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Comparison of regimes of active two-port networks with stabilization of load voltages

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Abstract. The problem of normalization of circuit regime parameters, choice of characteristic regimes and scale values is considered. Disadvantages of traditional normalization are shown. The method of projective coordinates reveals the characteristic regimes and proves the normalized expressions. The examples of recalculation of regime parameters are given for comparable circuits. The obtained results allow to carry out the analysis and to compare regimes of similar active two-port networks. The given approach is applicable to "flowed" form processes of different engineering areas.

Keywords: Normalization; similarity; characteristic regimes; active two-port; projective coordinates.

1 Introduction

In the theory of electric circuits one of the analysis problems is the calculation of actual or absolute values of regime parameters. Besides such calculation, in case of electric circuits with changeable parameters of elements, the representation of running regime parameters by a normalized form, with respect to some characteristic (for example, maximum) values, is important. In practice, in power supply systems (for example, direct current) the normalized forms of regime parameters give necessary information on qualitative characteristics or efficiency of steady state circuit regime. This allows comparing or setting regimes of different systems by the theory of similarity and simulation [1], [2]. For example, for a circuit in the form of an active two-pole, open circuit voltage, short circuit SC current, internal resistance can be the scales for the voltage, current and resistance of load. Let us specially note these values turn out at the expense of manipulations from the load terminals [3], [4].

However, the consideration of more complex circuits reveals no triviality of this problem, because a number of characteristic values are increased and well-founded approaches to the formation of normalized expressions are missing. The distributed power supply system with limited capacity voltage source and point of load regulators can be an example of such circuits [5], [6]. The low- dropout linear regulators can be used as voltage stabilizers [7], [8], [9].

For definiteness, it is possible to accept that such system, in sense of the circuit theory, represents the active multi-port network with set- up number of output terminals.

There are important features of such circuit – an interference of load currents on the voltage stabilizers regimes takes place; the SC regime has no physical sense and can't be accepted as a characteristic regime.

This brings up the problem of choice of the characteristic regimes, justification of type of the normalized expressions for parameters and equations of circuit, comparison of regimes between loads in the given circuit.

In a number of papers of the author, the approach is developed for interpretation of changes or "kinematics" of the circuit's regimes on the basis of projective geometry [10], [11], [12]. The load characteristic bunches define the system of projective coordinates with use of the characteristic values of regime parameters. Therefore, the coordinates of running regime point are expressed concerning of these points of characteristic regimes.

In this work projective geometry approach is applied to the resolve of the above problem.

2 Analysis of the traditional approach to normalizing of regime parameters

For an illustration of the assigned task, we consider two simple active two-poles with changeable loads in Fig.1.

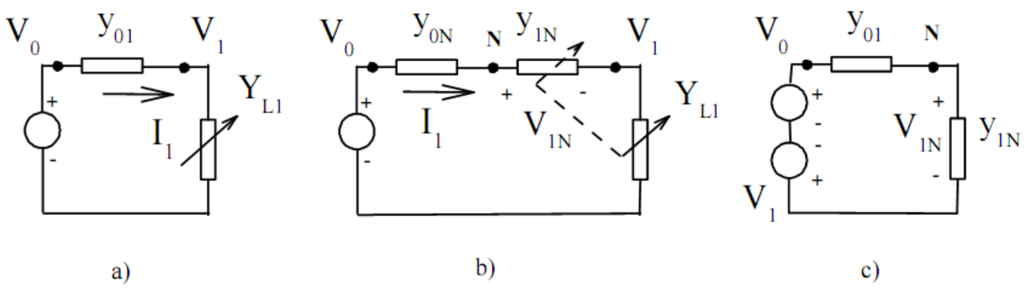

Fig.1. Two simple active two-poles with changeable loads:
a) - without voltage V_1 stabilization, b) - with voltage stabilization,
c) - equivalent circuit of active two-pole with voltage stabilization

The equation of the load straight line or *I-V* characteristic of the first active two-pole without voltage stabilization in Fig.1,a is given by

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$$I_1 = (V_0 - V_1)y_{01} = y_{01}V_0 - y_{01}V_1 \ ,$$

where conductivity y_{01} corresponds to the internal resistance of the voltage source V_0.

The normalization by the SC current $I_{0M} = y_{01}V_0$ permits to evaluate a qualitative characteristic of a running regime with a changeable load conductivity Y_{L1}. Then, we obtain the normalized expression of *I-V* characteristic

$$\frac{I_1}{I_{0M}} = 1 - \frac{V_1}{V_0}. \tag{1}$$

This expression contains two normalized values. The regimes of two similar circuits with running values of currents $I_1, \bar{I}_1$ and voltages $V_1, \overline{V_1}$ will be equivalent, identical or equal to each other if the normalized values of currents and voltages are equal to each other too

$$\frac{I_1}{I_{0M}} = \frac{\bar{I}_1}{\bar{I}_{0M}}, \quad \frac{V_1}{V_0} = \frac{\overline{V_1}}{\overline{V_0}}. \tag{2}$$

From these expressions, it is possible to define or set equal changes of regimes both for equal initial regimes, and for different initial regimes.

The equation of the load line of the second active two-pole with voltage V_1 stabilization in Fig.1,b is given by

$$I_1 = \frac{y_{0N} y_{1N}}{y_{0N} + y_{1N}} (V_0 - V_1) \ , \tag{3}$$

where conductivity y_{1N} corresponds to the conductivity of linear voltage regulator.

