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A New Optimization of Noise Transfer Function of Sigma-delta-modulator with Supposition Loop Filter Stability

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***Abstract*—**In this paper a discrete time sigma-delta ADC with new assumptions in optimization of noise transfer function (NTF) is presented, that improve SNR and accuracy of ADC. Zeros and poles of sigma-delta's loop filter is optimized and located by genetic algorithm with assumption loop filter stability and final quantization noise density of modulator will be significantly decrease. Supposition density of quantization noise as default of optimization result without need to additional circuit or filter, the folded noise in pass band due to down sampling, has been minimized so SNR will be more increase. The circuit is designed and implemented using MATLAB. The simulator result of sigma-delta ADC demonstrates this methodology has 7db (equivalent more than 1bit) improvement in SNR.

***Index Terms*— sigma-delta modulator, optimization, loop filter, decimation filter, quantization noise, NTF.**

1. INTRODUCTION

There is a large demand in wired and wireless communication systems for high-performance analog to- digital converters which have a wide signal bandwidth and high resolution. Oversampling ADCs use more digital signal processing to perform analog-to-digital conversion compared with Nyquist-rate ADCs. The advantage is significantly relaxed matching requirements on analog components, while still achieving medium to high resolution [1]. Furthermore, oversampling ADCs don't need steep roll-off anti-alias filtering typically required in Nyquist-rate ADCs. Conceptually, delta-sigma ADCs provide high resolution and linearity while using a low-resolution quantizer by taking advantage of oversampling and noise shaping. There are several design parameters: quantizer resolution, loop filter order, oversampling ratio (OSR). Increasing any of these parameters improves the SQNR [1].Most of the publications in the past are just regarding the NTF of the system so that the focus of most of the optimization methods tends to increase the signal to noise ratio (SNR) or the dynamic range (DR) to a maximum level.[2]

In [2] presented an optimization method by considering stability and STF peaking and doesn't discussion about NTF and minimizing quantization noise. [3] presented an optimization process by non linear decoding for obtaining minimum quantization noise and doesn't specially investigate and optimize STF and NTF in sigma delta modulator. [4] optimized NTF of sigma delta and doesn't discussion about STF and loop filter stability. [5] designed a sigma delta modulator with stability considerations and designed NTF by Chebyshev and Butterworth filter. There is no reason with using this filters we obtain optimal NTF. In [2],[3],[4],[5] and other paper until presented now, for optimizing sigma delta loop filter don't consider density of quantization noise of sigma-delta output. Studying quantization noise of sigma-delta output doesn't attenuate quite in transient band of digital filter and this noise combination with signal and lead to decrease SNR and when we down sample output, this noise several time folded in pass band and SNR more decreases.

In this paper for optimizing NTF and loop filter zeros and poles of sigma-delta modulator, calculated quantization noise of sigma-delta output. Also it's considered STF and loop filter's pole for stability. By this methodology we improve SNR at least 10db compare with same order sigma-delta.

2. SIGMA-DELTA MODULATOT

2.1 Review Sigma-Delta

Sigma-delta analog to digital convertor contain sigma-delta modulator and some digital low pass filter and decimation filter after modulator. The block diagram sigma-delta modulator ADC is shown in Fig. 1

The sigma-delta modulation is based on a negative feedback loop in which a low quality quantization is performed at a high sampling frequency, and a big amount of the quantization noise is moved into a superior area of the input signal frequency band [6]. The block diagram of first order one bit quantizer sigma-delta modulator is shown in Fig.2. Digital low pass filter and decimation filter, filter the high frequency shaped noise by sigma-delta modulator. Modulator output determine by equation (1). Whatever order of modulator's loop filter goes up, noise shaping would be better and signal to noise ratio (SNR) would be higher (Fig.3).

