

# The Effect of He4 Fusion on Primordial Deuterium

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viXra: 1404.0465 revised January, 2017

## Abstract

It is well known that approximately 23% to 25% of nucleons found throughout space are in the form of Helium 4 atoms. The distribution uniformity indicates that these atoms were formed in the very early universe. In addition, trace amounts of Deuterium, Lithium 3 and Beryllium 7 are also uniformly distributed. These elements are evidence of a process known as primordial nucleosynthesis based on historical work by G. Gamow, H. Bethe and A. Sakharov and more recently by N.D. Schramm [10].

Residual deuterium is a sensitive test for this period and the goal of the work is to determine when residual primordial deuterium originated and re-evaluate limits on cosmological parameters. Specifically, the WMAP [3] and PLANCK [13] missions concluded that baryons could not make up more than 0.046 of current density. The primary variable is the baryon/photon ratio that is a function of expansion temperature and radius. PLANCK concluded that the baryon/photon ratio was  $6e-10$  and WMAP's value was slightly lower. The author explored an expansion curve called R1+R3 based on values found in a model of the proton [5][7][Appendix 1]. The expansion curve is similar to the concordance model [4][3]. The temperature decreases from big bang values until He4 forms at  $8e8K$  but He4 fusion energy causes the temperature to spike and this affects the baryon/photon ratio. The temperature spike is accompanied by a radius increase. Both of these affect the baryon/photon ratio. The radius increase allows the baryon/photon ratio to be  $6e-10$  with a baryon fraction of 0.5 of current density. The other half of current density is dark matter. Temperature and radius histories that include He4 fusion energy appear to be missing from the literature.

## Discussion

Fusion in stars is from hydrogen. The hydrogen contributes protons that must be converted to neutrons by energetic electrons. This is quite a different situation than exists for the first few minutes. In this environment there were still a large fraction of neutrons that had not decayed and when deuterium formed, He4 was quickly formed. It is widely accepted that He4 fusion occurred at a temperature of  $8e8K$ . WMAP results [12] are important to cosmology but the temperature response to He4 fusion appears to be absent from their analysis. WMAP results support the existence of dark matter and are widely quoted for the discovery that most of the expected matter in the universe is missing. WMAP results also claim that cosmologies with more than 0.046 baryon fraction [4][6] are ruled out by the residual deuterium criteria. The view

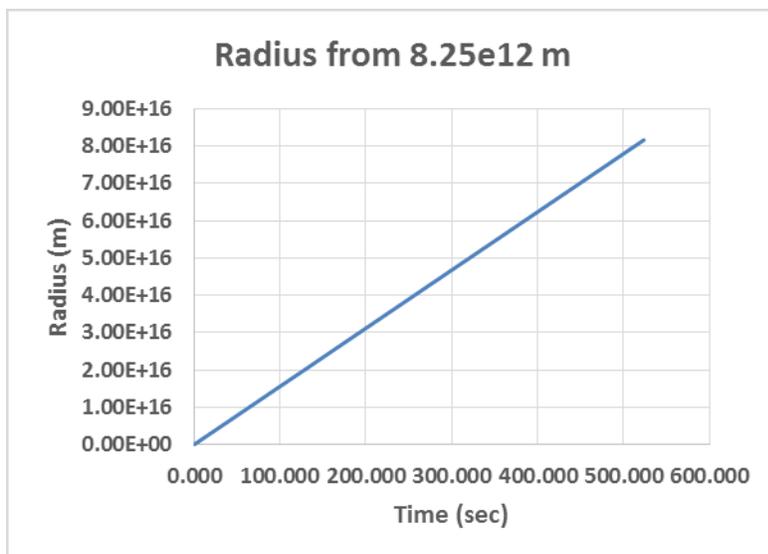
that conventional mass is only 4% of the observed universe, with the remainder “missing” is reluctantly becoming accepted. WMAP and PLANCK reports do not explicitly report the temperature and radius they use to make broad claims regarding the baryon fraction. These values are extremely critical and should have been reported. Literature is reviewed [8][9] that does not account for the fusion energy of He4. In particular the temperature spike and radius increase associated with fusion are missing from the literature.

## Radius and temperature history from beginning to He4 fusion

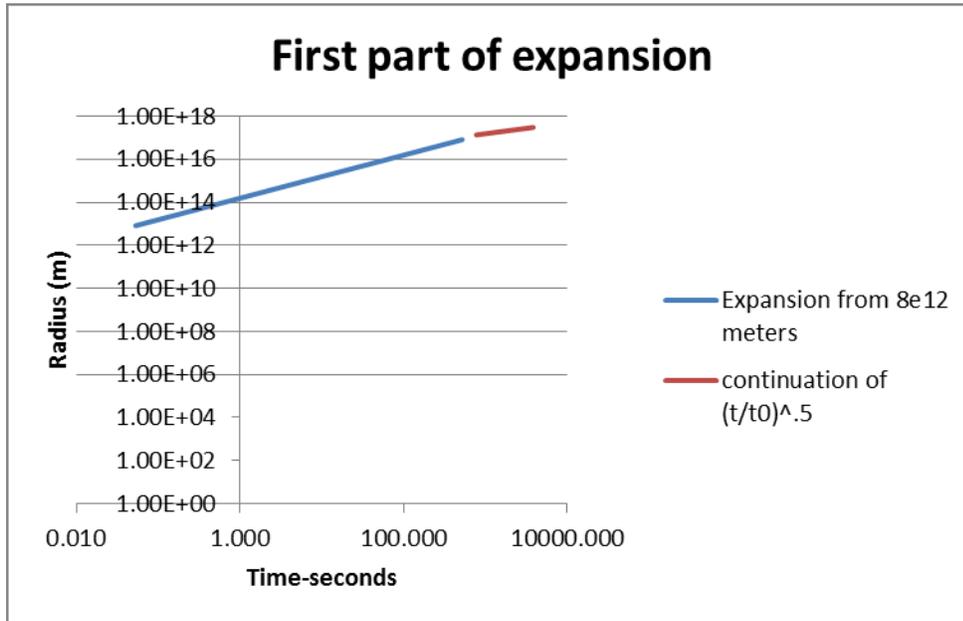
We find values in the proton mass model that give the beginning radius. It is related to values from the proton mass model, specifically  $E=2.732$  MeV in the equations below.  $R_0=7.22e-14 \cdot \exp(60)=8.25e12$  meters (the value  $\exp(60)$  is to scale one cell radius to full radius assuming  $\exp(180)$  neutrons and three dimensions).

