Model-independent solution to dark energy problem

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Abstract. The proposed model-independent solution to the dark energy problem requires only redefining the redshift parameter z with no special assumptions about the mechanism of cosmological redshifting. The new definition of the redshift parameter finds its justification in cosmological models based on both special and general relativity. In this paper a Hubble diagram is produced from a sample of the SNe1a observational data, after recalculating measured redshifts according to the new definition with no change in corresponding magnitudes (apparent luminosities). A linear fit of the observational data obtained with the newly-defined redshift z* can be considered as an evidence for the non-accelerating Universe with no need for any dark energy.

Keywords: Dark Energy, Redshift, Supernovae

1. Introduction

As is known, the assumption of expansion of the Universe has been made on the basis of the observational fact that the spectral lines of light emitted by a distant galaxy are shifted to longer wavelengths. Quantitative characteristic of the observed increase in wavelength is the redshift parameter z. This dimensionless parameter z is defined to be the fractional change in the wavelength of the emitted and detected light with respect to the one at emission. The equivalent definition of z in terms of the photon frequency is not used in this article

We consider two alternative approaches to explaining the nature of the cosmological redshift:

1. Standard cosmological model [1]. In this model, based on the general relativity (GR), the growth of the wavelength of light from a cosmological source is a consequence of the expansion of space itself in the Universe.

2. Kinematic cosmology by Milne [2]. The origin of the observed cosmological redshift is the Doppler effect caused by the actual recession of galaxies in static space. The Kinematic cosmology is based on the special relativity (SR) that makes it alternative to the Standard cosmological model.

We distinguish these two approaches from other models because each of them naturally makes its theoretical justification for the important property of the observed parameter z, namely, that for an individual cosmological source it has the same value over the entire spectrum of emitted electromagnetic waves. It is important to note that the redshift parameter z describing the expansion rate of the Universe does not depend on the intended mechanism of redshifting. It can be considered as a measurable value.

According to the empirical Hubble law [3] opened in 1929, the redshift is correlated with the distance to a galaxy considered as a cosmological light source. Graphical expression of the Hubble law is the diagram named after him.

The linearity of the Hubble diagram means that the Universe is expanding uniformly, with a constant rate, so that the wavelength of light from a individual cosmological source increases proportionally with its distance from Earth.

At the end of the last century redshift measurements were to be significantly advanced toward higher z. It became technically possible to observe supernovae of Type 1a (SNe1a) in distant galaxies. In 1998 it was discovered that the Hubble diagram on which the supernova magnitudes M are plotted as a function of their redshift z (in logarithmic scale) deviates appreciably from the simple linear law for rather distant galaxies. The behavior of this deviation is that at large distances (for large z) galaxies look dimmer than expected at a constant rate of expansion (assuming no luminosity evolution in the look-back time). This discovery was the reason to put forward the hypothesis that the Universe was expanding slower in the past than is now. The cause of the acceleration of the Universe rate expansion has been referred to as dark energy. The origin of dark energy is unknown. And this is the problem.

There are many attempts to explain the SNe1a Hubble diagram and thus to solve the problem of dark energy [4]. Most of them involve an ad hoc hypothesis to explain the non-linearity of the Hubble diagram by the existence of a special mechanism of extra redshifting for electromagnetic radiation and(or) additional decrease in its luminosity (flux) at large cosmological distances.

The Standard cosmological model, for example, is forced to return the cosmological constant Λ to the Einstein's equations, thus leading to anti-gravity effect between cosmological objects against the backdrop of gravitational attraction been weakening over time. However, this model gives rise to two serious problems known as the fine tuning and the cosmic coincidence [5].

As for the Kinematic cosmology, then, an analytical expression fitting well the observational SNe1a data has been obtained recently in the framework of this theory even without invoking the concept of dark energy [6]. Unfortunately some additional assumptions made in this SR-model are unacceptable for models based on general relativity.

The purpose of this paper is to show that there is a solution to the problem of dark energy which does not depend on the intended mechanism of cosmological redshifting.

In our view, the desired solution of such a cosmological problem can be model-independent if it is constructed by using only measurable values without any free

theoretical parameters and it admits physical interpretations on the ground of alternative theories as are general and special theories of relativity.

In the search for such a solution we have found that the conventional definition of redshift z used in practice both in theoretical and experimental studies contains an inaccuracy.

Assuming that it is this blunder which has led to the discovery of cosmic acceleration and to the hypothesis of dark energy we give a different definition of redshift parameter.

This newly-defined parameter, which we denote as z^* and propose to call "squared redshift" or "red-square", finds its justification in GR-models (e.g. the Standard cosmological model) as well as in alternative SR-models (e.g. Milne's Kinematic cosmology).