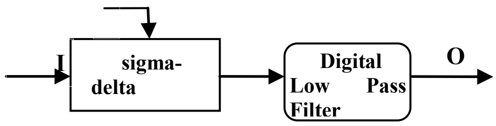

Figure 1: block diagram of sigma-delta convertor

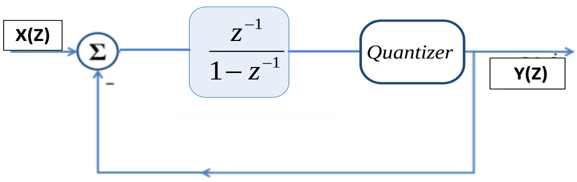

Figure 2: block diagram of first order one bit quantizer sigma-delta modulator

$$\boldsymbol{Y(Z) = STF(Z)X(Z) + NTF(Z)E(Z)} \qquad (1)$$

$$STF(Z) = \frac{Y(Z)}{X(Z)} = \frac{\frac{1}{Z-1}}{1+\frac{1}{Z-1}} = Z^{-1} \qquad (2)$$

$$NTF(Z) = \frac{Y(Z)}{E(Z)} = \frac{1}{1 + \frac{1}{Z-1}} = 1 - Z^{-1} \qquad (3)$$

SNR in output of sigma-delta modulator is depended on density of output quantization noise. Whatever density of output quantization noise is lower, SNR would be higher.

Location of loop filter's zeros and poles in first and second order of sigma-delta modulator has been fixed. But in high order loop filter, by changing location of zeros and poles, density of output quantization noise will be changed. In this paper zeros and poles of 5 order sigma-delta modulator has been located that output quantization noise density of sigma-delta ADC has been minimized. For calculating the output quantization noise density of sigma-delta ADC, noise transfer function of modulator and low pass digital filter after the modulator and down sampling effects of down sampler are considered. Then final function of density noise has been calculated and then has been optimized by genetic algorithm.

2.2 Choosing Sigma-Delta Loop Filter

For designing sigma-delta loop filter, CRFF structure in fig4 has been chosen. NTF zeroes of this structure are on unite circle (Z=1) and it makes density of output modulator noise be decrease [8]. For canceling high frequency quantization noise from modulator’s output and getting high resolution, modulator’s output must passed away through decimation. Output modulator rate has been increased by over-sampling must be decreased by down-sampler block after modulator, to nyquist rate. In fig 5 digital low pass filter and down sampler block has been shown.

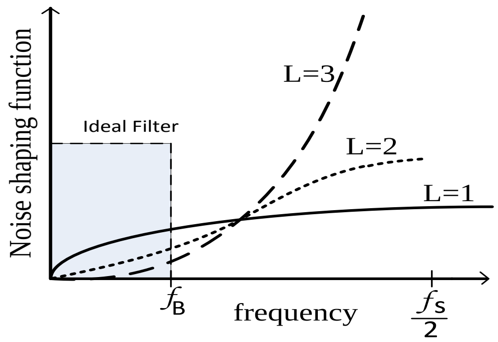

Figure 3: comparing noise shaping in NTF of first and second and third order of sigma-delta modulator

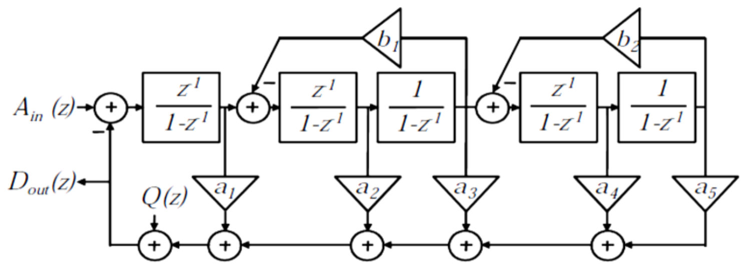

Figure 4: 5th order CRFF structure sigma-delta modulator [8]

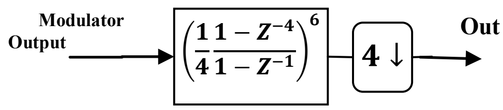

Figure 5: decimation filter and down sampler block after the sigma-delta modulator

In digital low pass filter, a comb filter has been used and in order down sampling modulator output down-sampler block has been. Equation (4) shows Z transfer of designed decimation filter. In fig.6, frequency response of designed decimation filter is shown.