It is possible to carry out also normalization by the SC current $I_{0M} = y_{0N}V_0$ of the voltage source if at experimental investigation there is access to this source. Then

$$\frac{I_1}{I_{0M}} = \frac{y_{1N} / y_{0N}}{1 + y_{1N} / y_{0N}} \left(1 - \frac{V_1}{V_0}\right). \tag{4}$$

This expression contains three normalized values. Therefore, a possible condition of equal regimes corresponds to the equalities

$$\frac{I_1}{I_{0M}} = \frac{\bar{I}_1}{\bar{I}_{0M}}, \quad \frac{V_1}{V_0} = \frac{\overline{V_1}}{\overline{V_0}}, \quad \frac{y_{1N}}{y_{0N}} = \frac{\bar{y}_{1N}}{\bar{y}_{0N}}. \tag{5}$$

If regimes differ, how we may express this difference in a convenient view? It is not clear, how we can work with a set of these six different values. It would be convenient to work with one value which characterizes this difference.

If access to the voltage source is missing, what then we must choose as a normalizing value? It is possible to normalize by value of the maximum current of load $I_{1M} = y_{0N}(V_0 - V_1)$, when the linear regulator is completely closed. Then we have

$$\frac{I_1}{I_{1M}} = \frac{y_{1N} / y_{0N}}{1 + y_{1N} / y_{0N}}. \quad (6)$$

Therefore, the possible condition of equal regimes corresponds to the equalities

$$\frac{I_1}{I_{1M}} = \frac{\bar{I}_1}{\bar{I}_{1M}}, \quad \frac{y_{1N}}{y_{0N}} = \frac{\bar{y}_{1N}}{\bar{y}_{0N}}. \quad (7)$$

We again obtain two normalized values. But there is a contradiction with condition (5) for currents.

In the equation (6) we pass from value y_{1N} to the voltage V_{1N} of linear regulator. Then, we obtain the normalized expression of *I-V* characteristic of regulator

$$\frac{I_1}{I_{1M}} = 1 - \frac{V_{1N}}{V_0 - V_1}.$$

The equivalent circuit in Fig.1,c corresponds to this expression.

Even for such a simple circuit there is an uncertainty, how correctly or reasonably we may present the normalized expression of a regime.

The problem becomes complicated even more for a case of two and more loads with the voltage stabilization. For this, we consider Fig.2. In case of two loads (without conductivity y_N) the system of two equations turns out

$$I_1 \frac{y_{0N} + y_{1N}}{y_{1N}} = (V_0 - V_1) y_{0N} - I_2, \quad I_2 \frac{y_{0N} + y_{2N}}{y_{2N}} = (V_0 - V_2) y_{0N} - I_1. \quad (8)$$

It is possible also to carry out the normalization by the SC current of the voltage source. These expressions contain six normalized values. If regimes differ, we have a set of twelve different values. On the other hand, the normalization by values of the maximum currents of loads $I_{1M} = y_{0N}(V_0 - V_1)$, $I_{2M} = y_{0N}(V_0 - V_2)$ leads to appearance of reciprocal components

$$\frac{I_2}{I_{1M}}, \quad \frac{I_1}{I_{2M}}.$$

These components also raise the number of normalized values.

Therefore, the shown examples of the formal normalization do not allow comparing the regimes of different systems.

3 Projective coordinates of an active two- port network

Let us use the projective coordinate's method [10], [11], [12]. Now, we consider an active two- port network with changeable loads Y_{L1}, Y_{L2} in Fig.2.

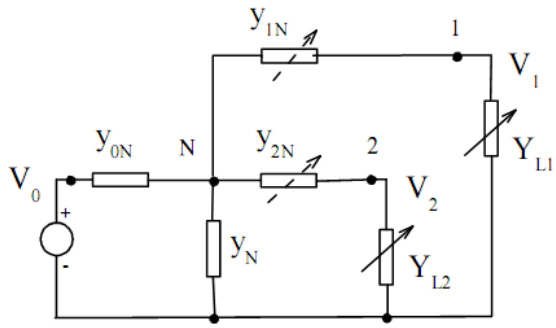

Fig. 2. Active two- port network with changeable loads Y_{L1}, Y_{L2}

This network is described by the following system of equations

$$\begin{aligned} I_1 \frac{y_{0N} + y_N + y_{1N}}{y_{1N}} &= y_{0N}V_0 - V_1(y_{0N} + y_N) - I_2 \\ I_2 \frac{y_{0N} + y_N + y_{2N}}{y_{2N}} &= y_{0N}V_0 - V_2(y_{0N} + y_N) - I_1 \end{aligned} \quad . \tag{9}$$

These expressions are the equations of two bunches of straight lines $(I_1, I_2, y_{1N}) = 0$, $(I_1, I_2, y_{2N}) = 0$ with parameters y_{1N}, y_{2N}. These bunches are presented in Fig.3. The bunch center, a point G_2, corresponds to the straight lines with parameter y_{1N}. Physically, the bunch center corresponds to such a characteristic regime of the load Y_{L1} which (that is regime) does not depend on value y_{1N}. It is carried out for the current $I_1 = 0$ at the expense of choice of the second load current I_2^{G2}. In this case, the voltage $V_N = V_1$. We obtain the characteristic value of the second load current using the first equation of system (9)

$$I_2^{G2} = y_{0N}V_0 - V_1(y_{0N} + y_N) \ . \tag{10}$$

The characteristic value of the first load current, which defines the bunch center G_1 of the straight lines with parameter y_{2N}, is expressed similarly

$$I_1^{G1} = y_{0N}V_0 - V_2(y_{0N} + y_N) \ . \tag{11}$$

In this case, the current $I_2 = 0$, and voltage $V_N = V_2$.