Section 2 shows what is wrong with use of the conventional redshift parameter z within the Standard cosmological model. We give the other definition of the redshift parameter (z^*) suitable for models based on general relativity. An argumentation for using the newly-defined parameter z^* in the alternative Kinematic cosmology by Milne is also provided.

In Section 3 the Hubble diagram is constructed for some sample of observational data on SNe1a. Two relations $M(z)$ and $M(z^*)$ are superimposed for comparison.

2. Redefining the redshift parameter

2.1. About redshift parameter in theory and in practice

The redshift parameter z is a quantitative characteristic of wavelength growth of light that occurs in the expanding universe between two events: its emission by a cosmological source and its registration by the observer on Earth.

Assuming that the detected photon with the measured wavelength λ_{obs} had a wavelength λ_{em} at the time of emission the parameter z is commonly determined as follows:

$$z = \frac{\lambda_{obs} - \lambda_{em}}{\lambda_{em}}, \tag{1}$$

or

$$z = 1 - \frac{\lambda_{obs}}{\lambda_{em}}. \tag{2}$$

According to the definition (1) it is necessary for calculating the redshift z to measure the increase in the photon wavelength arisen during its movement from the moment of emission to that of registration and to divide the result by the initial (emitted) value of wavelength.

Strictly speaking, only λ_{obs} is the directly measured quantity. The wavelength λ_{em} at the moment of emission t_{00} (cosmic time from the moment of the Big Bang) is unknown. It can not be measured. Only assumptions can be made with respect to its value. That is why in the formula (1) an unknown wavelength λ_{em} which characterizes some observable spectral line as to be at the time of emission of the registered light (i.e.

in the distant past epoch) is replaced by the laboratory value λ_0 measured on Earth at the present time t_0.

Thus, the parameter z used in practice in both spectral measurements and theoretical models is defined as follows:

$$z = \frac{\lambda_{obs} - \lambda_0}{\lambda_0}. \tag{3}$$

In particular, redshifts z which are included in the tables of observational data on Type 1a supernovae are counted according to this formula (3).

We note that both the observed wavelength λ_{obs} and the laboratory wavelength λ_0 are measured in the reference frame associated with the Earth and the present time t_0. As for the initial wavelength λ_{em} it corresponds to the time in the past t_{00} (the emission time) and to another place (a supernova, in our case).

It can be said λ_{em} is identically equal to some standard (laboratory) wavelength λ_{00} if it would be possible to conduct laboratory measurements in that distant epoch t_{00} near the supernova which light we register now (t_0).

Parameter z calculated by the formula (3) is a combination of the experimental quantities, so it can be attributed to measurable parameters. In the context of this article, it is important to note that for the same reason the values of z given as the observational data do not depend on any cosmological model.

Redshift z calculated by the formula (3) is a combination of two experimental (measured) quantities, so its value does not depend on any cosmological model. However, justifications for the replacement $\lambda_0 \to \lambda_{em}$ leading to the conventional definition (3) are different in two alternative models.

As already noted, in GR-models (in particular, in the Standard cosmological model) the light wavelength growth is caused by the expansion of space and occurs gradually at the motion of photons from the source to the receiver.

Of importance is to emphasize that in expanding space any standard measure of length (a ruler) varies over time synchronously with the lengths to be measured (wavelengths, in our case). So that the numerical values of these lengths expressed in scale units of a certain ruler remain unchanged.

Hence it follows that despite the expansion of space, the direct measurement of the wavelength of a spectral line produced near the supernova at the time of emission t_{00} would give a numerical value $N_{00}(\lambda_{00})$ to be exactly equal to the numerical value $N_0(\lambda_0)$ of the standard reference wavelength of this line measured in the laboratory on Earth at the present time t_{00}.

So, we really have:

$$N_{00}(\lambda_{00}) = N_0(\lambda_0). \tag{4}$$

The Standard cosmological model argues that equality (4) allows to make the substitution $\lambda_0 \to \lambda_{em}$ in the original determination of the redshift parameter (1). In our opinion, some misunderstanding lies here.

2.2. Misunderstanding with conventional redshift z (in GR-models)

Let us express the redshift parameter z as a result of measurements of the observed (λ_{obs}) and the standard (λ_0) wavelengths (as is used in practice):

$$z = \frac{N_0(\lambda_{obs}) - N_0(\lambda_0)}{N_0(\lambda_0)}. \tag{5}$$

This is the formula (3) in which both magnitudes of the wavelengths to be compared $(\lambda_{obs}$ and $\lambda_0)$ are expressed in terms of a length unit $[l]_0$ of the same ruler N_0 to be applied for measurements on Earth at the present time t_0.

As mentioned above the redshift parameter must actually evaluate the growth of the observed wavelength (λ_{obs}) compared with the initial (emitted) wavelength (λ_{em}) rather than with the standard one (λ_0).