<b>Identify the radius and time for the gravitational orbit described above</b>	
<b>Fundamental radius=<math>1.93e-13/(2.732 \cdot 2.732)^{.5}=7.224e-14</math> meters</b>	
<b>Fundamental time=<math>7.224e-14 \cdot 2 \cdot \pi \cdot \gamma(3e8)=h/E=4.13e-21/2.732</math></b>	
<b>Fundamental time</b>	<b>1.514E-21 seconds</b>

It gives the energy required to expand the radius (10.15 MeV labeled above as expansion kinetic energy (ke). What we don't know is the relationship between time and radius. However direct expansion with time; i.e.  $R=R_0 \cdot (\text{time}/\text{time}_0)^1$  works perfectly. I use a time scale that starts at the natural log value 45. But we must also know the units. The time I call cosmological time is exactly one time around the circle  $7.22e-14$  meters at velocity C. Cosmological time equals  $2 \cdot \pi \cdot 7.22e-14/3e8=1.51e-21$  seconds.  $\text{Time}_0=\exp(45) \cdot 1.51e-21=0.059$  seconds. The time scale is constructed by adding small constant increments to 45. This defines the expansion curve from the beginning  $R_0=8.25e12$  to  $R_h=2e17$  meters. Here is the relationship between radius and time:



But to be accurate it must match the curve at Rh (helium production) that was constructed backwards from the present time. The curve before and after Rh is shown below:



## Radius increase associated with He4 fusion

### Forces that determine expansion

We all use time ratios for expansion but what are the actual forces that cause particles to expand away from each other? I used cellular cosmology to calculate forces. The derivation below shows a different way to write equations that obey Newtonian gravity. The coupling constant for gravity is a published value  $1.16e-51$  Mev M (Wiki). The equation  $G=F r^2/M^2$  can also be written in terms of kinetic energy. That equation would be:

derive coupling constant $c^2$		
$G/1.603e-13=2$	ke R/Mm	
$G*1.67e-27^2/1.603e-13=2$	ke R/Nn	
Nn=1 for coupling constant		
$1.16045E-51$	mev m	
$1.16716E-51$	Mev m	Published
$nt m^2/kg^2*kg^2 mev/(nt m)$		
Mev m		
$1.16e-51*exp(90)/2$		
$7.08107E-13$	Ke r	(MeV m)

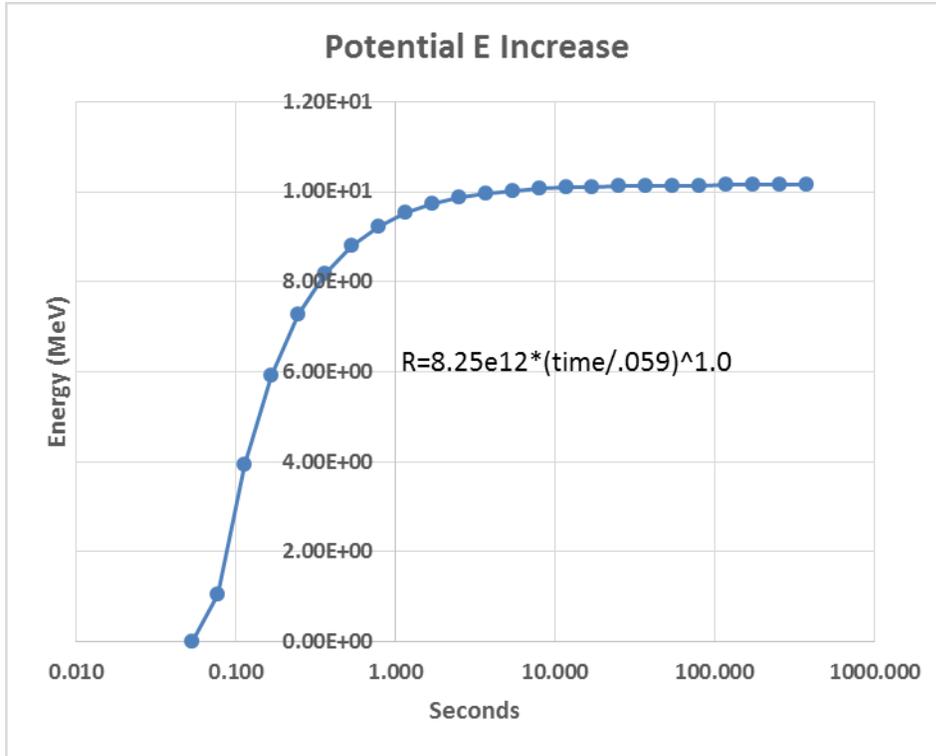
The coupling constant is scaled down to one proton orbiting a central mass of one proton at KE by applying  $exp(90)/2$ . The 2 makes it kinetic energy and  $exp(90)$  scales the calculation to one proton orbiting another proton. Kinetic energy (MeV) for a known radius r is  $7.08e-13/r$  with r in meters.

