y, σ) be a somewhat semi-open function. Let U ∈ τ and U ≠ϕ. Since f is somewhat semi-open there exists a semi-open set V in Y such that V ≠ ϕ and V ⊂ f(U). But every semi-open set is πgb-open. Therefore there exists a πgb-open set V in Y such that V ≠ ϕ and V ⊂ f(U), which implies that f is somewhat πgb-open function.

Remark 5.4: Converse of the above theorem need not be true in general,which follows from the following example.

Example 5.5 Let X = {a, b, c, d}. Let τ = {X, ϕ, {a, b}} and σ = {X, ϕ}. Define a function f : (X, τ) → (X, σ) by f(a) = c, f(b) = a, f(c) = c. Then clearly f is somewhat πgb-open function which is not a somewhat semi-open function.

Theorem 5.6 Every somewhat open function is somewhat πgb-open function.

Proof. Let f : (X, τ) → (X, σ) be somewhat open function. Let U ∈ τ and U ≠ ϕ. Since f is somewhat open function, there exists an open set V in Y such that V ≠ ϕ and V ⊆ f(U). But every open set is πgb-open. So there exists a πgb-open set V in Y such that V ≠ ϕ and V ⊆f(U). Thus f is somewhat πgb-open function.

Remark 5.7 Converse of the above theorem need not be true in general, which follows from the following example.

Example 5.8 Let X = {a, b, c, d}, τ = {X, ϕ, {a} , {b} {a, b}} and σ ={X,{a},{a,c}, ϕ}. Define a function f : (X, τ) → (X, σ) as follows f(a) = b, f(b) = c, f(d) = a, f(c) = a.Then clearly f is somewhat πgb-open function, but not a somewhat open function.

Theorem 5.9 If f : (X, τ) → (Y, σ) is an open map and g : (Y, σ) → (Z, η) is somewhat πgb-open map then g ∘ f : (X, τ) → (Z, η) is somewhat πgb-open map.

Proof. Let U ∈ τ . Suppose that U ≠ ϕ. Since f is an open map f(U) is open and f(U) ≠ϕ. Thus f(U) ∈ σ and f(U) ≠ ϕ.Since g is somewhat πgb-open map and f(U) ∈ σ such that f(U) ≠ϕ there exists a πgb-open set V ∈ η, V ⊂ g(f(U)), which implies g ∘ f is somewhat πgb-open function.

Theorem 5.10 If f : (X, τ) → (Y, σ) is a one-one and onto mapping, then the following conditions are equivalent.

(i) f is somewhat πgb-open map.

(ii) If C is a closed subset of X such that f(C) ≠ Y , then there is a πgb-closed subset D of Y such that D ≠ Y and D ⊃ f(C)

Proof. (i) ⇒ (ii) :Let C be any closed subset of X such that f(C) ≠ Y .Then X−C is open in X and X −C ≠ ϕ. Since f is somewhat πgb-open, there exists a πgb-open set V ≠ ϕ in Y such that V ⊂ f(X − C). Put D = Y − V . Clearly D is πgb-closed in Y and we claim that D ≠ Y . For if D = Y , then V = ϕ which is a contradiction.Since V ⊂ f(X − C), D = Y − V ⊃ Y − [f(X − C)] = f(C).

(ii) ⇒ (i)**:**Let U be any non-empty open set in X. Put C = X − U. Then C is a closed subset of X and f(X − U) = f(C) = Y − f(U) implies f(C) ≠ ϕ. Therefore, by(ii) there is a πgb-closed subset D of Y such that D ≠ Y and f(C) ⊂ D. Put V = Y − D. Clearly V is a πgb-open set and V ≠ ϕ. Further,

$V = X - D \subset Y - f(C) = Y - [Y - f(U)] = f(U)$.

Theorem 5.11 Let $f : (X, \tau) \to (Y, \sigma)$ be somewhat πgb-open function and A be any open subset of X. Then $f/A : (A, \tau/A) \to (Y, \sigma)$ is also somewhat πgb-open function.

