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ON ALMOST $b-I$ - CONTINUOUS FUNCTIONS

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Abstract. The aim of this paper is to introduce and characterize a new class of functions called almost $b-I$-continuous functions in ideal topological spaces by using $b-I$-open sets.

Keywords: Ideal topological spaces, $b-I$-open sets, almost $b-I$-continuous functions.

1 Introduction

The concept of ideals in topological spaces has been introduced and studied by Kuratowski [12] and Vaidyanathaswamy,[21]. An ideal I on a topological space (X,τ) is a nonempty collection of subsets of X which satisfies (i) $A \in I$ and $B \subset A$ implies $B \in I$ and (ii) $A \in I$ and $B \in I$ implies $A \cup B \in I$. Given a topological space (X,τ) with an ideal I on X and if P(x) the set of all subsets of X , a set operator $(.)^\star$ $P(X) \to P(X)$, called the local function [21] of A with respect to τ and I, is defined as follows: for $A \subset X$, $A^\star(\tau,I) = \{x \in X \mid U \cap A \notin I$ for every $U \in \tau(x)\}$, where $\tau(x) = \{U \in \tau \mid x \in U\}$. A Kuratowski closure operator $C1^\star(.)$ for a topology $\tau^\star(\tau,I)$ called the $*-$topology, finer than τ is defined by $C1^\star(A) = A \cup A^\star(\tau,I)$ when there is no chance of confusion, $A^\star(I)$ is denoted by $A^\star$. If I is an ideal on X, then (X,τ,I) is called an ideal topological space. The aim of this paper is to introduce and characterize a new class of functions called almost $b-I$ continuous functions in ideal topological spaces by using $b-I$-open sets.

2 PRELIMINARIES

Let A be a subset of a topological space (X,τ). We denote the closure of A and the interior of A by Cl(A) and Int(A), respectively. A subset A of a topological space (X,τ) is said to be regular open [20] if A= Int(Cl(A)). A set $A \subset X$ is said to be $\delta-$open [22] if it is the union of regular open sets of X. The complement of a regular open (resp. $\delta-$open) set is called regular closed (resp. $\delta-$closed). The intersection of all $\delta-$closed sets of (X,τ) containing A is called the $\delta-$closure [22] of A and is denoted by $\mathrm{Cl}_\delta(A)$. A point $x \in X$ is called a $\theta-$closure of A if $Cl(A) \cap A \neq \emptyset$ for every open set V of X containing x. The set of all $\theta-$cluster points of A is called the $\theta-$closure of A [22] and is denoted by $\mathrm{Cl}_{\theta}(A)$, If A= $\mathrm{Cl}_{\theta}(A)$, then A is said to be $\theta-$closed [22].

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The complement of $\theta-$closed set is said to be $\theta-$open [22]. A subset A of a topological space (X,τ) is said to be $b-$open [4] (resp. semiopen [13], preopen[14], $\beta-$open [1]) if $A\subset \mathrm{Int}(\mathrm{Cl(A)})\cup\mathrm{Cl}(\mathrm{Int(A)})$ (resp. $A\subset$Cl(Int(A)), $A\subset\mathrm{Int}(\mathrm{Cl(A)})$, $A\subset$Cl(Int(Cl(A)))). The set of all regular open (resp. regular closed, $\delta-$open, $\delta-$closed, $b-$open preopen) sets of (X,τ) is denoted by RO(X) (resp. RC(X), δO(X), δC(X), BO(X), PO(X)). A subset S of an ideal topological space (X,τ,I) is called $b-I$-open [7] if $S\subset$Int (Cl*(S)) $\cup$ Cl*(Int(S)). The complement of a $b-I$-open set is called a $b-I$-closed set [7]. The intersection of all $b-I$-closed sets containing S is called the $b-I$- clouser of S and is denoted by bI Cl(S). The $b-I$ interior of S is defined by the union of all $b-I$-open sets contained in S and is denoted by bI Int(S). The set of all $b-I-$open sets of (X,τ,I) is denoted by BIO(X). The set of all $b-I$-open sets of (X,τ,I) containing a point $x\in X$ is denoted by BIO(X,x).

Definition 2.1 A function $f:(X,\tau)\to(Y,\sigma)$ is said to be:

(1) b-continuous [4] if $f^{-1}(V)$ is b-open in X for every open set V of Y;

(2) almost continuous [18] if $f^{-1}(V)$ is open in X for every regular open set V of X;

(3) R-map [8] if $f^{-1}(V)$ is regular open in X for every regular open set V of X.

(4) almost b-continuous [19] if $f^{-1}(V)$ is b-open in X for every regular open set V of Y.

Definition 2.2 A function $f:(X,\tau,I)\to(Y,\sigma,I)$ is said to be $b-I$-irresolute if $f^{-1}(V)$ is $b-I$-open in X for every $b-I$-open subset V of Y.

Definition 2.3 A function $f:(X,\tau,I)\to(Y,\sigma)$ is said to be:

(1) $b-I$-continuous [7] if $f^{-1}(V)$ is $b-I$-open in X for every open set V of Y,

(2) weakly $b-I$-continuous [5] if for each $x\in X$ if for each open subset V in

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Y containing f(x), there exists $U \in BIO(X,x)$ such that $f(U) \subset Cl(V)$.