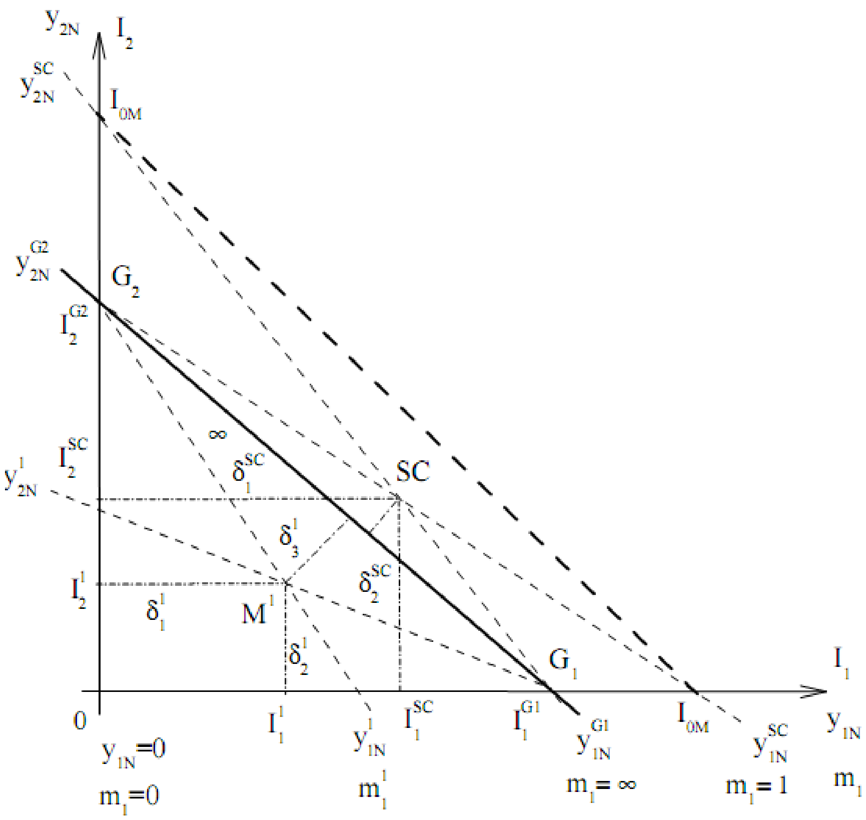

Fig. 3. Two bunches of straight lines with parameters y_{1N}, y_{2N}

We find the characteristic value of the first regulator y_{1N}^{G1} using the first equation of system (9) and current (11)

$$y_{1N}^{G1} = I_1^{G1} \frac{y_{0N} + y_N}{y_{0N}V_0 - V_1(y_{0N} + y_N) - I_1^{G1}} = I_1^{G1} \frac{y_{0N} + y_N}{I_2^{G2} - I_1^{G1}} = \frac{I_1^{G1}}{V_2 - V_1} \ . \tag{12}$$

Similarly, the characteristic value of the second regulator is

$$y_{2N}^{G2} = I_2^{G2} \frac{y_{0N} + y_N}{I_1^{G1} - I_2^{G2}} = \frac{I_2^{G2}}{V_1 - V_2}. \tag{13}$$

Let the initial or running regime corresponds to a point M^1 which is set by values of loads Y_{L1}^1, Y_{L2}^1 or currents $I_1^1 = Y_{L1}^1 V_1$, $I_2^1 = Y_{L2}^1 V_2$. The corresponding values of regulator conductivities are defined by the system (9). However, we obtain the more convenient relationships using (10), (11)

$$y_{1N}^1 = I_1^1 \frac{y_{0N} + y_N}{I_2^{G2} - (I_1^1 + I_2^1)}, \quad y_{2N}^1 = I_2^1 \frac{y_{0N} + y_N}{I_1^{G1} - (I_1^1 + I_2^1)}. \tag{14}$$

Also, this point M^1 is defined by projective non-uniform m_1^1, m_2^1 and homogeneous $\xi_1^1, \xi_2^1, \xi_3^1$ coordinates which are set by a triangle of reference $G_1\ 0\ G_2$ and a unit point [13], [14]. The point *0* is the origin of coordinates and the straight line $G_1\ G_2$ is a line of infinity ∞. As the unit point, we must also choose a some characteristic regime using the condition of stability of voltages V_1, V_2. As mentioned above, the SC load currents regime is not such one. However, the SC current regime of voltage source, when the voltage $V_N = 0$, allows finding such a characteristic regime. In this case, the SC current of voltage source is

$$I_{0M} = y_{0N} V_0 = I_1 + I_2.$$

This expression corresponds to an equation of straight line. In Fig.3, this line intersects the axes in the points $I_1 = I_{0M}$, $I_2 = I_{0M}$. The straight lines with parameters y_{1N}^{SC}, y_{2N}^{SC} correspond to these points. Let us determine the values y_{1N}^{SC}, y_{2N}^{SC}. We assume the current $I_2 = 0$ in the first and $I_1 = 0$ second equation (9). Then, the values

$$y_{1N}^{SC} = -y_{0N} V_0 / V_1, \quad y_{2N}^{SC} = -y_{0N} V_0 / V_2. \tag{15}$$