Proof. Let $U \in \tau/A$ such that $U \neq \phi$. Since U is open in A and A is open in (X, τ), U is open in (X, τ) and since by hypothesis $f : (X, \tau) \to (Y, \sigma)$ is somewhat πgb-open function, there exists a πgb-open set V in Y , such that $V \subset f(U)$. Thus, for any open set U in $(A, \tau/A)$ with $U \neq \phi$, there exists a πgb-open set V in Y such that $V \subset f(U)$ which implies f/A is somewhat πgb-open function.

Theorem 5.12 Let (X, τ) and (Y, σ) be any two topological spaces and $X = A \cup B$ where A and B are open subsets of X and $f : (X, \tau) \to (Y, \sigma)$ be a function such that f/A and f/B are somewhat πgb-open, then f is also somewhat πgb-open function.

Proof. Let U be any open subset of (X, τ) such that $U \neq \phi$. Since $X = A \cup B$, either $A \cap U \neq \phi$ or $B \cap U \neq \phi$ or both $A \cap U \neq \phi$ and $B \cap U \neq \phi$. Since U is open in (X, τ), U is open in both $(A, \tau/A)$ and $(B, \tau/B)$.

Case (i): Suppose that $U \cap A \neq \phi$ where $U \cap A$ is open in τ/A. Since by hypothesis f/A is somewhat πgb-open function, there exists a πgb-open set $V \in (Y, \sigma)$ such that $V \subset f(U \cap A) \subset f(U)$, which implies f is somewhat πgb-open function.

Case (ii): Suppose that $U \cap B \neq \phi$, where $U \cap B$ is open in $(B, \sigma/B)$. Since by hypothesis f/B is somewhat πgb-open function, there exists a πgb-open set V in (Y, σ) such that $V \subset f(U \cap B) \subset f(U)$, which implies that f is also somewhat πgb-open function.

Case (iii): Suppose that both $U \cap B \neq \phi$ and $U \cap A \neq \phi$. Then obviously f is somewhat πgb-open function from the case (i) and case (ii). Thus f is somewhat πgb-open function.

6.Somewhat almost πgb-open functions

Definition 6.1: A function f is said to be somewhat almost πgb-open provided that if $U \in RO(\tau)$ and $U \neq \phi$, then there exists a non-empty πgb-open set V in Y such that $V \subseteq f(U)$.

Example 6.2: Let $X = \{a, b, c\}$, $\tau = \{\varphi, \{a\}, X\}$ and $\sigma = \{\varphi, \{a\}, \{b, c\}, X\}$. The function $f: (X, \tau) \to (X, \sigma)$ defined by $f(a) = a$, $f(b) = c$ and $f(c) = b$ is somewhat almost πgb-open, somewhat πgb- open and somewhat open.

Theorem 6.3: For a bijective function f, the following are equivalent:

(i) f is somewhat almost πgb-open.

(ii) If C is regular closed in X, such that $f(C) \neq Y$, then there is a πgb- closed subset D of Y such that $D \neq Y$ and $D \supset f(C)$.

Proof: (i) ⇒ (ii): Let $C \in RC(X)$ such that $f(C) \neq Y$. Then $X\text{-}C \neq \phi \in RO(X)$. Since f is somewhat almost πgb-open, there exists $V \neq \phi \in \pi GBO(Y)$ such that $V \subset f(X\text{-}C)$. Put $D = Y\text{-}V$. Clearly $D \neq \phi \in \pi GBC(Y)$. If $D = Y$, then $V = \phi$, which is a contradiction. Since $V \subset f(X\text{-}C)$, $D = Y\text{-}V \supset (Y\text{-} f(X\text{-}C)) = f(C)$.

(ii) ⇒ (i): Let U ≠ ϕ ∈ RO(X). Then C = X-U∈ RC(X) and f(X-U) = f(C) = Y- f(U) implies f(C) ≠ Y. Then by (ii), there is D ≠ϕ∈πGBC(Y) and f(C) ⊂ D. Clearly V=Y-D ≠ϕ∈ πGBO(Y). Also, V = Y-D ⊂ Y- f(C) = Y- f(X-U) = f(U).

Theorem 6.4: The following statements are equivalent:

(i) f is somewhat almost πgb-open.

(ii) If A is a πgb-dense subset of Y, then $f^{-1}(A)$ is a dense subset of X.