Definition 2.4 An ideal topological space (X,τ,I) is said to be:

(1) $b-I$-T1 [6] (resp. $r-T_1$ [10]) if for each pair of distinct points x and y of X, there exists $b-I$-open (resp.regular open) sets and U and V such that $x \in U, y \notin U$ and $x \notin V, y \in V$.

(2) $b-I$-T2 [6] (resp. $r-T_2$ [10]) if for each pair of distinct points x and y of X, there exists $b-I$-open (resp. regular open) sets U and V such that $x \in U,\ y \in V$ and $U \cap V = \varnothing$.

Lemma 2.5. The following statements are true:

(1) Let A be a subset of a space (X,τ). Then $A \in PO(X)$ if and only if $s\,Cl(A) = Int(Cl(A))$ [11].

(2) A subset A of a space (X,τ) is $\beta-$open if and only if Cl(A) is regular closed [3].

3. ALMOST $b-I$-CONTINUOUS FUNCTIONS

Definition 3.1. A function $f:(X,\tau,I) \to (Y,\sigma)$ is said to be:

(1) almost $b-I$-continuous at a point $x \in X$ if for each open subset V of Y Containing f(x), there exists $U \in BIO(X,x)$ such that $f(U) \subset Int(Cl(V))$;

(2) almost $b-I$-continuous if it has this property at each point of X.

Remark 3.2. almost $b-I$-continuity implies weak $b-I$-continuity and it is obvious that almost $b-I$-continuity implied by $b-I$-continuity. However, the converses of these implications is not true in general as the following examples show.

Example 3.3. Let $X=\{a,b,c\}, \tau=\{\varnothing,\{a\},\{b\},\{a,b\},X\}, \sigma=\{\varnothing,\{a\},\{b\},\{a,b\},X\}$ and $I=\{\varnothing,\{a\}\}$. Define a function $f:(X,\tau,I) \to (Y,\sigma)$ by $f(a)=b,\ f(b)=c$ and $f(c)=a$. Then f is almost $b-I$-continuous but not $b-I$-continuous.

Example 3.4. Let $X=\{a,b,c\},\ \tau=\{\varnothing,\{a\},\{b,c\}\}, \sigma=\{\varnothing,\{a\},\{b\},\{a,b\},X\}$ and $I=\{\varnothing,\{a\}\}$. Then the identity function $f:(X,\tau,I) \to (Y,\sigma)$ is weakly $b-I$-continuous but not almost $b-I$-continuous.

Theorem 3.5. For a function $f:(X,\tau,I) \to (Y,\sigma)$, the following statements are equivalent:

(1) f is almost $b-I$-continuous at $x\in X$;

(2) $x\in$ Int(Cl⋆ $(f^{-1}(s\text{Cl}(V)))) \cup$ Cl⋆ (Int$(f^{-1}(s\text{Cl}(V))))$ for every open set V of Y containing f(x);

(3) $f^{-1}(V)\subset bI\,\text{Int}(f^{-1}(s\text{Cl}(V)))$ for every open set V of Y;

(4) $bI\,\text{Cl}(f^{-1}(s\,\text{Int}(F)))\subset f^{-1}(F)$ for every closed set F of Y.

Proof. (1) ⇒ (2): Let V be an open set of Y containing f(x). Then there exists $U\in BIO(X,x)$ such that $f(U)\subset\text{Int}(\text{Cl}(V))=s\text{Cl}(V)$. Then $U\subset f^{-1}(s\text{Cl}(V))$. Since $U\in BIO(X,x)$, $x\in U\subset$ Int(Cl⋆ $(f^{-1}(U))) \cup$ Cl⋆Int $(f^{-1}(U))))$ ⊂Int(Cl⋆ $(f^{-1}(s\text{Cl}(V)))) \cup$ Cl⋆(Int $(f^{-1}(s\text{Cl}(V))))$.

(2) ⇒ (3): Let V be open set of Y containing f(x) and U an open set of X containing x. Since $x\in$ Int(Cl⋆ $(f^{-1}(s\text{Cl}(V)))) \cup$ Cl⋆(Int $(f^{-1}(s\text{Cl}(V))))$,we have $x\in f^{-1}(s\text{Cl}(V)) \cap$ Int(Cl⋆ $(f^{-1}(s\text{Cl}(V)))) \cup$ Cl⋆(Int $(f^{-1}(s\text{Cl}(V)))))= bI\,\text{Int}(f^{-1}(s\text{Cl}(V)))$ by [16], Theorem 2.4]. Hence $f^{-1}(V)\subset bI\,\text{Int}(f^{-1}(s\text{Cl}(V))))$.

(3) ⇒ (1): Let V be an open set of Y containing f(x), then $x\in f^{-1}(V)\subset bI\,\text{Int}(f^{-1}(s\text{Cl}(V)))$ .Set U= $bI\,\text{Int}(f^{-1}(s\text{Cl}(V)))$, then $U\in BIO(X,x)$ such that $f(U)\subset s\text{Cl}(V)$. This shows that f is almost $b-I$-continuous at x.

(3) ⇒ (4): Clear.