Now, we can define the load currents I_1^{SC}, I_2^{SC} using (15) and the equations (9)

$$I_1^{SC} = V_1 \frac{I_1^{G1}}{V_2 + V_1 I_1^{G1} / I_{0M}}, \quad I_2^{SC} = V_2 \frac{I_2^{G2}}{V_1 + V_2 I_2^{G2} / I_{0M}}. \tag{16}$$

These current correspond to the point SC in Fig.3. We consider this point SC as a unit point.

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Let us return, now, to the determination of projective coordinates of a running regime point. The non-uniform projective coordinate m_1^1 is set by a cross-ratio of four points. Three of these points correspond to the points of characteristic regimes, and the fourth point corresponds to the point of a running regime

$$m_1^1 = (0\ y_{1N}^1\ y_{1N}^{SC}\ y_{1N}^{G1}) = \frac{y_{1N}^1 - 0}{y_{1N}^1 - y_{1N}^{G1}} \div \frac{y_{1N}^{SC} - 0}{y_{1N}^{SC} - y_{1N}^{G1}}, \tag{17}$$

There, the points $y_{1N} = 0,\ y_{1N} = y_{1N}^{G1}$ correspond to the extreme or base ones. The point y_{1N}^{SC} is the unit one. The values of m_1 are shown in Fig.3. The non-uniform projective coordinate m_2^1 is expressed similarly

$$m_2^1 = (0\ y_{2N}^1\ y_{2N}^{SC}\ y_{2N}^{G2}) = \frac{y_{2N}^1}{y_{2N}^1 - y_{2N}^{G2}} \div \frac{y_{2N}^{SC}}{y_{2N}^{SC} - y_{2N}^{G2}}. \tag{18}$$

The expressions (17), (18) demonstrate the conductivities $y_{1N},\ y_{2N}$ in a relative form. This essentially differs from the formal normalization of these values by the conductivity y_{0N} or $y_{0N} + y_N$ according to (4).

The homogeneous projective coordinates ξ_1, ξ_2, ξ_3 set the non-uniform coordinates as follows

$$m_1 = \frac{\xi_1}{\xi_3} = \frac{\rho\xi_1}{\rho\xi_3},\ m_2 = \frac{\xi_2}{\xi_3} = \frac{\rho\xi_2}{\rho\xi_3}, \tag{19}$$

where ρ is a coefficient of proportionality.

The homogeneous coordinates are defined by a ratio of distances of the points M^1, SC to the sides of the triangle of reference

$$\rho\,\xi_1^1 = \frac{\delta_1^1}{\delta_1^{SC}} = \frac{I_1^1}{I_1^{SC}},\quad \rho\,\xi_2^1 = \frac{\delta_2^1}{\delta_2^{SC}} = \frac{I_2^1}{I_2^{SC}},\quad \rho\,\xi_3^1 = \frac{\delta_3^1}{\delta_3^{SC}}, \tag{20}$$

$$\mu_3\delta_3^{SC} = \left(\frac{I_1^{SC}}{I_1^{G1}} + \frac{I_2^{SC}}{I_2^{G2}} - 1\right),\ \mu_3\delta_3^1 = \left(\frac{I_1^1}{I_1^{G1}} + \frac{I_2^1}{I_2^{G2}} - 1\right), \tag{21}$$

where μ_3 is normalized factor.

Then, the homogeneous coordinates obtain the matrix form

$$\begin{pmatrix} \rho\xi_1 \\ \rho\xi_2 \\ \rho\xi_3 \end{pmatrix} = \begin{pmatrix} \frac{1}{I_1^{SC}} & 0 & 0 \\ 0 & \frac{1}{I_2^{SC}} & 0 \\ \frac{1}{I_1^{G1}\mu_3\delta_3^{SC}} & \frac{1}{I_2^{G2}\mu_3\delta_3^{SC}} & -\frac{1}{\mu_3\delta_3^{SC}} \end{pmatrix} \cdot \begin{pmatrix} I_1 \\ I_2 \\ 1 \end{pmatrix} = [C]\cdot[I] . \quad (22)$$

The inverse transformation is

$$\begin{pmatrix} \rho_I I_1 \\ \rho_I I_2 \\ \rho_I 1 \end{pmatrix} = \begin{pmatrix} I_1^{SC} & 0 & 0 \\ 0 & I_2^{SC} & 0 \\ \frac{I_1^{SC}}{I_1^{G1}} & \frac{I_2^{SC}}{I_2^{G2}} & -\mu_3\delta_3^{SC} \end{pmatrix} \cdot \begin{pmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \end{pmatrix} = [C]^{-1}\cdot[\xi] . \quad (23)$$

From here, we pass to the currents

$$I_1 = \frac{\rho_I I_1}{\rho_I 1} \frac{I_1^{SC}\cdot m_1}{\frac{I_1^{SC}}{I_1^{G1}}\cdot m_1 + \frac{I_2^{SC}}{I_2^{G2}}\cdot m_2 - \mu_3\delta_3^{SC}}$$
$$I_2 = \frac{\rho_I I_2}{\rho_I 1} = \frac{I_2^{SC}\cdot m_2}{\frac{I_1^{SC}}{I_1^{G1}}\cdot m_1 + \frac{I_2^{SC}}{I_2^{G2}}\cdot m_2 - \mu_3\delta_3^{SC}} . \quad (24)$$

Therefore, it is possible to consider that non-uniform and homogeneous projective coordinates reasonably represent the running regime of a circuit by a relative form.