	He4 Fusion energy addition (MeV)							→ 0.51	4.52E-02
R (meters)	8.24E+12	1.21E+13	1.78E+13	2.62E+13	3.96E+16	5.82E+16	2.07E+17	2.51E+17	
r=R/exp(60) m	7.22E-14	1.06E-13	1.56E-13	2.29E-13	3.47E-10	5.09E-10	1.82E-09	2.20E-09	
coup*ph/pr	5.57E-02	7.09E-13							
ke=coup/r	7.71E+11	6.68E+00	4.54E+00	3.09E+00	2.05E-03	1.39E-03	3.70E-01	1.41E-01	
g=(939/(939+ke))	1.2166E-09	9.9293E-01	9.9518E-01	9.9671E-01	1.0000E+00	1.0000E+00	9.9961E-01	9.9985E-01	
V=(1-(g)^2)^0.5*C (m)	2.9979E+08	3.5592E+07	2.9405E+07	2.4280E+07	6.2612E+05	5.1640E+05	8.4172E+06	5.2051E+06	
F=mV^2/r (Nt)	1.7037E-36	1.6335E-38	7.5845E-39	3.5177E-39	1.5478E-45	7.1624E-46	5.3396E-44	1.6841E-44	
E=Fdr (MeV)		4.23E+00	2.89E+00	1.97E+00	1.31E-03	8.90E-04	5.32E-01	4.96E-02	
de from Rh	0.00E+00	5.92	7.26	8.18	10.15		1.76E-09	2.17E-09	

Each column of calculations is a radius increment. R is the expansion curve and T is the temperature curve reported in the section above entitled “Constructing the expansion radius”. The radius r is R/exp(60), again to scale the calculation down to the proton-proton level. Next we determine the orbital ke related to gravity (keg) by the definition of coupling constant above, i.e. Coup=keg\*r. We know r and can determine keg. But we know that ke cannot fall below the energy contributed by photons because inertial forces *and* impact by photons drive expansion. The photon energy is kep=T\*1.5\*B where Boltzmann’s constant B=8.6e-11 MeV/K. With this we put (keg+kep) in the equation for gamma and then determine orbital velocity. From here we can calculate the force F=mV^2/r. Above it is 5.3e-44 Nt at the point that 0.51 MeV He4 fusion occurs (more on this energy value below). This causes the radius to increase (dR=E/F) from 5.82e16 meters to 2.07e17 meters.

The remainder of the expansion curve to the present time does not affect our discussion of He4 formation and is included in Appendix 1.

The proton mass model has a value 10.15 MeV associated with expansion Kinetic Energy. The following chart shows the kinetic energy being converted to potential energy as a function of time.



### He4 fusion energy

Important values can be calculated at  $R_h$  where primordial helium4 forms. Helium4 formation [11] occurs when the SAHA equation for deuterium indicates that its probability is one. This is known to occur at  $8e8K$  but the exact radius where this temperature occurs is critical. We can calculate the fusion energy added at that point. The value 0.25 in the figure below represents the fraction of normal matter fused to He4 and the value 4 represents the number of nucleons required to form He4 atoms. The value 7.07 MeV is He4 binding energy [3].

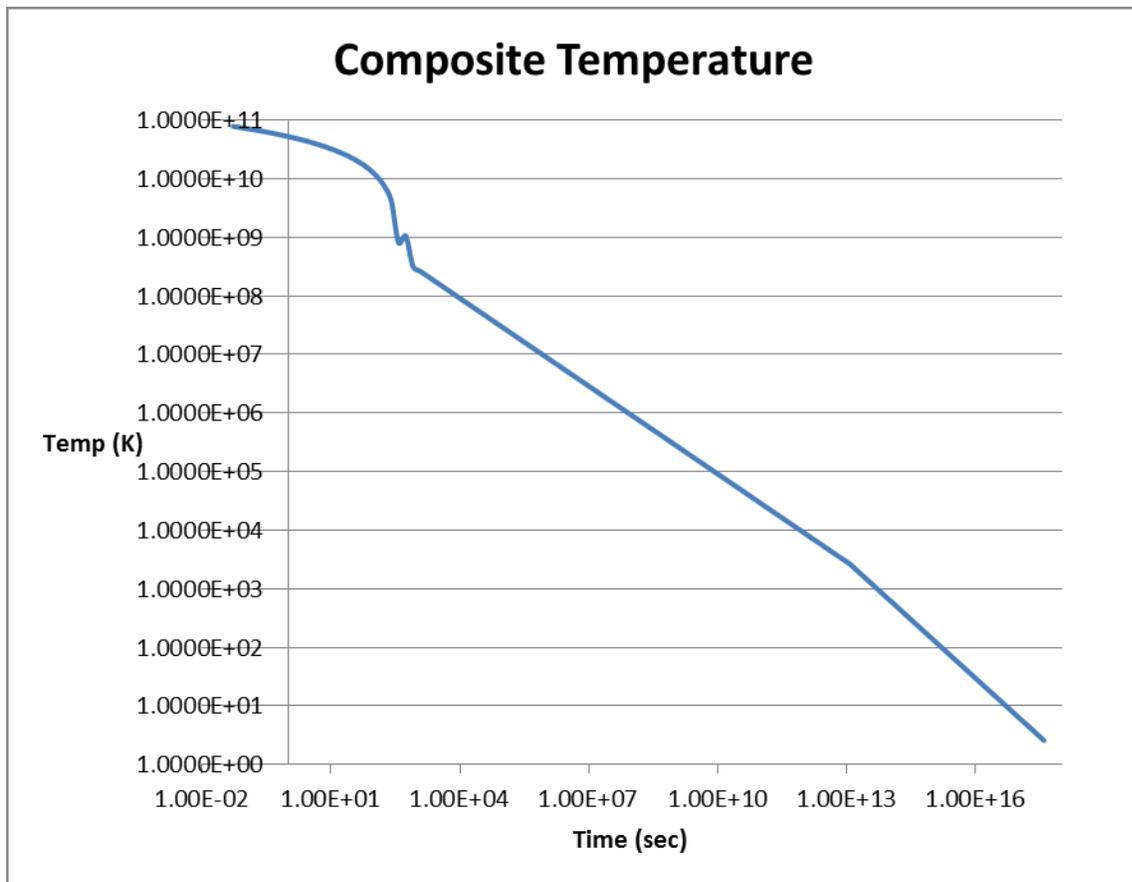
Binding Energy							
MeV	Number	dq MeV					
7.07	4.65E+76	3.29E+77	He4 binding energy*0.5*exp(180)*.25/4				
0.11	7.45E+77	8.19E+76	Energy remaining from 10.15 MeV initial energy				
		4.11E+77	sum dq MeV				
		0.552	MeV/proton				

[descanso.jpl.nasa.gov/SciTechBook/series1/Goebel\\_03\\_Chap3\\_plasphys.pdf](https://descanso.jpl.nasa.gov/SciTechBook/series1/Goebel_03_Chap3_plasphys.pdf)