Theorem 3.6. For a function $f:(X,\tau,I)\to(Y,\sigma)$, the following statements are equivalent:

(1) f is almost $b-I$-continuous;

(2) $f^{-1}(\text{Int}(\text{Cl}(V))\in \text{BIO}(X)$ for every open set V of Y;

(3) $f^{-1}(\text{Cl}(\text{Int}(V))\in \text{BIO}(X)$ for every closed set V of Y;

(4) $f^{-1}(V)\in \text{BIO}(X)$ for every $V\in \text{RO}(Y)$;

(5) $f^{-1}(F)\in \text{BIC}(X)$ for every $F\in \text{RC}(Y)$;

(6) For each $x\in X$ and each open set V of Y containing f(x) there exists $U\in BIO(X,x)$ such that $f(U)\subset s\text{Cl}(V)$;

(7) $bI\,\text{Cl}(f^{-1}(\text{Cl}(\text{Int}(F)))\subset f^{-1}(F)$ for every closed set F of Y;

(8) $bI\,\text{Cl}(f^{-1}(A))\subset f^{-1}(\text{Cl}(A))$ for every $A\in \text{BO}(Y)$;

(9) $bI\,\text{Cl}(f^{-1}(A))\subset f^{-1}(\text{Cl}(A))$ for every $A\in SO(Y)$;

(10) $f^{-1}(V)\subset bI\,\text{Int}(f^{-1}(\text{Int}(\text{Cl}(V))))$ for every open set $V\in \text{PO}(Y)$;

(11) $f(bI\,\text{Cl}(A))\subset \text{Cl}_\delta(f(A))$ for every subset A of X;

(12) $bI\,\mathrm{Cl}(f^{-1}(B)) \subset f^{-1}\mathrm{Cl}_{\delta}(\mathrm{f(B)})$ for every subset B of Y;

(13) $f^{-1}(\mathrm{F}) \in \mathrm{BIC(X)}$ for every $F \in \delta\mathrm{C(Y)}$;

(14) $f^{-1}(\mathrm{V}) \in \mathrm{BIO(X)}$ for every $\mathrm{V} \in \delta\mathrm{O(Y)}$.

Proof. (4) $\Rightarrow$ (5): Let $\mathrm{F} \in \mathrm{RC(Y)}$. Then $\mathrm{Y} \setminus \mathrm{F} \in \mathrm{RO(Y)}$. Take $x \in f^{-1}(Y \setminus F)$, then $f(x) \in \mathrm{Y} \setminus \mathrm{F}$ and since f is almost $b-I$ -continuous, there exists $\mathrm{W_x} \in \mathrm{BIO}(\mathrm{X},x)$ such that $x \in \mathrm{W_x}$ and $f(\mathrm{W_x}) \subset Y \setminus F$. Then $x \in \mathrm{W_x} \subset f^{-1}(Y \setminus F)$ so that $f^{-1}(Y \setminus F) = \bigcup_{x \in f^{-1}(Y \setminus F)} \mathrm{W_x}$

Since any union of $b-I$ -open sets is $b-I$ -open [2], $f^{-1}(Y \setminus F)$ is $b-I$ -open in X and hence $f^{-1}(\mathrm{F}) \in \mathrm{BIC(X)}$.

(5) $\Rightarrow$ (11): Let A be a subset of X. Since $\mathrm{Cl}_{\delta}(\mathrm{f(A)})$ if $\delta-$closed in Y , it is equal to $\cap\{F_{\alpha} : F_{\alpha}$ is regular closed in $Y, \alpha \in \Lambda\}$, where Λ is an index set. From (5), we have $A \subset f^{-1}(\mathrm{Cl}_{\delta}(\mathrm{f(A)})) = \cap\{f^{-1}(F_{\alpha}) : \alpha \in \Lambda\} \in \mathrm{BIC(X)}$ and hence $bI\,\mathrm{Cl(A)} \subset f^{-1}(\mathrm{Cl}_{\delta}$ $(\mathrm{f(A)}))$. Therefore, we obtain $\mathrm{f}(bI\,\mathrm{Cl(A)}) \subset \mathrm{Cl}_{\delta}(\mathrm{f(A)})$.

(11) $\Rightarrow$ (12): Set $\mathrm{A} = f^{-1}(B)\, in$ (11) ,then $f(bI\,\mathrm{Cl}(f^{-1}(B))) \subset \mathrm{Cl}_{\delta}(f(f^{-1}(B)))$ $\subset \mathrm{Cl}_{\delta}(B)$ and hence $bI\,\mathrm{Cl}(f^{-1}(B)) \subset f^{-1}(\mathrm{Cl}_{\delta}(B))$.

(12) $\Rightarrow$ (13): Let F be $\delta-$closed set of Y, then $bI\,\mathrm{Cl}(f^{-1}(F)) \subset f^{-1}(F)$ so $f^{-1}(F) \in \mathrm{BIC(X)}$.

(13) $\Rightarrow$ (14): Let V be $\delta-$open set of Y, then Y\V is $\delta-$closed set in Y. This gives $f^{-1}(\mathrm{Y\backslash V}) \in \mathrm{BIC(X)}$ and hence $f^{-1}(V) \in \mathrm{BIO(X)}$.