4 Recalculation of regime parameters of comparable circuits

Let us consider two similar circuits with different values of element parameters and regime parameters. It is necessary to prove an approach to comparison or recalculation of running regimes of such circuits. The characteristics of these comparable circuits are given in Fig.4. The condition of regime comparison is the conformity of characteristic regimes, how this is shown by arrows and that permits to regard this conformity as a projective transformation.

Case of equal regimes. The running regimes of these circuits (a point M_1 corresponds to a point $\overline{M}_1$) will be equivalent or equal to the each other if their non- uniform coordinates are equal, and homogeneous projective coordinates are proportional, that is

$$m_1 = \overline{m}_1 \;,\; m_2 = \overline{m}_2 \,,\, [\rho\xi] = [\overline{\xi}] \,. \tag{25}$$

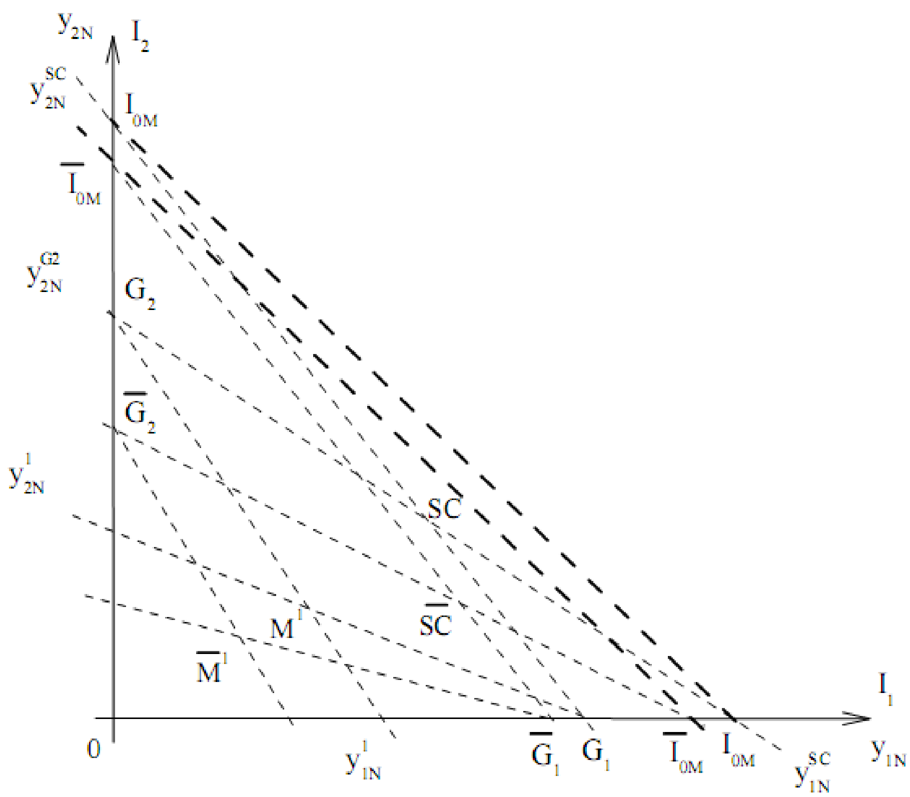

Fig. 4. Characteristics of similar circuits

We find the conformity of load currents using the relationships (22), (23)

$$[\rho\overline{\mathbf{I}}] = [\overline{\mathbf{C}}]^{-1} \cdot [\overline{\xi}] = [\overline{\mathbf{C}}]^{-1} \cdot [\mathbf{C}] \cdot [\mathbf{I}] = [\mathbf{M}] \cdot [\mathbf{I}] \,. \tag{26}$$

The resulting matrix $[\mathbf{M}]$ has the view

$$[\mathbf{M}] = \begin{bmatrix} M_{11} & 0 & 0 \\ 0 & M_{22} & 0 \\ M_{31} & M_{32} & M_{33} \end{bmatrix}, \qquad (27)$$

$$M_{11} = \frac{\bar{I}_1^{SC}}{I_1^{SC}}, \quad M_{22} = \frac{\bar{I}_2^{SC}}{I_2^{SC}}, \quad M_{33} = \frac{\bar{\delta}_3^{SC}}{\delta_3^{SC}},$$

$$M_{31} = \frac{\bar{I}_1^{SC}}{\bar{I}_1^{G1}} \frac{1}{I_1^{SC}} - \frac{\bar{\delta}_3^{SC}}{I_1^{G1}\delta_3^{SC}}, \quad M_{32} = \frac{\bar{I}_2^{SC}}{\bar{I}_2^{G2}} \frac{1}{I_2^{SC}} - \frac{\bar{\delta}_3^{SC}}{I_2^{G2}\delta_3^{SC}}.$$