There are four components to the plasma; protons, dark matter, photons and free electrons (and massless neutrinos). An equation is found in the above reference for the energy of three components. The table below is for  $1.06e9 K$ :

Radius (meters)	3.96E+16	5.82E+16	2.07E+17	2.51E+17
Temp (K)	4.1932E+09	8.1559E+08	1.0599E+09	3.4968E+08
	$v=(8kT/m\pi)^{.5}$ Protons		1.16E-01	9.59E-02
		Dark matter	1.16E-01	9.59E-02
	$KE=T*1.5 B$ Photons		1.37E-01	4.52E-02
	$v=(8kT/m\pi)^{.5}$ Electrons		1.16E-01	9.59E-02
		0.515	0.515	

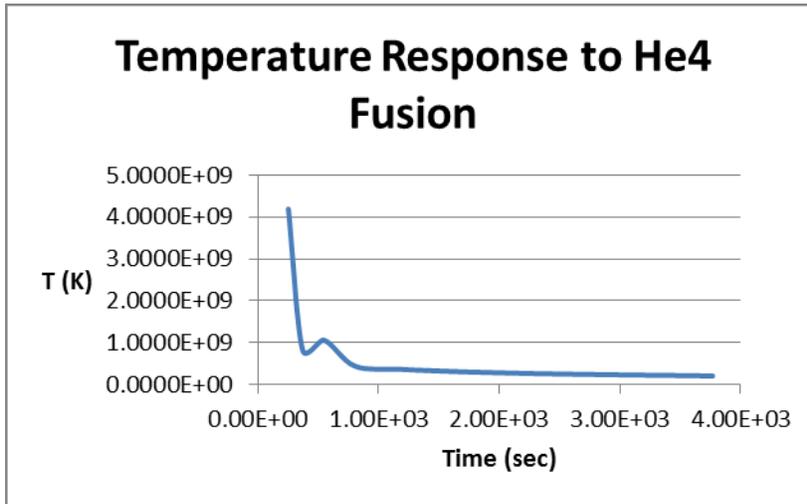
The total energy for the plasma components compares with the He4 energy release above (0.515 MeV/proton). When He4 fuses the temperature spikes to 1.06e9K. Dark matter saps 0.0954 MeV from the total at 2.07e17 meters and continues to take smaller amounts as expansion progresses. As expansion occurs the temperature falls as Rh/R and yields 2.73K at the current time. Photon KE (MeV) determines the temperature ( $T=KE/(1.5B)$ ) where B is Boltzmann's constant 8.6e-11 MeV/K. The curve is interesting.



The beginning temperature (3.92e10 K at 10.15 MeV) starts to fall and dives when the kinetic energy is nearly depleted. When the temperature hits 8e8 K the SAHA equation for deuterium initiates He4 fusion. This causes a spike in temperature to 1.06e9 K but then continues to fall according to Rh/R. The break in the curve at 3e13 seconds is decoupling where expansion follows a 2/3 power rather than the earlier 1/2 power.

## Temperature spike from He4 fusion

The temperature history decreases initially but as He4 fusion occurs, the temperature increases before finally decreasing to the Cosmic Background Temperature (CBR) due to expansion. The temperature spike caused by the release of 0.55 MeV/proton is shown below. Although protons release fusion energy, they are only 0.5 of the total mass.



## Freeze-out Mechanism

Initially the number of neutrons is controlled by a Boltzmann relationship:  $n'/N = \exp(-1.293/Ke)$  where  $Ke$  is in MeV and  $p' = N - n'$ . As the kinetic energy falls with expansion, neutrons become less prevalent until a condition known as freeze-out occurs. At this condition, temperature reduction due to expansion is high compared to the reaction rates and the reaction favoring protons stops progressing. The relative forward and reverse reactions determine the balance of neutrons and protons as early expansion occurs. An excerpt from Pebbles [4] Table 6.2 pg. 185 is included below. Reference 12 contains an excellent review of these reactions. They are a function of the density of the reactants. If dark matter is present, the radius will need to be larger to give the same number of reactions. This is missing from both Pebbles and reference 12.

T(10 <sup>10</sup> K)	lam t	lamb t	n'/(n'+p')
22.5	4700	5000	0.483
13.1	900	1010	0.471
7.6	170	208	0.451
4.45	31	43	0.418
2.59	5.4	9.7	0.363
1.51	0.85	2.3	0.292

When  $n'/(n'+p')$  takes on the approximate value 0.2, freeze-out has occurred because the forward reaction (lam t) cannot keep up with the reverse reaction rate (lamb t). This keeps more protons from forming from this mechanism fixing the ratio.

## Decay mechanism

There is another mechanism that forms protons. The neutrons decay to protons with the relationship:

$$n'/N = (\text{EXP}(-0.693 \cdot t / 866))$$

where  $t$  is time and 866 sec is the decay half time.

## Photo-disintegration of Deuterium

Deuterium fraction is limited by photo-disintegration [4][6]. It is well known that deuterium readily fuses to He4 after the temperature falls to approximately  $8e8$  K.

The SAHA equation [4] is utilized to give the early deuterium fraction.

SAHA value = $\text{LN}(4/3 \cdot ((1 \cdot 0.8) / ((1.5E+67) / (0.5 \cdot \text{EXP}(180))))^{(3/2)}) + \text{LN}((0.5 \cdot 0.697^2) \cdot (T/1e10)^{(3/2)}) - (2.58 / (8.16e8 / 10000000000))$	
part1 = $\text{LN}(4/3 \cdot ((1 \cdot 0.8) / ((1.57e67) / (0.5 \cdot \text{EXP}(180))))^{(3/2)})$	
part2 = $\text{LN}((0.5 \cdot 0.697^2) \cdot (T/1e10)^{(3/2)})$	
part3 = $(2.58 / (T/1e10))$	
SAHA value = part1 + part2 - part3	
SAHA fraction = $1 / \exp(\text{SAHA value}) = D N / (n' p')$	
Example at $1.16e8$ K	
SAHA value	1.04E-01
SAHA fraction	9.02E-01