(14) $\Rightarrow$ (1): Let V be any regular open set of Y. Since V is $\delta-$open in Y, $f^{-1}(V) \in \mathrm{BIO(X)}$ and hence from $f(f^{-1}(V)) \subset V = \mathrm{Int(Cl(V))}$. then f is almost $b-I$ - continuous.

(5) $\Rightarrow$ (8): Let A be any $b-$open set in Y. Since Cl(A) is regular closed, $f^{-1}(\mathrm{Cl(A)})$ is $\delta-$closed and $f^{-1}(A) \subset f^{-1}(\mathrm{Cl(A)})$. Hence, $bI\,\mathrm{Cl}(f^{-1}(A)) \subset f^{-1}(\mathrm{Cl(A)})$.

(8) $\Rightarrow$ (9): obvious.

(9) $\Rightarrow$ (10): Let V be a preopen set. Then we have $\mathrm{V} \subset \mathrm{Int(Cl(V))}$ and $\mathrm{Cl(Int(Y\backslash V))}$ $\subset \mathrm{Y\backslash V}$. Moreover, since the set Cl(Int(Y\V)) is semi open, it follows that $X \setminus bI\,\mathrm{Int}(f^{-1}(\mathrm{Int(Cl(V))})) = bI\,\mathrm{Cl}(\mathrm{X} \backslash f^{-1}(\mathrm{Int(Cl(V))})) = bI\,\mathrm{Cl}(f^{-1}(Y \setminus \mathrm{Int(Cl(V))}))$ $= bI\mathrm{Cl}(f^{-1}(\mathrm{Cl(Int(Y\backslash V))})) \subset f^{-1}(\mathrm{Cl(Int(Y\backslash V))}) \subset f^{-1}(\mathrm{Y\backslash V}) \subset \mathrm{X} \backslash f^{-1}(V)$. Hence, we obtain $f^{-1}(V) \subset bI\,\mathrm{Int}(f^{-1}(\mathrm{Int(Cl(V))}))$.

(10) $\Rightarrow$ (4): Let V be a regular open set. Since V is preopen, we get $f^{-1}(V) \subset bI\,\mathrm{Int}(f^{-1}(\mathrm{Int(Cl(V))})) = bI\,\mathrm{Int}(f^{-1}(V))$. Hence $f^{-1}(V) \in \mathrm{BIO(X)}$.

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The other implications are obvious.

Theorem 3.7. For a function $f:(X,\tau,I)\to(Y,\sigma)$, the following statements are equivalent:

(1) f is almost $b-I$ -continuous;

(2) $bI\,\mathrm{Cl}(f^{-1}(\mathrm{Cl}(\mathrm{Int}(\mathrm{Cl}(B)))))\subset f^{-1}(\mathrm{Cl}(B))$ for every open subset B of Y.

(3) $bI\,\mathrm{Cl}(f^{-1}(\mathrm{Cl}(\mathrm{Int}(F))))\subset f^{-1}(F)$ for every closed subset F of Y;

(4) $bI\,\mathrm{Cl}(f^{-1}(\mathrm{Cl}(V)))\subset f^{-1}(\mathrm{Cl}(V))$ for every open subset V of Y;

(5) $f^{-1}(V)\subset bI\,\mathrm{Int}(f^{-1}(s\mathrm{Cl}(V)))$ for every open subset V of Y;

(6) $f^{-1}(V)\subset \mathrm{Int}(\mathrm{Cl}^\star(f^{-1}(s\mathrm{Cl}(V))))\cup \mathrm{Cl}^\star(\mathrm{Int}(f^{-1}(s\mathrm{Cl}(V))))$ for every Open subset V of Y;

(7) $f^{-1}(V)\subset \mathrm{Int}(\mathrm{Cl}^\star(f^{-1}(\mathrm{Int}(\mathrm{Cl}(V)))))\cup \mathrm{Cl}^\star(\mathrm{Int}(f^{-1}(\mathrm{Int}(\mathrm{Cl}(V)))))$ for every open subset V of Y.

proof. (1) $\Rightarrow$ (2): Let B be any subset of Y. Assume that $x\in X\setminus f^{-1}(\mathrm{Cl}(B))$. Then $f(x)\in Y\setminus \mathrm{Cl}(B)$ and there exists an open set V containing f(x) such that $V\cap B=\varnothing$; hence $\mathrm{Int}(\mathrm{Cl}(V))\cap \mathrm{Cl}(\mathrm{Int}(\mathrm{Cl}(B)))=\varnothing$. since f is almost $b-I$ -continuous, there exists $U\in \mathrm{BIO}(X,x)$ such that $f(U)\subset \mathrm{Int}(\mathrm{Cl}(V))$. Therefore, we have $U\cap f^{-1}(\mathrm{Cl}(\mathrm{Int}(\mathrm{Cl}(B))))=\varnothing$ and hence $x\in X\setminus bI\,\mathrm{Cl}(f^{-1}(\mathrm{Cl}(\mathrm{Int}(\mathrm{Cl}(B)))))$. Thus, we obtain $bI\,\mathrm{Cl}(f^{-1}(\mathrm{Cl}(\mathrm{Int}(\mathrm{Cl}(B)))))\subset f^{-1}(\mathrm{Cl}(B)))$.