$$H(Z) = \left(\frac{1}{4}\frac{1 - Z^{-4}}{1 - Z^{-1}}\right)^6 \qquad (4)$$

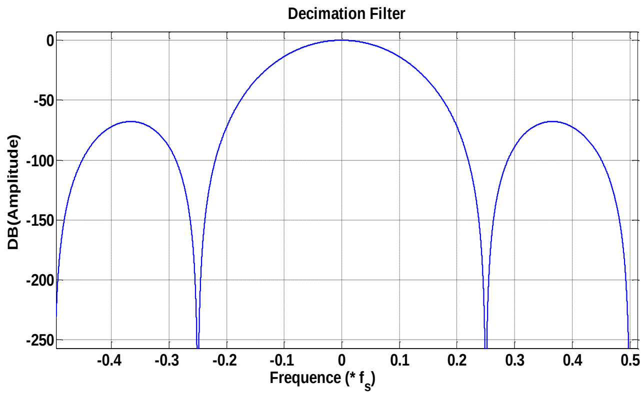

Figure 6: frequency response of decimation filter designed

3. Optimizing SIGMA-DELTA

3.1 Genetic Algorithm

GAs are search and optimization algorithms based on the mechanics of natural selection and natural genetics. They make use of structured but randomized information exchange and concept of the survival of the fittest. The algorithm starts with an initial population which consists of a collection of chromosomes i.e. possible solutions coded in the form of strings. The chromosome which produces the minimum error function value represents the best solution. The chromosomes which represent the better solutions are selected using roulette wheel selection technique. Genetic operators like crossover, mutation, elitism etc. are applied over the selected chromosomes. As a result a new set of chromosome is produced. This process is repeated until a fit solution appears. In essence, a population of chromosomes is always available to get the desired result. Occasionally a new part is added to a chromosome to make it more robust. Genetic algorithms exploit past to extrapolate new search points to provide improved performance. A robust method like GA works well across a wide range of problems and also is more efficient. The traditional derivatives based approach, enumerative schemes and simple random walks are not that good for all classes of problems. On the other hand, heuristics approaches, such as genetic algorithms (GAs), differ from the traditional ones in that there exists a high probability that the global optimal solution will be reached. Fig.7 shows the flowchart of the binary GA [7].

3.2 Using GA in ΣΔ ADC Design

In the design of ΣΔ ADCs, we need to optimize a large set of parameters including the overall structures and the performance of the building blocks to achieve the minimum density of noise in output and required signal-to-noise ratio. Therefore, behavioral simulations were carried out using a set of Simulink models in MATLAB Simulink environment in order to verify the performance The most important parameter to be optimized in a sigma-delta modulator are the gain coefficients in order to achieve the desired signal-to-noise ratio. GA is one of the best optimization techniques which find a global optimum solution without taking much of the computational power. The steps involved in the process of optimization using GA is shown in Fig.7 In the first step genetic algorithm cost function or fitness has to be calculated. The cost function is the density noise function of sigma-delta ADC output. After defining the fitness, the gain coefficients have been defined as chromosome and with considering linear and non linear condition, genetic algorithm has been started.

3.3 Calculating density noise of sigma-delta as fitness function

For calculating density noise of sigma-delta first step, noise transfer function (NTF) of sigma-delta modulator must be calculated then density of quantization noise from NTF must be calculated. NTF transfer function of fig.4 is calculated by Meison rule:

$$NTF = \frac{num_{NTF}}{den_{NTF}} \qquad (6)$$

$$num_{NTF} = 1 + \frac{b_1 Z}{(Z-1)^2} + \frac{b_2 Z}{(Z-1)^2} + \frac{b_1 b_2 Z^2}{(Z-1)^4} \qquad (7)$$