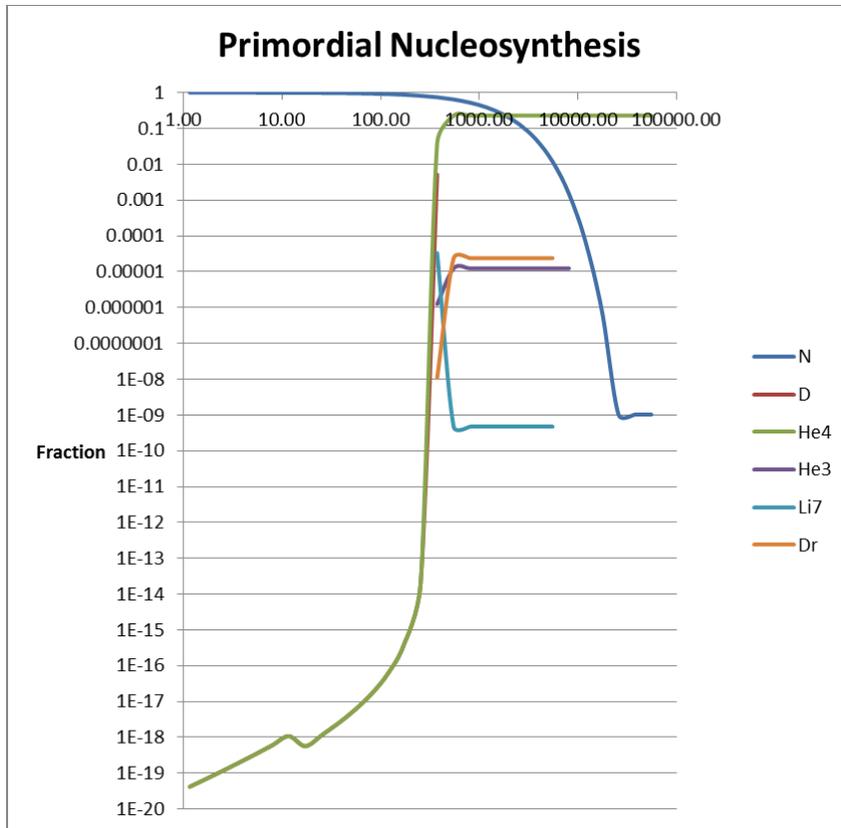
Where: Ln stands for natural logarithm, D=deuterium, N=total number of nucleons,  $p'$ =protons,  $n'$ =neutrons, Omega baryons=0.5 and  $T/1e10$  is the temperature in degrees K divided by  $1e10$ . The value  $\text{exp}(180)$  is the number of particles in the universe [7] and the value 0.697 is  $h$ , the best value of the Hubble constant [3].

SAHA fraction =  $D N / (p' n') = 1 / \exp(\text{SAHA value})$ . For example at equilibrium SAHA value=0 and fraction =  $1 / \exp(0) = 1$ .

## Deuterium and He4 formation

He4 formation can be calculated by applying the freeze-out mechanism to determine  $n'/N$ , then applying decay to  $n'/N$ . As temperature decreases D and He4 fraction is predicted by  $1 / \exp(\text{SAHA})$  but when  $T = 8e8$  K the SAHA fraction  $(DN) / (p' n') = 1.0$ . At this condition, literature states that the deuterium D rapidly converts to He4. The SAHA equation is unity at 373 seconds and  $5.82e16$  meters radius. The reaction equilibrium occurs at SAHA fraction=1,  $D N / (p' n') = 1$ . Calculation of He4 fraction: At this point  $N = 0.5 \cdot \text{exp}(180) = 7.47e77$  and the number of neutrons  $n' = 2.35e77$ . The He4 fraction =  $(2 \cdot n' / N) / (1 + n' / N) = 0.24$ .

## Details of primordial nucleosynthesis



Initially the universe is mainly neutrons. They readily form deuterium (D, the red curve) but its abundance is limited by photodisintegration since it has a low binding energy. The horizontal axis above is seconds from the beginning. The SAHA equation predicts the increasing fractions of He4 and deuterium and also predicts the temperature at which probability becomes 1 for the reactions that produce He4 (from free neutrons and deuterium). This occurs at  $8e8K$  and  $1e17$  meters. After that point 25% of normal mass exists in the form of He4 atoms (the green curve). The blue curve shows that neutrons continue to decay. The yellow, purple and light blue curves are the residuals related to baryon/photon ratio. After the SAHA value becomes positive (maximum = 1.0), the D fraction becomes fixed. Appendix 2 contains two literature analyses of this period. Although they contain more dynamics of the individual reactions, they do not include the temperature spike that is critical to the correct baryon/photon ratio.

## Baryon/photon ratio and deuterium residual

We are now in a position to calculate the important baryon/photon ratio and from the ratio calculate deuterium residual, He3 residual and Li7 residuals [20]. The baryon/photon ratio equation is below; all one has to do is put in the radius and temperature at that radius (R&T). This is exactly the point where temperature spikes. The point where He4 forms is  $1.06e9K$  and  $2.07e17$  meters.

$$\text{Baryon/photon} = \frac{0.5 \cdot \text{EXP}(180) / (4/3 \cdot \text{PI}() \cdot R^3)}{(8 \cdot \text{PI}()) / (4.31e-21 \cdot 3e8)^3 \cdot (1.5 \cdot 8.62e-11 \cdot T)^3}$$

	1.0971E-09	7.7371E-10	7.3656E-10	1.3646E-09	5.8370E-08	5.8755E-10	1.1803E-08	1.1803E-08
Radius (meters)			1.0212E+00	1.0000E+00	5.82E+16	2.07E+17	2.51E+17	3.05E+17
Temperature (K)				1.35E+17	8.16E+08	1.06E+09	3.22E+08	2.65E+08
baryon/photon ratio				1.0400E+09	5.84E-08	5.88E-10	1.18E-08	1.18E-08
Time (seconds)				Measured	373	549	807	1186
$D=4.6e-4*(B/P*1e10)^{-1.67}*1/exp(SAHA)$				2.37E-05	1.10E-08	2.39E-05	2.39E-05	2.39E-05
$He3=3e-5*(B/P*1e10)^{-0.5}$				3.3e-5 to 1e-4	1.24E-06	1.24E-05	1.24E-05	1.24E-05
$Li7=5.2e-10*(B/P*1e10)^{-2.43}+6.3e-12*(B/P*1e10)^{2.43}$				6.00E-09	3.32E-05	4.73E-10	4.73E-10	4.73E-10
<a href="http://cds.cern.ch/record/262880/files/9405010.pdf">http://cds.cern.ch/record/262880/files/9405010.pdf</a>				SAHA	1 (equilibrium)	3.01E+00	-5.47E+01	-7.20E+01

Measured values and calculated values for He3 and Lithium7 depend on the baryon/photon ratio. Reference 10 equations are included in the table. The deuterium residual agrees with measured values at baryon/photon ratio 6e-10. After the temperature spike, the temperature falls but the residual values are already frozen at a value close to the measured value 2.3e-5.