(2) $\Rightarrow$ (3): Let F be any closed set of Y. Then we have $bI\,\mathrm{Cl}(f^{-1}(\mathrm{Cl}(\mathrm{Int}(\mathrm{Cl}(\mathrm{Int}(F))))))$ $=bI\,\mathrm{Cl}(f^{-1}(\mathrm{Cl}(\mathrm{Int}(F))))\subset f^{-1}(\mathrm{Cl}(\mathrm{Int}(F)))\subset f^{-1}(F)$.

(3) $\Rightarrow$ (4): For any open set V of Y , Cl(V) is regular closed in Y and we have $bI\,\mathrm{Cl}(f^{-1}(\mathrm{Cl}(V)))=bI\mathrm{Cl}(f^{-1}(\mathrm{Cl}(\mathrm{Int}(\mathrm{Cl}(V)))))\subset f^{-1}(\mathrm{Cl}(V))$.

(4) $\Rightarrow$ (5): Let V be any open set of Y. Then Y\Cl(V) is open in Y and we have $X\setminus bI\,\mathrm{Int}(f^{-1}(s\mathrm{Cl}(V)))=bI\,\mathrm{Cl}(f^{-1}(Y\setminus(s\mathrm{Cl}(V)))\subset f^{-1}(\mathrm{Cl}(Y\setminus\mathrm{Cl}(V)))\subset X\setminus f^{-1}(V)$. Therefore, we obtain $f^{-1}(V)\subset bI\,\mathrm{Int}(f^{-1}(s\mathrm{Cl}(V)))$.

(5) $\Rightarrow$ (6): Let V be any open set of Y. Then we obtain $f^{-1}(V)\subset bI\,\mathrm{Int}(f^{-1}(s\mathrm{Cl}(V)))$ $\subset\mathrm{Int}(\mathrm{Cl}^\star(f^{-1}(s\mathrm{Cl}(V))))\cup \mathrm{Cl}^\star(\mathrm{Int}(f^{-1}(s\mathrm{Cl}(V))))$.

(6) $\Rightarrow$ (1): Let x be any point of X and V any open set of Y containing f(x). Then $x\in f^{-1}(V)\subset\mathrm{Int}(\mathrm{Cl}^\star(f^{-1}(s\mathrm{Cl}(V))))\cup \mathrm{Cl}^\star(\mathrm{Int}(f^{-1}(s\mathrm{Cl}(V))))$. It follows from Theorem 3.5 that f is almost $b-I$ -continuous at any point x of X. Therefore, f is almost $b-I$ -continuous at any point x of X. (6) $\Rightarrow$ (7): Clear.

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Theorem 3.8. (1) A function $f:(X,\tau,\{\varnothing\})\to(Y,\sigma)$ is almost $b-I$-continuous if and only if it is almost $b-I$-continuous.
(2) A function $f:(X,\tau,\mathrm{N})\to(Y,\sigma)$ is almost b-continuous if and only if it is almost $b-I$-continuous (N is the ideal of all nowhere dense sets).
(3) A function $f:(X,\tau,p(X))\to(Y,\sigma)$ is almost $b-I$-continuous if and only if it is almost continuous.

Proof. It follows from proposition 2 of [7].

Definition 3.9. [9] Let A and B be subsets of an ideal topological space (X,τ,I) such that $A\subset B\subset X$. Then $(B,\tau_{|B},I_{|B})$ is an ideal topological space with an ideal $I_{|B}= =\{I\in I\,|\,I\subset B\}=\{I\cap B\,|\,I\in I\}$.

Lemma 3.10. [7] Let A and B be subsets of an ideal topological space (X,τ,I). If $A\in$ BIO(X) and B is open in (X,τ), then $A\cap B\in BIO(B)$.

Theorem 3.11. Let $f:(X,\tau,I)\to(Y,\sigma)$ be an almost $b-I$-continuous function and $A\subset X$. If $A\in\tau$, then $f_{|A}:(A,\tau_{|A},I_{|A})\to(Y,\sigma)$ is almost $b-I_{|A}-$continuous.
Proof. It follows from Lemma 3.10.

Theorem 3.12. Let $f:(X,\tau,I)\to(Y,\sigma)$ be a function and $\Lambda=\{U_i:i\in I\}$ be a Family such that $U_i\in\tau$ for each $i\in I$. If $f\,|\,U_i$ is almost $b-I$-continuous for each $i\in I$, then f is almost $b-I$-continuous.

Proof. Suppose that V is any regular open subset of (Y,σ). Since $f\,|\,U_i$ is almost $b-I$-continuous for each $i\in I$, it follows that $(f\,|\,U_i)^{-1}(V)$ is $b-I$-open in U_i. We have $f^{-1}(V)=\cup_{i\in I}(f^{-1}(V)\cap U_i)=\cup_{i\in I}(f\,|\,U_i)^{-1}(V)$. Since any union of $b-I$-open sets is $b-I$-open, $f^{-1}(V)\in$ BIO(X). Hence f is $b-I$-continuous.