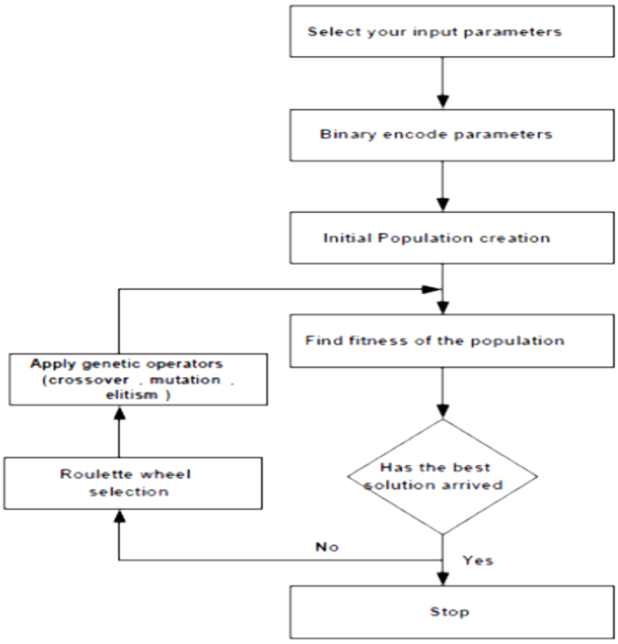

Figure 7: genetic algorithm stage [7]

$$den_{NTF} = 1 + \frac{a_1}{Z-1} + \frac{a_2}{(Z-1)^2} + \frac{a_3 Z}{(Z-1)^3} + \frac{a_4 Z}{(Z-1)^4} + \frac{a_5 Z^2}{(Z-1)^5} + \frac{b_1 Z}{(Z-1)^2} + \frac{b_2 Z}{(Z-1)^2} + \frac{b_1 a_1 Z}{(Z-1)^3} + \frac{b_2 a_1 Z}{(Z-1)^3} + \frac{b_1 b_2 Z^2}{(Z-1)^4} + \frac{a_2 b_2 Z}{(Z-1)^4} + \frac{a_3 b_2 Z^2}{(Z-1)^5} + \frac{a_1 b_1 b_2 Z^2}{(Z-1)^5} \quad (8)$$

$$P_{e\_total} = \int_{-f_{bound}/2}^{f_{bound}/2} S_e^2(f) |NTF(f)|^2 \, df \quad (9)$$

That $S_e(f)$ is density of white noise in sigma-delta modulator and it is constant and equal to $\Delta^2/_{12}$ that Δ is quantization step.

3.5 Considered genetic algorithm conditions

In all stage of genetic optimizing modulator loop filter must be stable and coefficient gain must be determined that either density noise of sigma-delta be minimum or modulator loop filter be remained stable. For remaining loop filter stable, poles of NTF must be located in unique circle (Z=1) during the optimization.

3.6 Genetic algorithm result

After set all genetic algorithm settings in MATLAB and running it, it has gotten to this point for gain coefficient:

$$\begin{cases} a_1 = 2.61341289830207830 \\ a_2 = 1.70344071624279030 \\ a_3 = 1.34238372790813450 \\ a_4 = 0.47294158601760866 \\ a_5 = 0.06448528103828430 \\ b_1 = 0.020690429687499998 \\ b_2 = 0.03582616508007050 \end{cases}$$

After that, result is put in loop filter and result of modulator output can be shown.

4. SIMULATION RESULT

In fig.8 frequency response of NTF, STF and loop filter are shown for the loop filter's gain coefficients are obtained from genetic algorithm. Fig.9 shows zeros and poles of NTF. As same as expected, NTF's zeros are on unite circle (Z=1) and NTF's poles are in unite circle then filter is stable.