With  $0.5 * \exp(180)$  protons, the baryon/photon ratio is 6e-10. Several isotopes are part of the primordial spectrum but once established they can't change at the lower temperatures. The literature equations for predicting deuterium, helium3 and lithium7 are on the left [20]. At the calculated baryon/photon ratio the deuterium residual is in the range and the other two measurements are close to the measured values. The Planck mission's baryon/photon ratio result was 6e-10. A review of the reaction rates in reference 12 indicates that the density at the temperature for He4 formation is critical. Rh above is  $2e17$  meters. The reason this Rh has to be larger than the equivalent Planck mission radius  $9.06e16$  meters is that a larger radius is required to enclose the required He4, D, He3 and Li7 reactions because dark matter is part of the density and it does not react. This is shown by a ratio of the densities (3/2 because there are three species). Both sets of data below give 6e-10 baryon/photon but the set labelled Planck are for a lower radius and a compensating decrease to 0.046 baryons fraction of critical density. The line labelled "with dark matter" below is for 0.5 baryon fraction of critical density.

T (K)	8.16E+08	1.06E+09	3.22E+08	2.65E+08	2.19E+08	1.80E+08	1.49E+08	1.23E+08
		$Baryon/photon=(0.5*EXP(180))/(4/3*PI*(R^3))/(8*PI)/(4.31e-21*3e8)^3*(1.5*8.62e-11*T)^3)$						
		baryon/photon number density		radius (m)				
		$.5*exp(180)/vol$						
	1.685	6.1985E-10	2.9797E+25	9.0686E+16		n+p	Planck	
		5.8755E-10	1.9941E+25	2.0736E+17	1.49E+00	n+p+d	with dark matter	

### Result of deuterium abundance possible limitation

The baryon/photon ratio and deuterium abundance should not cause baryons to be severely limited like WMAP [4] and other documents suggest (0.046 fraction of critical density). The number of baryons associated with 0.5 fraction of critical density is okay with respect to this possible limitation

## Conclusions

Primordial fusion of He4 releases a significant amount of energy and must be included when determining temperature curves associated with expansion. After formation of He4, the temperature spikes and the radius increases. At this point the baryon/photon ratio is  $6e-10$  and residual values agree with measurements. The photon/baryon ratio reported for WMAP is as reported  $4.4e-10$  at the end of expansion but it was higher at the temperature spike.

There are two reasons that the baryon fraction was historically reduced to 0.046 fraction of final density. Firstly, the reactions reported in the literature did not include dark matter which is also 0.5 of final density. The radius at He4 formation has to be increased by 1.5 to include 3 species, not two in the reaction equations. Secondly, literature does not include the temperature spike to  $1.06e9K$ . This is exactly the point that the residual fractions of D, He3 and Li7 form. With these two corrections the baryon fraction is 0.5 of current density and dark matter is also 0.5 of current density.

## References:

1. Barbee, Gene H., A top down approach to fundamental interactions, FQXi essay June 2012, revised Feb 2014, viXra:1307.0082. Reference Microsoft ® spreadsheet entitled Unifying concepts.xls, Barbee.
2. Barbee, Gene H., Application of information in the proton mass model to cosmology, viXra:1307.0090 revised January, 2015. Reference Microsoft ® spreadsheet entitled simple1c.xls, Barbee.
3. Bennett, C.L. et al. First Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Preliminary Maps and Basic Data, Astrophysical Journal, 2001.
4. Peebles, P.J.E., Principles of Physical Cosmology, Princeton University Press, 1993.
5. Barbee, Gene H., On the Source of the Gravitational Constant, Prespacetime Journal, Volume 5 No 3, April, 2014. Originally viXra:1307.0085.
6. Bergstrom, L. and Goobar, A., Cosmology and Particle Astrophysics, 2nd Edition, Springer-Praxis Books in Astrophysics and Astronomy, 2004.
7. Barbee, Gene H., *On Expansion Energy, Dark Energy and Missing Mass*, Prespacetime Journal pending publication, Volume 5 No 5, May, 2014. This paper was revised January, 2017. viXra:1307.0089v7.
8. [http://burro.astr.cwru.edu/Academics/Astr222/Cosmo/Early/nucleosynth\\_fig.jpg](http://burro.astr.cwru.edu/Academics/Astr222/Cosmo/Early/nucleosynth_fig.jpg).
9. <http://www.phys.utk.edu/witek/NP621/Greene.pdf>
10. <http://cds.cern.ch/record/262880/files/940501.pdf> Review of work by N. D. Schramm.
11. Barbee, Gene H., *Dark Energy*, viXra:1511.0185v3, January, 2017.
12. Patrick Peter, Jean Philippe Uzan, *Primordial Cosmology*, OUP Oxford, Feb 14, 2013.
13. Planck Collaboration, P.A.R. Ade, et.al., <https://arxiv.org/pdf/1502.01589v3.pdf>