We believe that at t_{00} the wavelength of the emitted photon was a reference ("past-standard") wavelength λ_{00}, i.e. $\lambda_{em} \equiv \lambda_{00}$. Really, we can consider λ_{00} to be a standard laboratory wavelength (similar to λ_0) in the event that its measurement was provided at the moment t_{00} near the supernova with a ruler N_{00}. This is the meaning of the equality (4).

Using (4) we can obtain from (5) another expression for the parameter z:

$$z = \frac{N_0(\lambda_{obs}) - N_{00}(\lambda_{00})}{N_{00}(\lambda_{00})}. \tag{6}$$

In this formula, the registered wavelength λ_{obs} is compared with the emitted wavelength λ_{em} $(\equiv \lambda_{00})$ as required by the physically correct definition (1). However, the result of this comparison (numerical value of z) is at least metrologically incorrect since the quantities to be compared are measured by non-identical rulers. The fact is that, as we found out earlier, in GR-models the ruler N_{00} on a supernova should be different from the Earth ruler N_0 because of the expansion of space. The unit of length $[l]_{00}$ of the ruler N_{00} appears to be shorter than the corresponding length unit $[l]_0$ of the ruler N_0, i.e. $[l]_{00}$ ¡ $[l]_0$. This means that the numerator of expression (6) contains the difference between two values which are obtained by measuring with non-identical rulers and, for this reason, are expressed in the length units being equal in name only but different physically.

So, from the metrological point of view the number obtained with expression (6) is physically meaningless. Consequently, replacement $\lambda_0 \to \lambda_{em}$, leading to expression (5) which gives the same result (6) can not be considered as allowable.

If this mistake is ignored, the formula (3) used in practice will lead to underestimating the redshift parameter z and, hence, the expansion rate of the Universe. So that the luminosity distance to a galaxy will be greater than that determined by its redshift. A reasonable conclusion will be that supernovae in distant galaxies look dimmer. And such illusion can be a basis for the dark energy hypothesis which has arisen in 1998 [7]. In short, we believe that the dark energy problem has been raised by the improperly defined redshift parameter z.

2.3. Eliminating misunderstandings with redshift

To assess properly the value of redshift we should measure the initial wavelength λ_{em} ($\equiv \lambda_{00}$) by the same ruler N_0 which is used for measuring the observed wavelength (λ_{obs}) and laboratory wavelength (λ_0).

In our notation it looks like this:

$$z^* = \frac{N_0(\lambda_{obs}) - N_0(\lambda_{00})}{N_0(\lambda_{00})}. \tag{7}$$

Or

$$z^* + 1 = \frac{N_0(\lambda_{obs})}{N_0(\lambda_{00})}. \tag{8}$$

We have designated this new redshift parameter by star because it differs from conventional z by its numerical value and its physical meaning. Unfortunately, we are unable to carry out such measurements to be in the distant past and in the vicinity of the supernova.

But this is not required. As can be seen from (7), (8) it is sufficient to express the initial wavelength λ_{em} ($\equiv \lambda_{00}$) in the current length units $[l]_0$ which as we assume are greater than the relevant units $[l]_{00}$ at the emission time t_{00}, namely:

$$[l]_0 = [l]_{00}(z + 1). \tag{9}$$

The relation (4) is equivalent to the following expression:

$$\frac{\lambda_{00}}{[l]_{00}} = \frac{\lambda_0}{[l]_0}. \tag{10}$$

Whence taking into account (9) and using only the current units of length $[l]_0$ we get

$$\frac{\lambda_{00}(z + 1)}{[l]_0} = \frac{\lambda_0}{[l]_0}. \tag{11}$$

Finally, in current units of length the desired value of the supernova standard wavelength λ_{00} of an emitted photon is expressed in terms of its Earth laboratory wavelength λ_0 as follows:

$$\lambda_{00} = \lambda_0/(z + 1). \tag{12}$$

Using (4), (8) and (12), we find a link between the old (z) and new (z^*) redshift parameters:

$$z^* + 1 = (z + 1)^2. \tag{13}$$

For practical purposes it is desirable to express the newly defined parameter z^* through the directly measured values. Such values are the observed (λ_{obs}) and standard laboratory (λ_0) wavelengths i.e. the same terms that are included in the conventional definition (3).

From the expression (8) accounting for (12) and (3) we obtain

$$z^* + 1 = \lambda_{obs}^2/\lambda_0^2, \tag{14}$$

or

$$z^* = \frac{\lambda_{obs}^2 - \lambda_0^2}{\lambda_0^2}. \tag{15}$$

So, the dimensionless parameter z^* can be determined, for example, as follows: the relation of the difference of squares of the observed wavelength and the corresponding laboratory wavelength to the square of the laboratory wavelength. This newly-defined parameter z^* can be called as squared redshift or, simply, red-square parameter.