Proof: (i) ⇒ (ii): Let A be a πgb- dense set in Y. If $f^{-1}(A)$ is not dense in X, then there exists B ∈ RC(X) such that $f^{-1}(A)$ ⊂ B ⊂ X. Since f is somewhat almost πgb-open and X-B ∈ RO(X), there exists C ≠ ϕ ∈ πGBO(Y) such that C ⊂ f(X-B). Therefore, C ⊂ f(X-B) ⊂ $f(f^{-1}(Y-A))$ ⊂ Y-A. That is, A ⊂ Y-C ⊂ Y. Now, Y-C is a πgb-closed set and A ⊂ Y-C ⊂ Y. This implies that A is not a πgb-dense set in Y, which is a contradiction. Therefore, $f^{-1}(A)$ is a dense set in X.

(ii) ⇒ (i): If A ≠ ϕ ∈ RO(X). We want to show that int(πgb- (f(A))) ≠ φ. Suppose int(πgb- (f(A))) = φ. Then, πgb-cl{(f(A))} = Y. Then by (ii), $f^{-1}(Y - f(A))$ is dense in X. But $f^{-1}(Y- f(A))$ ⊂ X-A. Now, X-A ∈ RC(X). Therefore, $f^{-1}(Y- f(A))$ ⊂ X-A gives X = cl{($f^{-1}(Y- f(A))$)} ⊂ X-A. Thus A = ϕ, which contradicts A ≠ φ. Therefore, int(πgb-(f(A))) ≠ ϕ. Hence f is somewhat almost πgb-open.

Theorem 6.5: Let (X, τ) and (Y, σ) be any two topological spaces, X =A ∪ B where A and B are regular open subsets of X and f : (X, τ) → (Y, σ) be a function such that f/A and f/B are somewhat almost πgb-open functions. Then f is somewhat almost πgb-open.

Proof. Let U be any regular open set in X .Since X=A ∪ B, either A∩U ≠ ϕ or B∩U ≠ ϕ or both A∩U ≠ ϕ and B∩U ≠ ϕ Since U is regular open in X, U is regular open in both A and B.

Case (i): If A∩U ≠ φ ∈RO(A). Since f/A is somewhat almost πgb-open, there exists a πgb-open set V of Y such that V ⊂f(U∩A) ⊂ f(U), which implies that f is a somewhat almost πgb-open. Case (ii): If B∩U ≠ φ∈RO(B). Since f/B is somewhat almost πgb-open, there exists a πgb-open set V in Y such that V⊂ f(U∩B) ⊂ f(U), which implies that f is somewhat almost πgb-open. Case (iii): Suppose that both A∩U ≠ φ and B∩U ≠ φ. Then by case (i) and (ii) f is somewhat almost πgb-open.

7.Somewhat M- πgb-open function

Definition 7.1: A function *f* is said to be somewhat M- πgb-open provided that if U∈πGBO(τ) and U≠ φ , then there exists a non-empty πgb-open set V in Y such that V⊂ *f*(U).

Example 7.2: Let X = {a, b, c}, τ = {φ, {a},{a,b} X} and σ = {φ, {a}, {b, c}, X}. The function f: (X, τ) →(X, σ) defined by f(a) = a, f(b) = c and f(c) = b is somewhat quasi πgb-open.

Theorem 7.3: For a bijective function f, the following are equivalent:

(i)f is somewhat M- πgb-open.

(ii) If C ∈ πGBC(X), such that f(C) ≠ Y, then there is a πgb- closed subset D of Y such that D ≠ Y and D ⊃ f(C).

Proof: (i) ⇒ (ii): Let C∈ πGBC(X), such that f(C) ≠ Y. Then X-C ≠ ϕ∈ πGBO(X), Since f is somewhat M-πgb-open, there exists V≠ ϕ∈ πGBO(Y) such that V⊂ f(X-C). Put D = Y-V. Clearly D ≠ ϕ ∈ πGBC(Y). If D = Y, then V = ϕ, which is a contradiction. Since V ⊂ f(X-C), D = Y-V ⊃ (Y- f(X-C)) = f(C).