## Appendix 1 Expansion history

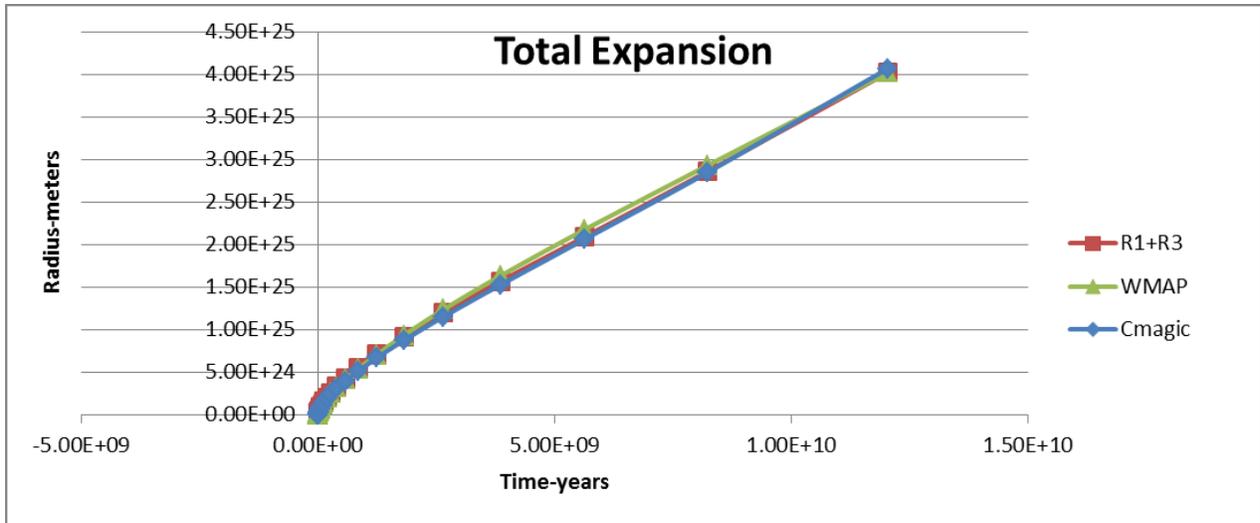
## Constructing the expansion radius from He4 to the present time

There is uncertainty in current literature regarding the initial radius of the universe. Some say it was a point and an exponential expansion known as inflation quickly increased the radius. WMAP [4] indicates that they use an expansion radius consisting of two parts. We will construct the expansion curve starting at the current radius and work backwards in time to minimize uncertainty. To know the current radius, we must know the number of neutrons in nature. Based on probabilities for the neutron components a calculation for the number of neutrons can be performed. [Appendix 1 topic entitled "The number of neutrons in nature"]. At the current time the universe density is  $9.14 \times 10^{-27} \text{ kg/m}^3$ . This is also considered critical density. The volume that would contain  $\exp(180) \times 1.67 \times 10^{-27} \text{ Kg} = 2.48 \times 10^{51} \text{ Kg}$  is  $2.48 \times 10^{51} / 9.14 \times 10^{-27} = 2.72 \times 10^{77} \text{ m}^3$ . Assuming a sphere, the current radius is  $4.02 \times 10^{25}$  meters. This includes both expansion components. The first expansion component is scalable with time ratios if we take out the controversial second component. We will use radius  $3.14 \times 10^{25}$  meters as the current radius (this does not include the lambda component but we will add back photon energy) to bring the total radius to  $4.02 \times 10^{25}$  meters.

The plasma dissipates at decoupling and from decoupling radius ( $R_d$ ) to the present time the expansion is determined by  $R = R_d \cdot (\text{time}/\text{time}_d)^{2/3}$ . Time ( $d$ ) at decoupling was  $3.19 \times 10^{13}$  seconds and time now is  $3.79 \times 10^{17}$  seconds.  $R_d = 3.14 \times 10^{25} / (3.94 \times 10^{17} / 3.19 \times 10^{13})^{2/3} = 3.06 \times 10^{22}$  meters. (Note: don't worry too much about the times quoted. The radius is related to a time ratio and there will be more on this later).

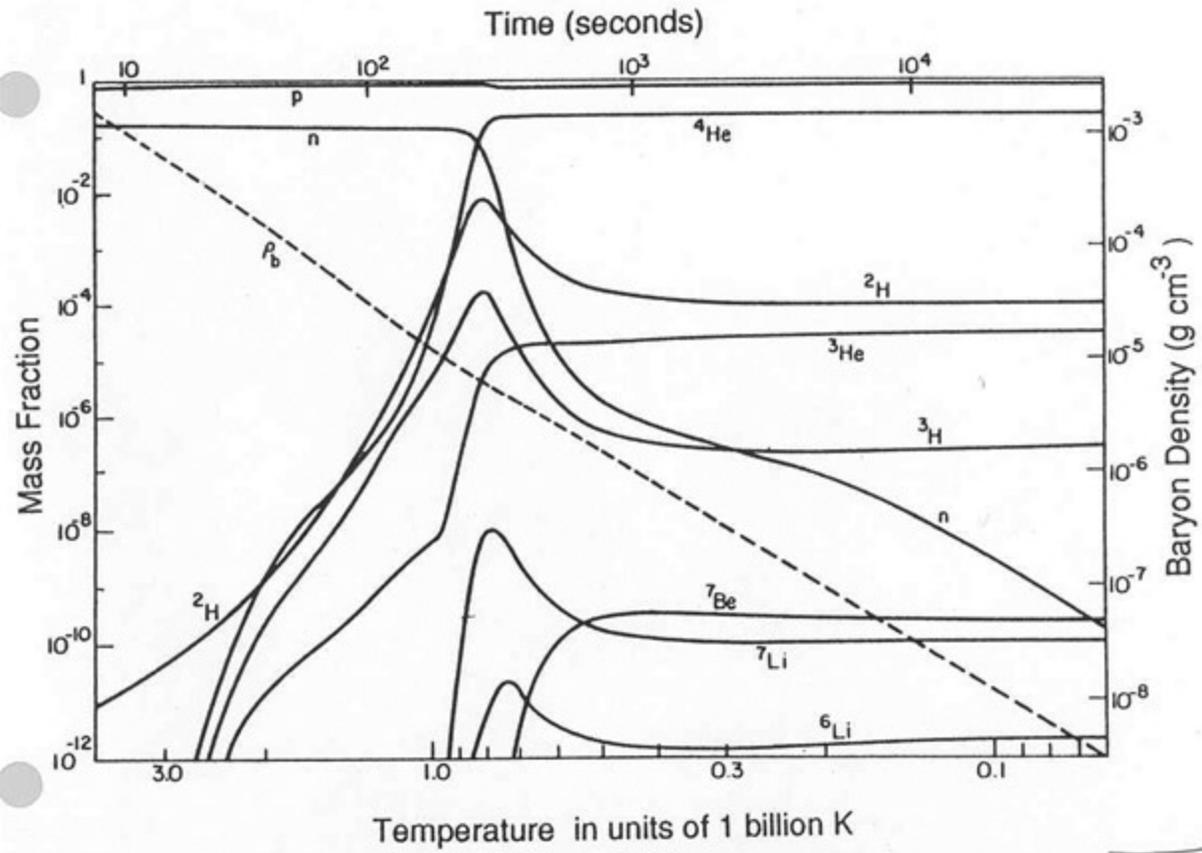
Again working backward we construct the earlier part of the curve from  $R_d$  back to  $R_h$ , the point where primordial He formed. This is predicted by the SAHA value 1 for deuterium which occurs at  $8 \times 10^8 \text{ K}$ . There is agreement that after inflation, plasma exists and expansion is radiation dominated [21]. The physics of radiation driven expansion is a function of time to the 0.5 power [10]. That is,  $R_h = 3.06 \times 10^{22} / (3.19 \times 10^{13} / 549)^{0.5} = 2.07 \times 10^{17}$  meters. He4 forms right after the temperature falls to  $8 \times 10^8 \text{ K}$  and increases the temperature to  $1 \times 10^9 \text{ K}$  [Appendix topic "Details of primordial nucleosynthesis"]. Calculation of baryon density depends on radius, especially the radius when residual deuterium formed.