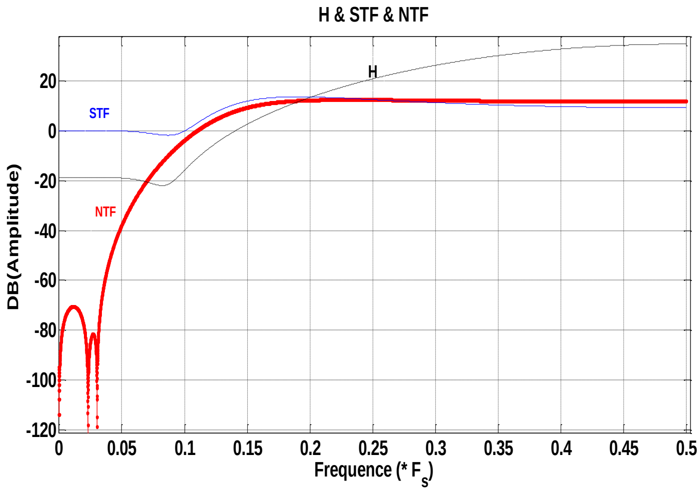

Figure 8: frequency response of NTF and STF and loop filter designed

Designed Analog to digital converter (ADC) has been tested for a sinuous signal with $f_{in} = 0.5\ kHz$ and sampling frequency $f_s = 500\ kHz$. Fig.10 shows modulator output before decimation filter and Fig.11 shows final output of ADC after decimation filters and down samplers. By calculating fast Fourier transform (FFT) from modulator output and final output of ADC, signal to noise ratio (SNR) and *effective number of bits* (ENOB) have been calculated from equation (10) to (15). In Fig.12 and Fig.13 show FFT of modulator output and final output of ADC.

$$SNR = 10 \log\left(\frac{|P_{signal}|}{|P_{noise}|}\right) = 10 \log\left(\frac{|P_{signal}|}{|P_{total} - P_{signal}|}\right) \quad (10)$$

$$ENOB = {}^{(SNR - 1.76)}/_{6.02} \quad (11)$$

$$SNR_{\text{modulator output}} = 117.12\ db \quad (12)$$

$$ENOB_{\text{modulator output}} = 19.162 \quad (13)$$

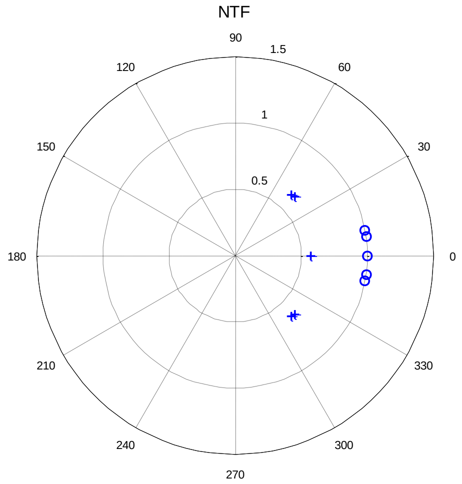

Figure 9: poles and zeros of NTF of sigma-delta modulator designed

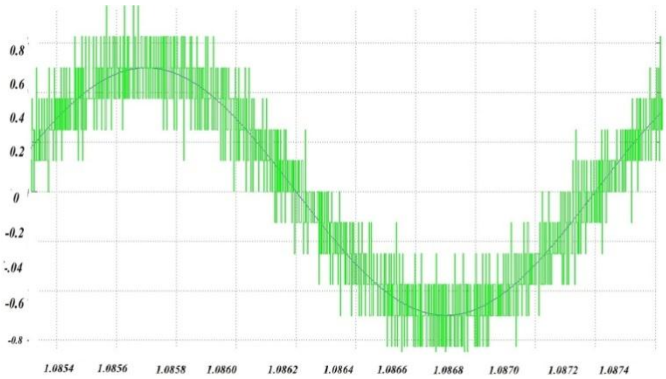

Figure 10: time response of sigma-delta modulator output

$$SNR_{decimation\ \mathrm{output}} = 117.07\ db \quad (14)$$

$$ENOB_{decimation\ \mathrm{output}} = 19.154 \quad (15)$$

5. CONCLUSION

In this paper a five order sigma-delta analog to digital convertor (ADC) is designed and optimized that density of quantization noise in ADC output be minimized then SNR and ENOB be improved. For calculating density of quantization noise in ADC output, quantization noise of sigma-delta modulator output has been considered. Then final cost function calculated, has been optimized by genetic algorithm and gain coefficients of loop filter have been calculated that density of quantization noise in ADC has been minimized then SNR and ENOB has been improved.