Definition 3.13. A filter base Λ is said to be
(1) $b-I$-convergent to a point x in X if for any U $\in$ BIO(X,x), there exists $B\in\Lambda$ such that $B\subset U$.
(2) r – convergent to a point x in X if for any regular open set U of X containing x, there exists $B\in\Lambda$ such that $B\subset U$.

Theorem 3.14. If a function $f:(X,\tau,I)\to(Y,\sigma)$ is almost $b-I$-continuous, then for each point $x\in X$ and each filter base Λ in X $b-I$- converging to x, the filter base $f(\Lambda)$ is r – convergent to $f(x)$.

Proof. Let $x \in X$ and Λ be any filter base in X $b-I$ - converging to x. Since f is $b-I$ -continuous, then for any open set V of (Y,σ) containing f(x), there exists U $\in$ BIO(X,x) such that $f(U) \subset V$. Since Λ is $b-I$ - converging to x, there exists $B \in \Lambda$ such that $B \subset U$. This means that $f(B) \subset V$ and hence the filter base $f(\Lambda)$ is convergent to f(x).

Definition 3.15. A sequence (x_n) is said to be $b-I$ -convergent to a point x if for Every $b-I$ -open set V containing x, there exists an index η_0 such that for $n \geq \eta_0, x_n \in V$.

Theorem 3.16. If a function $f:(X,\tau,I) \to (Y,\sigma)$ is almost $b-I$ -continuous, then for each point $x \in X$ and each net (x_n) which is $b-I$ -convergt to x, the net $(f(x_n))$ is r-convergent to f(x).

Proof. The proof is similar to that of Theorem 3.14.

Theorem3.17. If an injective function $f:(X,\tau,I) \to (Y,\sigma)$ is almost $b-I$ - continuous and (Y,σ) is $r-T_1$,then (X,τ,I) is $b-I-T_1$.

Proof. Suppose that Y is $r-T_1$. For any distict points x and y in X, there exist regular open sets V and W such that $f(x) \in V$, $f(y) \notin V$, $f(x) \notin W$ and $f(y) \in W$. Since f is almost $b-I$ -continuous, $f^{-1}(V)$ and $f^{-1}(W)$ are $b-I$ -open subsets of (X,τ,I) such that $x \in f^{-1}(V)$, $y \notin f^{-1}(V)$, $x \notin f^{-1}(W)$ and $y \in f^{-1}(W)$. This shows that (X,τ,I) is $b-I-T_1$.

Theorem 3.18. If $f:(X,\tau,I) \to (Y,\sigma)$ is a almost $b-I$ -continuous injective function and (Y,σ) is $r-T_2$, then (X, τ) is $b-I-T_2$.

Proof. For any pair of distinct points x and y in X, there exist disjoint regular open sets U and V in Y such that $f(x) \in U$ and $f(y) \in V$ .Since f is almost $b-I$ - continuous, $f^{-1}(U)$ and $f^{-1}(V)$ are $b-I-$ open sets in X containing x and y, respectively. Therefore, $f^{-1}(U) \cap f^{-1}(V) = \varnothing$. because $U \cap V = \varnothing$. This shows that (X,τ,I) is $b-I-T_2$.

Theorem 3.19. If $f:(X,\tau,I)\rightarrow(Y,\sigma)$ is a almost continuous function and $g:(X,\tau,I)\rightarrow(Y,\sigma)$ is almost $b-I$-continuous function and Y is a $r-T_2-$ space, then the set E = $\{x\in X : f(x)=g(x)\}$ is $b-I$-closed set in (X,τ,I).

Proof. If $x\in X\setminus$E, then it follows that $f(x)\neq g(x)$. Since Y is $r-T_2$, there exist disjoint regular open sets V and W of Y such that $f(x)\in V$ $g(x)\in W$. Since f is almost continuous and g is almost $b-I$-continuous ,then $f^{-1}(V)$ is open and $g^{-1}(W)$ is $b-I$-open in X with $x\in f^{-1}(V)$ and $x\in g^{-1}(W)$.
Put A = $f^{-1}(V)\cap g^{-1}(W)$. By Lemma 3.10, A is $b-I$-open in X. Therefore, $f(A)\cap g(A)=\varnothing$ and it follows that $x\notin bI\,Cl(E)$. This shows that E is $b-I$-closed in X.

Definition 3.20 A function $f:(X,\tau,I)\rightarrow(Y,\sigma)$ is said to be faintly $b-I$-continuous if for each $x\in X$ and each $\theta-$open set V of Y containing $f(x)$,there exists U $\in BIO(X,x)$ such that $f(U)\subset V$.

Theorem 3.21. A function $f:(X,\tau,I)\rightarrow(Y,\sigma)$ is faintly $b-I$-continuous if and only if for every $\theta-$closed set V of Y $f^{-1}(V)\in BIC(X)$.

Theorem 3.22. The implications $(1)\Rightarrow(2)\Rightarrow(3)\Rightarrow(4)\Rightarrow(5)$ hold for the following properties of a function $f:(X,\tau,I)\rightarrow(Y,\sigma)$:

(1) f is $b-I$-continuous.
(2) $f^{-1}(Cl_\partial(B))$ is $b-I$-closed in X for every subsets B of Y.
(3) f is almost $b-I$-continuous.
(4) f is weakly $b-I$-continuous.
(5) f is faintly $b-I$-continuous.