Interpretation of WMAP gives the following expansion history:



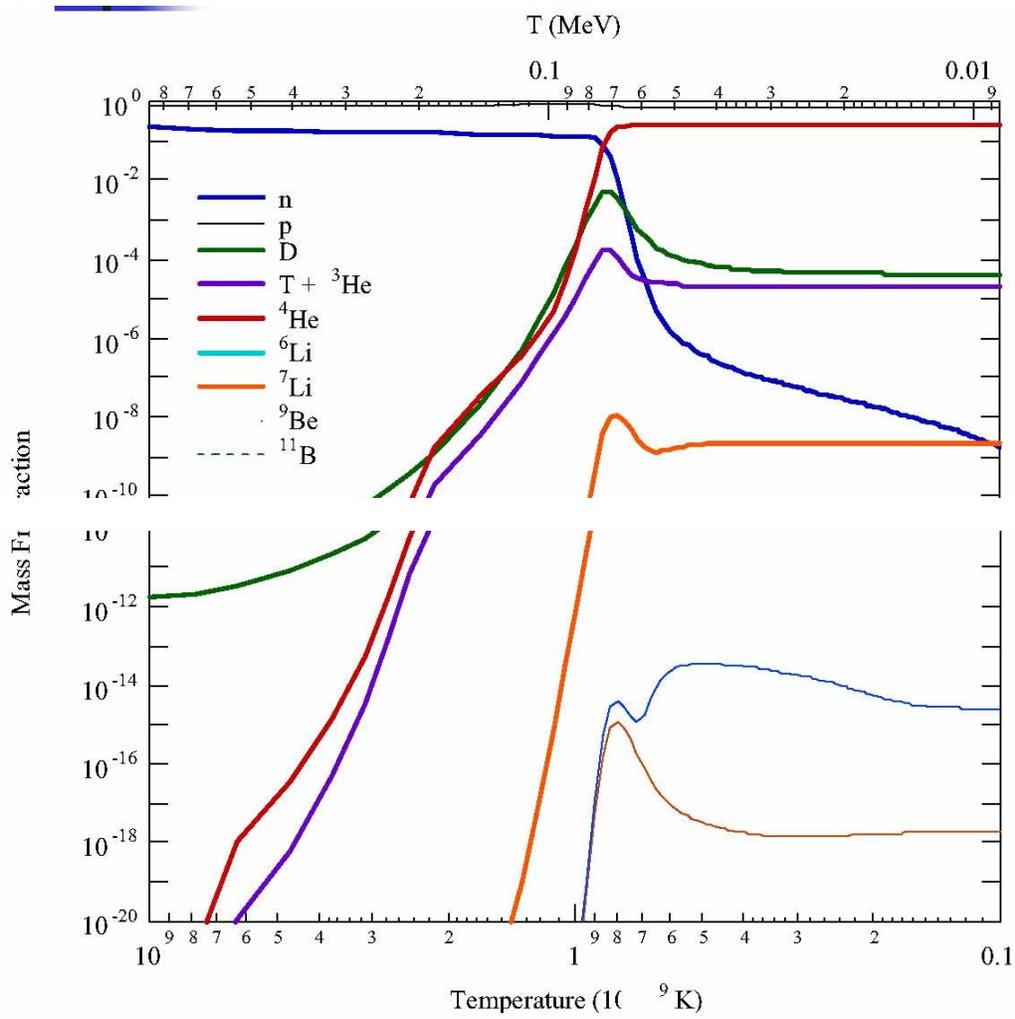
## Appendix 2 Literature primordial nucleosynthesis

The temperature in the graph below is about  $3e9$  K at 12 seconds. The kinetic energy associated with this temperature is  $1.5 \cdot B \cdot T = 0.39$  MeV, where  $B$  is Boltzmann's constant  $8.62e-11$  MeV/K. One can see from the smoothly decreasing temperature in the horizontal axis that as Helium 4 fuses, there is no increase in temperature. This amount of energy should increase the temperature to about  $1e9$  K as shown in the graph showing the temperature spike.



[http://burro.astr.cwru.edu/Academics/Astr222/Cosmo/Early/nucleosynth\\_fig.jpg](http://burro.astr.cwru.edu/Academics/Astr222/Cosmo/Early/nucleosynth_fig.jpg)

A similar graph from a different source [9] is shown below. Again, the temperature does not increase with He4 fusion.



<http://www.phys.utk.edu/witek/NP621/Greene.pdf>