The similar condition of the conformity of regulator conductivities follows from (17) and (18). In particular, the conductivity $\bar{y}_{1N}$ is defined by the equality

$$\frac{y_{1N}^1}{y_{1N}^1 - y_{1N}^{G1}} \div \frac{y_{1N}^{SC}}{y_{1N}^{SC} - y_{1N}^{G1}} = \frac{\bar{y}_{1N}^1}{\bar{y}_{1N}^1 - \bar{y}_{1N}^{G1}} \div \frac{\bar{y}_{1N}^{SC}}{\bar{y}_{1N}^{SC} - \bar{y}_{1N}^{G1}}. \qquad (28)$$

Case of not equal regimes. Let subsequent regime of the second circuit be given by a point $\bar{M}_2$ in Fig.4. The point M_2 of equal regime of the first circuit corresponds to this point $\bar{M}_2$. Therefore, it is possible to consider the points M_1, M_2 of the first circuit as a change of its regime, that is $M_1 \rightarrow M_2$. Then, the same change of regime will be for the second circuit, that is $\bar{M}_1 \rightarrow \bar{M}_2$. Therefore, this regime change defines the difference of regimes of comparable circuits.

Let us validate an expression of regime change via change of non- uniform projective coordinates. The non- uniform coordinate $\bar{m}_1^2$ of subsequent regime, the point $\bar{M}_2$, is given by (17)

$$\bar{m}_1^2 = (0 \; \bar{y}_{1N}^2 \; \bar{y}_{1N}^{SC} \; \bar{y}_{1N}^{G1}) = \frac{\bar{y}_{1N}^2}{\bar{y}_{1N}^2 - \bar{y}_{1N}^{G1}} \div \frac{\bar{y}_{1N}^{SC}}{\bar{y}_{1N}^{SC} - \bar{y}_{1N}^{G1}}.$$

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The change of regime are naturally expressed through the cross- ratio

$$\overline{m}_1^{21} = (0\ \overline{y}_{1N}^2\ \overline{y}_{1N}^1\ \overline{y}_{1N}^{G1}) = \frac{\overline{y}_{1N}^2}{\overline{y}_{1N}^2 - \overline{y}_{1N}^{G1}} \div \frac{\overline{y}_{1N}^1}{\overline{y}_{1N}^1 - \overline{y}_{1N}^{G1}} = \overline{m}_1^2 \div \overline{m}_1^1 . \quad (29)$$

This change is also equal to the change of regime of the first circuit $\overline{m}_1^{21} = m_1^{21} = m_1^2 \div m_1^1$, and $\overline{m}_1^1 = m_1^1$.

Let us express these changes through load currents. Using (19), (20), (21) we obtain

$$m_1^{21} = \frac{I_1^2}{I_1^1}\frac{\mu_3\delta_3^1}{\mu_3\delta_3^2},\ m_2^{21} = \frac{I_2^2}{I_2^1}\frac{\mu_3\delta_3^1}{\mu_3\delta_3^2}. \quad (30)$$

If the changes $m_1^{21} \neq 1, m_2^{21} \neq 1$, then the regimes of these circuits are different. In addition, if the $m_1^{21} \neq m_2^{21}$, then, for the given circuit, the load regimes are different.

Let the changes m_1^{21}, m_2^{21} and the initial equal regime, the points $M_1, \overline{M}_1$, be given. It is necessary to find load currents of these regime changes, the points $M_2, \overline{M}_2$.

Using the expression (28) of current change [5] we have for the second circuit

$$[\overline{\mathbf{I}}^2] = \begin{pmatrix} \rho\overline{I}_1^2 \\ \rho\overline{I}_2^2 \\ \rho 1 \end{pmatrix} = \begin{pmatrix} m_1^{21} & 0 & 0 \\ 0 & m_2^{21} & 0 \\ \frac{1}{\overline{I}_1^{G1}}(m_1^{21}-1) & \frac{1}{\overline{I}_2^{G2}}(m_2^{21}-1) & 1 \end{pmatrix} \cdot \begin{pmatrix} \overline{I}_1^1 \\ \overline{I}_2^1 \\ 1 \end{pmatrix} = [\mathbf{m}^{21}] \cdot [\overline{\mathbf{I}}^1]. \quad (31)$$

From here, we pass to the required currents

$$\overline{I}_1^2 = \frac{\overline{I}_1^1 \cdot m_1^{21}}{\frac{\overline{I}_1^1}{\overline{I}_1^{G1}} \cdot (m_1^{21}-1) + \frac{\overline{I}_2^1}{\overline{I}_2^{G2}} \cdot (m_2^{21}-1) + 1}$$
$$\overline{I}_2^2 = \frac{\overline{I}_2^1 \cdot m_2^{21}}{\frac{\overline{I}_1^1}{\overline{I}_1^{G1}} \cdot (m_1^{21}-1) + \frac{\overline{I}_2^1}{\overline{I}_2^{G2}} \cdot (m_2^{21}-1) + 1} . \quad (32)$$

Similar relationships are obtained for the first circuit.