If, in addition, Y is regular, then the five properties are equivalent of one another.

Proof. $(1)\Rightarrow(2)$: Since $Cl_\partial(B)$ is closed in Y for every subset B of Y, by Theorem 3.6, $f^{-1}(Cl_\partial(B))$ is $b-I$-closed in X.
$(2)\Rightarrow(3)$:For any subset B of Y, $f^{-1}(Cl_\partial(B))$ is $b-I$-closed in X and hence we have $bI\,Cl(f^{-1}(B))\subset bI\,Cl\ f^{-1}(Cl_\partial(B)) = f^{-1}(Cl_\partial(B))$. It follows from Theorem 3.6 that f is almost $b-I$-continuous.
$(3)\Rightarrow(4)$:This is obvious.

(4) $\Rightarrow$ (5) : Let F be any $\theta-$ closed set of Y. It follows from 3.21 that $bI\,C1\,(f^{-1}(F)) \subset f^{-1}(C1_{\theta}(F)) = f^{-1}(F)$ .Therefore, $f^{-1}(F)$ is $b-I$ -closed in X and hence f is faintly $b-I$ -continuous.

Suppose that Y is regular. We prove that (5) $\Rightarrow$ (1) .Let V be any open set of Y. Since Y is regular, V is $\theta-$ open in Y. By the faint $b-$ continuity of f , f^{-1} is $b-I$ -open in X. Therefore, f is $b-I$ -continuous.

Definition 3.23. A function $f:(X,\tau,I)\rightarrow(Y,\sigma)$ is said to be $b-I$ -preopen if $f(U) \in PO(Y)$ for every $b-I$ -open set U of X**.**

Theorem 3.24. If a function $f:(X,\tau,I)\rightarrow(Y,\sigma)$ is $b-I$ -preopen and weakly $b-I$ -continuous, then f is almost $b-I$ -continuous.

Proof. Let $x\in X$ and let V be an open set of Y containing $f(x)$**.** Since f is weakly $b-I$ -continuous, there exists U $\in BIO(X,x)$ such that $f(U)\subset C1(V)$**.** Since f is $b-I$ -preopen, $f(U)\subset$ Int(C1($f(U)$))$\subset$ Int(C1(V)); hence f is almost $b-I$ -continuous.

Theorem 3.25. Let $f:(X,\tau,I)\rightarrow(Y,\sigma)$ be a function and g: X$\rightarrow$X$\times$ Y the graph function defined by g(x) = (x, $f(x)$) for every $x\in X$ **.** Then g is almost $b-I$ -continuous if and only if f is almost $b-I$ -continuous.

Proof. Let x be any point of X and V any regular open set of Y containing $f(x)$. Then we have g(x) = (x, $f(x)$) $\in$ X$\times$ V is regular open in X$\times$ Y. Since g is almost $b-I$ -continuous, there exists U$\in$ $BIO(X)$ such that g(U) $\subset$X$\times$ Y. Therefore, we obtain $f(U)\subset V$; hence f is almost $b-I$ -continuous. Conversely, let $x\in X$ and W be a regular open set of X $\times$ Y containing g(x).There exist a regular open set U_1 in X and a regular open set V in Y such that $U_1\times V\subset W$. Since f is almost $b-I$ -continuous, there exist $U_2\in BIO(X,x)$ such that $f(U_2)\subset V$. Put $U=U_1\cap U_2$,then we obtain $x\in U\in BIO(X,x)$ and g(U) $\subset U\times V\subset W$. This shows that g is almost $b-I$ -continuous.

Theorem 3.26. Let $f:(X,\tau,I)\rightarrow(Y,\sigma,I)$ and g : $(Y,\sigma,I)\rightarrow(z,\eta)$ be functions. Then the composition g o $f:(X,\tau,I)\rightarrow(z,\eta)$ is almost $b-I-$continuous if f and g satisfy one of the following conditions:

(1) f is almost $b-I$ - continuous and g is R–map.
(2) f is $b-I$ -irresolute and g is almost $b-I$ - continuous
(3) f is $b-I$ - continuous and g is almost continuous.

Proof. Clear.

Definition 3. 27. A topological space (X,τ) is said to be:

(1) almost regular [17] if for any regular closed set F of X and any point $x \in X \setminus F$ there exist disjoint open sets U of V such that $x \in U$ and $F \subset V$.

(2) Semi-regular if for any open set U of X such that $x \in U$ there exists a regular open set V of X such that $x \in V \subset U$.

Theorem 3.28 . If $f:(X,\tau,I)\to(Y,\sigma)$ is a weakly $b-I$ - continuous function and Y is almost regular, then f is almost then $b-I$ - continuous .

Proof . Let $x \in X$ and let V be any open set of Y containing f(x). By the almost regularity of Y, there exists a regular open set G of Y such that $f(x) \in G \subset Cl(G) \subset \mathrm{Int}(Cl(V))$ [[17],Theorem 2.2]. Since f is weakly $b-I$ - continuous, there exists $U \in BIO(X,x)$ such that $f(U) \subset CI(G) \subset \mathrm{Int}(Cl(V))$. Therefore , f is almost $b-I$ - continuous.