A GA-based search engine is developed for the quick and easy design of sigma-delta modulators. The genetic algorithm based search engine can effectively search for solutions with different characteristics and enables tradeoffs between different designs considerations. It has been successfully used to improve the performance of a fifth order sigma-delta ADC

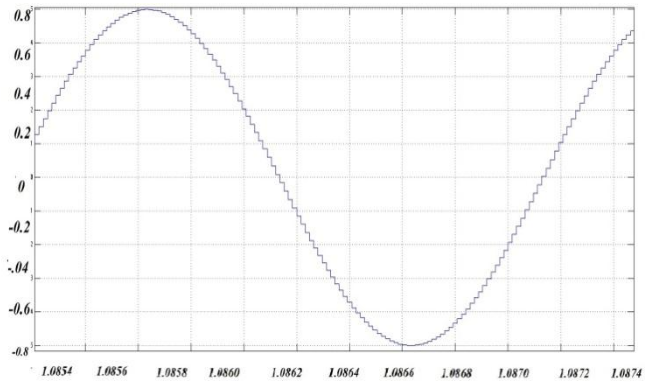

Figure 11: time response output of decimation and downs ampler block after sigma-delta modulator

which is considered decimation and digital low pass filer after the sigma delta modulator. The coefficients can be optimized using GA which results in extended dynamic range. It has also been applied to a traditional fifth order feedback topology to find peak SNR values with good stability.

In [8] loop filter gain coefficients have been optimized without considering decimation and digital filter and down sampler block effects after the sigma-delta modulator. Then final SNR and ENOB of [8] are:

$$SNR_{ref[8]} = 110.8686\, db \tag{16}$$

$$ENOB_{ref[8]} = 18.124 \tag{17}$$

Gain coefficient of [8] is presented below:

$$\begin{cases} a_1 = 2.612 \\ a_2 = 1.6731 \\ a_3 = 1.3380 \\ a_4 = 0.4590 \\ a_5 = 0.0643 \\ b_1 = 0.013 \\ b_2 = 0.035 \end{cases}$$

Then in this work 7 DB in SNR and more than 1 bit in ENOB have been improved.

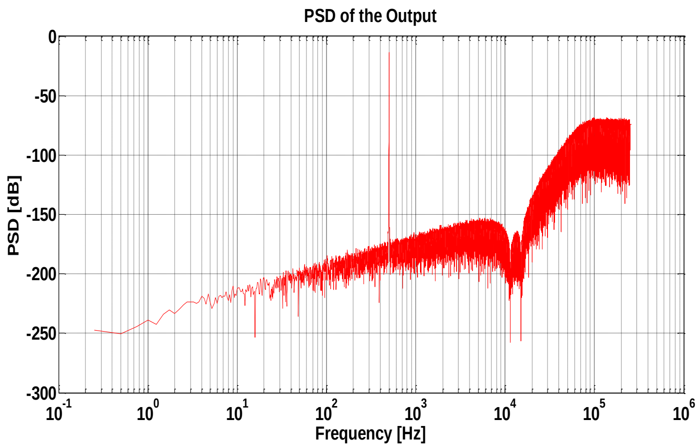

Figure 12: FFT of sigma-delta modulator output

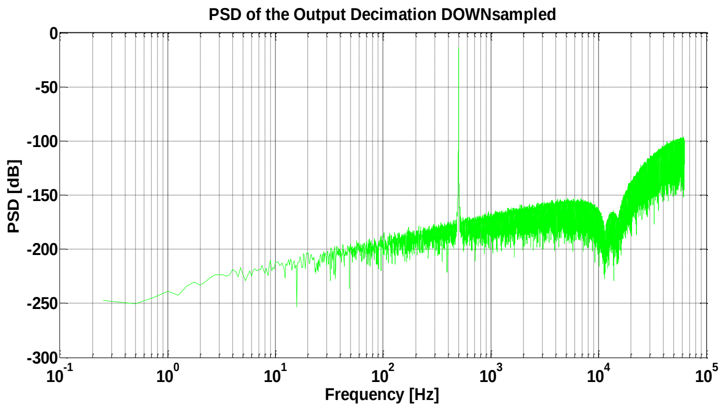

Figure 13: FFT of output of decimation and downs ampler block after sigma-delta modulator

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