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Now, we find an expression of recalculation of comparable circuit currents; the conformity $M_1 \rightarrow \overline{M}_2$. Using (31), (26) we obtain

$$[\bar{\mathbf{I}}^2]=[\mathbf{m^{21}}]\cdot[\bar{\mathbf{I}}^1]=[\mathbf{m^{21}}]\cdot[\mathbf{M}]\cdot[\mathbf{I^1}]=[\mathbf{J}]\cdot[\mathbf{I^1}].$$

The resultant matrix $[\mathbf{J}]$ has the view

$$[\mathbf{J}]=\begin{bmatrix} J_{11} & 0 & 0 \\ 0 & J_{22} & 0 \\ J_{31} & J_{32} & J_{33} \end{bmatrix}, \tag{33}$$

$$J_{11}=m_1^{21}\frac{\bar{I}_1^{SC}}{I_1^{SC}}, \quad J_{22}=m_2^{21}\frac{\bar{I}_2^{SC}}{I_2^{SC}}, \quad J_{33}=\frac{\mu_3\bar{\delta}_3^{SC}}{\mu_3\delta_3^{SC}},$$

$$J_{31}=m_1^{21}\frac{\bar{I}_1^{SC}}{\bar{I}_1^{G1}}\frac{1}{I_1^{SC}}-\frac{\mu_3\bar{\delta}_3^{SC}}{I_1^{G1}\mu_3\delta_3^{SC}}, \quad J_{32}=m_2^{21}\frac{\bar{I}_2^{SC}}{\bar{I}_2^{G2}}\frac{1}{I_2^{SC}}-\frac{\mu_3\bar{\delta}_3^{SC}}{I_2^{G2}\mu_3\delta_3^{SC}}.$$

From the obtained expressions, the procedure of regime comparison of two circuits follows. For this purpose, we consider an example.

Example. Let us consider a circuit with two loads in Fig.1. The element parameters have the following values, $U_0=5$, $y_{0N}=2.5$, $y_N=0.625$. Hereinafter, the dimensions of values are not specified.

The stabilized load voltages are equal to $V_1=2$, $V_2=3$.

The characteristic values of currents and conductivities of circuit for the first load by (11), (12), (15), (16) are

$$I_1^{G1}=3.125, \; y_{1N}^{G1}=3.125, \; y_{1N}^{SC}=-6.25, I_1^{SC}=1.785.$$

The characteristic values of currents and conductivities of circuit for the second load by (10), (13), (15), (16) are

$$I_2^{G2}=6.25, \; y_{2N}^{G2}=-6.25, \; y_{2N}^{SC}=-4.166, I_2^{SC}=5.357.$$

Let the currents of initial regime, point M_1, be equal to

$$I_1^1=1, \; I_2^1=1.$$

Then, the conductivities of the regulators by (14)

$y_{1N}^{1} = 0.735, \ y_{2N}^{1} = 2.777$.

The non- uniform projective coordinates by (17), (18)

$m_1^1 = -0.461, \ m_2^1 = -0.154$.

The negative values mean that the point M_1 of a running regime and the unit point SC are on different sides from the infinitely remote straight line $G_1 \, G_2$.

The distances of the points M_1 , SC to the straight line $G_1 \, G_2$ by (21)

$$\mu_3\delta_3^1 = \left(\frac{1}{3.125} + \frac{1}{6.25} - 1\right) = -0.52 \, , \ \mu_3\delta_3^{SC} = \left(\frac{1.785}{3.125} + \frac{5.357}{6.25} - 1\right) = 0.428 .$$

The homogeneous coordinates by (20)

$$\rho\,\xi_1^1 = \frac{1}{1.785} \ , \qquad \rho\,\xi_2^1 = \frac{1}{5.357} = 0.186 \, , \qquad \rho\,\xi_3^1 = \frac{-0.52}{0.428} .$$

The matrix $[\mathbf{C}]$ by (22)

$$[\mathbf{C}] = \begin{bmatrix} \frac{1}{1.785} & 0 & 0 \\ 0 & \frac{1}{5.357} & 0 \\ \frac{1}{3.125 \cdot 0.428} & \frac{1}{6.25 \cdot 0.428} & -\frac{1}{0.428} \end{bmatrix} .$$

The reverse matrix by (23)

$$[\mathbf{C}]^{-1} = \begin{bmatrix} 1.785 & 0 & 0 \\ 0 & 5.357 & 0 \\ \frac{1.785}{3.125} & \frac{5.357}{6.25} & -0.428 \end{bmatrix} .$$

Let us check up the currents by (24)

$$I_1^1 = \frac{-1.785 \cdot 0.461}{-\frac{1.785}{3.125} \cdot 0.461 - \frac{5.357}{6.25} \cdot 0.153 - 0.428} = \frac{-0.82}{-0.82} = 1,$$

$$I_2^1 = \frac{-5.357 \cdot 0.153}{-0.82} = 1.$$

We consider the second circuit. The element parameters have the same values, except the conductivity $\bar{y}_N = 0{,}25$.