(ii) ⇒ (i): Let U ≠ ϕ ∈ πGBO(X), Then C = X-U∈ πGBC(X), and f(X-U) = f(C) = Y- f(U) implies f(C) ≠ Y. Then by (ii), there is D ≠ϕ∈πGBC(Y) and f(C) ⊂ D. Clearly V=Y-D ≠ϕ∈ πGBO(Y). Also, V = Y-D ⊂ Y- f(C) = Y- f(X-U) = f(U).

Theorem 7.4: Let (X, τ) and (Y, σ) be any two topological spaces, X =A ∪ B where A and B are regular open subsets of X and f : (X, τ) → (Y, σ) be a function such that f/A and f/B are somewhat almost πgb-open functions. Then f is somewhat almost πgb-open.

Proof. Let U ≠ ϕ ∈ πGBO(X),.Since X=A ∪ B, either A∩U ≠ ϕ or B∩U ≠ ϕ or both A∩U ≠ ϕ and B∩U ≠ ϕ Since U is regular open in X,U is regular open in both A and B.

Case (i): If A∩U ≠ φ ∈ πGBO(A), Since f/A is somewhat almost πgb-open, there exists a πgb-open set V of Y such that V ⊂f(U∩A) ⊂ f(U), which implies that f is a somewhat almost πgb-open.

Case (ii): If B∩U ≠ φ∈ πGBO(B). Since f/B is somewhat almost πgb-open, there exists a πgb-open set V in Y such that V⊂ f(U∩B) ⊂ f(U), which implies that f is somewhat almost πgb-open.

Case (iii): Suppose that both A∩U ≠ φ and B∩U ≠ φ. Then by case (i) and (ii) f is somewhat M- πgb-open.

References:

[1] D. Andrijevic, On b-open sets, Mat. Vesnik 48 (1996), 59-64.
[2] S. S. Benchalli and Priyanka M. Bansali ,Somewhat b-Continuous and Somewhat b-Open Functions in Topological Spaces,Int. Journal of Math. Analysis, Vol. 4, 2010, no. 46, 2287 – 2296.
[3] M. Caldas, A separation axioms between semi-T_0 and semi-T_1, Mem. Fac. Sci.Kochi Univ. Ser. A Math. 181(1997) 37-42.
[4] J. Dontchev, On generalizing semi-preopen sets, Mem. Fac. Sci. Kochi Univ. Ser. A Math. 16 (1995), 35-48.
[5] J.Dontchev and T.Noiri, Quasi Normal Spaces and πg-closed sets ,Acta Math. Hungar., 89(3)(2000), 211-219.
[6] E. Ekici and M. Caldas, Slightly -continuous functions, Bol. Soc. Parana. Mat. (3) 22 (2004), 63-74. M. Ganster and M. Steiner, On some questions about b-open sets, Questions Answers Gen.Topology 25 (2007), 45-52.
[7] Gentry, K. R. and Hoyle, H. B., Somewhat continuous functions, Czech. Math. Jl., 21.(1971). No. 1, 5-12.
[8] M. Ganster and M. Steiner, On bτ-closed sets, Appl. Gen. Topol. 8 (2007), 243-247.
[9] N. Levine, Semi-open sets and semi-continuity in topological spaces, Amer. Math. Monthly 70(1963), 36-41.
[10] N. Levine, Generalized closed sets in topology, Rend. Circ. Mat. Palermo (2) 19 (1970), 89-96.
[11] D. Santhileela and G. Balasubramanian, Somewhat semi continuous and somewhat semi open functions , Bull. Cal. Math. Soc.,94 (1) (2002) 41-48.
[12] D. Sreeja and C. Janaki ,On πgb- Closed Sets in Topological Spaces,International Journal of Mathematical Archive-2(8), 2011, 1314-1320.
[13] D. Sreeja and C. Janaki , A New Type of Separation Axioms in Topological Spaces Asian Journal of Current Engineering and Maths1: 4 Jul – Aug (2012) 199 – 203.
[14] Youngblood .A. L., Weakly equivalent topologies, Master's Thesis (1965) University of Georgia, Georgia.
[15] Zdenek Frolik, Remarks concerning the Invariance of Baire Spaces under Mappings, Czech. Math. Jl., II (86). (1961) 381-385.