Theorem 3.29. If $f:(X,\tau,I)\to(Y,\sigma)$ is an almost. If $b-I$ - continuous function and Y is semi-regular, then f is $b-I$ - continuous.

Proof . Let $x \in X$ and let V be any open set of Y containing f(x). By the semi-regularity of Y, there exists a regular open set G of Y such that $f(x) \in G \subset V$. Since f is almost $b-I$ - continuous, there exists $U \in BIO(X,x)$ such that $f(U) \subset \mathrm{Int}(Cl(G)) = G \subset V$ and hence f is $b-I$ - continuous.

Definition 3.30. : A $b-I$ - frontier of a subset A of $f:(X,\tau,I)$ denoted by bI Fr(A), is defined be $bI\,Cl(A) \cap bI\,Cl(X \setminus A)$.

Theorem 3.31. The set of all points x ∈ X in which a function $f:(X,\tau,I)\to(Y,\sigma)$ is not almost $b-I$ - continuous is identical with the union of $b-I$ -frontier of the inverse images of regular open sets containing f(x).

Proof. Suppose that f is not almost $b-I$ -continuous at x ∈ X. Then there exists a regular open set V of Y containing f(x) such that $U \cap (X \setminus f^{-1}(V)) \neq \varnothing$ for every $U \in BIO(X,x)$. Therefore, we have x ∈ $bI\,Cl(X \setminus f^{-1}(V)) = X \setminus bI\,\mathrm{Int}(f^{-1}(V))$ and

$x \in f^{-1}(V)$. Thus, we obtain $x \in$ bI Fr($f^{-1}(U)$). Conversely, suppose that f is almost $b-I$ - continuous at $x \in X$ and let V be a regular open set of Y containing f(x). Then there exists $U \in BIO(X,x)$ such that $U \subset f^{-1}(V)$. That is. $x \in bI\, Int(f^{-1}(V))$ Therefore, $x \in X \setminus bIFr(f^{-1}(V))$.

Theorem 3.32. If $g:(X,\tau,I) \rightarrow (Y,\sigma)$ is almost $b-I$ - continuous and S is δ-closed set of $X \times Y$, then $_{px}(S \cap G(g))$ is $b-I$-closed in X, Where P_X represents the projection of $X \times Y$ onto X and G(g) denotes the graph of g.

Proof. Let S be any $\delta-$closed set of $X \times Y$ and $x \in bICl(_{p_x}(S \cap G(g))$. Let U be any open set of X containing x and V any open set of Y containing g(x). Since g is almost $b-I-$continuous, we have $x \in g^{-1}(V) \subset bI\ Int(g^{-1}(Int(Cl(V))))$ and $U \cap bI\ Int(g^{-1}(IntCl(V))))) \in BIO(X,x)$. Since $x \in bICl(_{p_x}(S \cap G(g))$, $U \cap bI\ Int$ $(g^{-1}(IntCl(V))))) \cap {}_{p_x}(S \cap G(g))$ contains some point u of X. This implies that (u, g(u)) $\in$ S and g(u) $\in$ Int (Cl(V)). Thus, We have $\varnothing \neq (U \times Int(Cl(V)) \cap S \subset Int$ $(Cl(U \times V)) \cap S$ and hence (x,g(x)) $\in Cl_{\delta}(S)$. Since S is $\delta-$closed,(x,g(x)) $\in$ $(_{p_x}(S \cap G(g))$ and $x \in_{p_x} (S \cap G(g))$.Then $(_{p_x}(S \cap G(g))$ is $b-I$ closed.

Corollary 3.33. If $f:(X,\tau,I) \rightarrow (Y,\sigma)$ has a $\delta-$closed graph and $g:(X,\tau,I)$ $\rightarrow (Y,\sigma)$ is almost $b-I-$continuous,then the set $\{x \in X : f(x) = g(x)\}$ is $b-I-$ closed in X.

Proof. Since $G(f)$ is $\delta-$closed and $_{px}(G(f) \cap G(g)) = \{x \in X : f(x) = g(x)\}$ it follows from theorem 3.32 that $\{x \in X : f(x) = g(x)\}$ is $b-I$-closed in X.

Theorem 3.34. If for each pair of distinct x_1 and x_2 in an ideal topological space (X,τ,I) there exists a function f of X into a Hausdorff space Y such that $f(x_1) \neq f(x_2)$, f is weakly $b-I-$continuous and f is almost $b-I-$continuous at x_2,then X is $b-I-T_2$.

Proof. Since Y is Hausdorff, if for each pair of distinct point x_1 and x_2 there exist disjoint open sets V_1 and V_2 of Y containing $f(x_1)$ and $f(x_2)$,respectively; hence $Cl(V_1) \cap Int(Cl(V_2)) = \varnothing$. Since f is weakly $b-I-$continuous at x_1,there exists $U_1 \in BIO(X,x)$ such that $f(U_1) \subset Cl(V_1)$. Since f is almost $b-I$ -

continuous at x_2 ,there exists $U_2 \in BIO(X, x_2)$ such that $f(U_2) \subset Int(Cl(V_2))$. Therefore ,We obtain $U_1 \cap U_2 = \varnothing$. This shows that X is $b-I-T_2$.

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