The characteristic values of currents and conductivities of circuit for the first load

$$\bar{I}_1^{G1} = 4.25,\ \bar{y}_{1N}^{G1} = 4.25,\ \bar{y}_{1N}^{SC} = -6.25,\ \bar{I}_1^{SC} = 2.309,$$

and for the second load

$$\bar{I}_2^{G2} = 7,\ \bar{y}_{2N}^{G2} = -7,\ \bar{y}_{2N}^{SC} = -4.166,\ \bar{I}_2^{SC} = 5.706.$$

The distance of point $S\bar{C}$ to straight line $\bar{G}_1\ \bar{G}_2$

$$\bar{\mu}_3 \bar{\delta}_3^{SC} = 0.358.$$

Case of equal regimes. It is necessary to define the values of load currents of the second circuit. For this purpose, we calculate elements of the matrix $[\mathbf{M}]$ according to (25)

$$M_{11} = \frac{2.309}{1.785} = 1.293,\ M_{22} = \frac{5.706}{5.357} = 1.0652,\ M_{33} = \frac{0.358}{0.428} = 0.836,$$

$$M_{31} = \frac{2.309}{4.25}\frac{1}{1.785} - \frac{0.358}{3.125 \cdot 0.428} = 0.0365,$$

$$M_{32} = \frac{5.706}{7}\frac{1}{5.357} - \frac{0.358}{6.25 \cdot 0.428} = 0.0183.$$

The values of currents by (26)

$$\bar{I}_1^1 = \frac{1.293 \cdot 1}{0.0365 \cdot 1 + 0.0183 \cdot 1 + 0.836} = \frac{1.293}{0.89} = 1.45,$$

$$\bar{I}_2^1 = \frac{1.0652 \cdot 1}{0.89} = 1.194\,.$$

Let us check up the equality of the non- uniform coordinates by (25). The conductivities of regulators by (14)

$$\bar{y}_{1N}^1 = \frac{1.45 \cdot 2.75}{7 - 2.645} = 0.916\,,\ \ \bar{y}_{2N}^1 = \frac{1.194 \cdot 2.75}{4.25 - 2.645} = 2.047\,.$$

According to (28), the non- uniform coordinates are identical

$$\bar{m}_1^1 = \frac{0.916}{0.916 - 4.25} \div \frac{-6.25}{-6.25 - 4.25} = -0.274 \div 0.595 = -0.461 = m_1^1\,,$$

$$\bar{m}_2^1 = \frac{2.0475}{2.0475 + 7} \div \frac{-4.166}{-4.166 + 7} = 0.226 \div (-1.47) = -0.154 = m_2^1\,.$$

Case of not equal regimes. Let us consider the second circuit. We believe that the regime corresponds to the point $\bar{M}_2$. The currents and conductivities are respectively $\bar{I}_1^2 = 1.5, \bar{I}_2^2 = 2$ and $\bar{y}_{1N}^2 = 1.179, \bar{y}_{2N}^2 = 7.333$.
Then, the change of regime by (29)

$$m_1^{21} = \frac{1.179}{1.179 - 4.25} \div \frac{0.9164}{0.9164 - 4.25} = 0.3839 \div 0.2749 = 1.396\,,$$

$$m_2^{21} = \frac{7.333}{7.333 + 7} \div \frac{2.0475}{2.0475 + 7} = 0.5116 \div 0.2263 = 2.261\,.$$

Let us check up the change of regime through currents by (30)

$$m_1^{21} = \frac{I_1^2}{I_1^1}\frac{\mu_3 \delta_3^1}{\mu_3 \delta_3^2} = \frac{1.5}{1.45}\frac{0.48}{0.3613} = 1.396,\ \ m_2^{21} = \frac{2}{1.194}\frac{0.48}{0.3613} = 2.261\,.$$

Let the changes $m_1^{21} = 1.396, m_2^{21} = 2.261$ and the initial equal regime, points $M_1, \overline{M}_1$, be given. The values of initial currents correspond to our example of equal regimes. It is necessary to find the load currents of these regime changes, the point $M_2, \overline{M}_2$.

The currents of the second circuit by (32)

$$\bar{I}_1^2 = \frac{1.45 \cdot 1.397}{1.45 \cdot 0.0935 + 1.194 \cdot 0.18 + 1} = \frac{2.02}{1.35} = 1.5, \ \bar{I}_2^2 = \frac{1.194 \cdot 2.261}{1.35} = 2.$$

For the first circuit

$$I_1^2 = \frac{1 \cdot 1.397}{1 \cdot 0.1271 + 1 \cdot 0.2018 + 1} = \frac{1.397}{1.329} = 1.051, \quad I_2^2 = \frac{1 \cdot 1.261}{1.329} = 1.701.$$

Let us check up the conformity of points $M_1 \to \overline{M}_2$. For this purpose, we calculate the matrix $[\mathbf{J}]$ by (33)

$$[\mathbf{J}] = \begin{bmatrix} 1.8073 & 0 & 0 \\ 0 & 2.4089 & 0 \\ 0.1574 & 0.2103 & 0.8368 \end{bmatrix}.$$

The currents of point $\overline{M}_2$

$$\bar{I}_1^2 = \frac{1.8073 \cdot 1}{0.1574 \cdot 1 + 0.2103 \cdot 1 + 0.836} = \frac{1.8073}{1.2045} = 1.5, \ \bar{I}_2^2 = \frac{2.4089}{1.2045} = 2.$$

Conclusions

1. The traditional approach of normalization does not allow comparing regimes of circuits with stabilization of load voltages.
2. The projective coordinates reasonably represent a running regime of circuit in a relative form.
3. The recalculation formulas of regime parameters are obtained for comparable circuits.

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