2.4. Misunderstanding with redshift parameter z in SR-models

For SR-models (in particular, for Milnes Kinematic cosmology) in which space is considered to be static the assumption that λ_0 (now) is equal to λ_{em} (then) seems to be natural since in such space the wavelength of the emitted light should not change as it propagates from the source to the detector. Unlike GR-models, in SR-models the ruler does not change its length but when measuring a standard wavelength (λ_0 or λ_{00}) the relative velocity of the source and the detector must be kept in mind. By definition, a wavelength to be standard is obtained by measuring in the fixed reference frame, moreover both the source and detector should be at rest with respect to each other.

As in the case of a GR-model two standard wavelengths corresponding to the observed wavelength λ_{obs} should be distinguished:

1. Earth-standard wavelength λ_0 is measured in the laboratory with both the source and detector being at rest on Earth (reference frame NV_0).

2. Supernova-standard wavelength λ_{00} refers to the case where both the source and detector of the light to be measured are assumed at rest relative to the supernova (reference frame NV_{00}).

Physically, these two quantities being measured each in its own frame of reference are equal. In our notation, this equation looks like this:

$$NV_{00}(\lambda_{em}) \equiv NV_{00}(\lambda_{00}) = NV_0(\lambda_0). \tag{16}$$

However, it does not follow that $\lambda_{00} = \lambda_0$ since these values are measured in different reference frames moving relative to each other with some non-zero velocity. When measuring the supernova-standard wavelength λ_{00} it must be borne in mind that the detector as well as the source should be fixed relative to the supernova. Since the source on the supernova recedes from Earth at a velocity V, and the detector on Earth must be stationary relative to the source, then in the reference frame associated with the Earth the detector should also move at the same velocity V in the direction of SNe. With the detector moving towards the light emitted by the supernova we find that thus measured wavelength λ_{00} is less than the Earth-standard wavelength λ_0 and is:

$$\lambda_{00} = \lambda_0/(z+1) \tag{17}$$

which is equivalent to (12) obtained in the case of GR-models.

Substituting λ_{00} from (17) into (1) instead of λ_{em} we again arrive at (15). Whence, as in the case of GR-models, we obtain the relationship (13) between the old and new redshift parameters which can be represented as:

$$z^* = 2z + z^2. \tag{18}$$

3. Fitting the SNe observational data

3.1. Hubble diagram in terms of squared redshift z^*

Hubble's law essentially means equality of the luminosity distance obtained from the inverse square law and the distance defined by the corresponding redshift. In the absence of luminosity evolution and/or variations of the expansion rate of the Universe, Hubble diagram should be a straight line. The Standard cosmological model recognizes deviation from the straight line in the diagram magnitude vs. redshift obtained with SNe1a data as an indication of the accelerated expansion [7]. We believe that the cause of the discovered effect of the accelerating Universe is the metrologically incorrect definition (3) of redshift parameter z which is used in practice.

Having made adjustments to the definition of redshift (see Section 2.3) we can expect that in its new terms (15) we will obtain a linear Hubble diagram $M(z^*)$. According to the scale of luminosity distances adopted in observational cosmology such linear relationship can be analytically represented as follows:

$$M(z^*) = M_0 + 5\log_{10}(z^*), \tag{19}$$

where M_0 is a constant.

In order to obtain the observational values of z^* we can take available redshifts z from the observational data and recalculate them by using the established connection (18) between these two parameters. Due to the same connection (18) the Hubble diagram in terms of old redshift z ceases to be linear and should look like this:

$$M(z) = M_0 + 5\log_{10}(2z + z^2). \tag{20}$$

Precisely this analytical relationship was obtained in the paper [6]. The derivation of this formula is based on the concepts of Milne's Kinematic cosmology and, notably, does not require the concept of dark energy. We will not comment on those additional assumptions that the author (F.Farley) had to for reducing the apparent luminosity (magnitude) and obtain the expression (20) which, as shown in the article [6], fits well the SNe1a observational data. It is important for us that this work lets have the numerical value of the constant M_0 at which this analytical formula (20) gives the best approximation of the observational data. This value is:

$$M_0 = 41.8. \tag{21}$$

We believe that the constant M_0 calculated for (20) should have the same value (21) for our new representation of Hubble diagram (19) because only redshifts are changed in this relation with remaining magnitudes unaltered. Thus, due to the results of [6], we

have the opportunity to build the Hubble diagram magnitude M vs. squared redshift z^* even without conducting statistical analysis of observational data to determine the constant M_0 which, incidentally, is the only free parameter in expressions (19) and (20).

3.2. Drawing the graph M(z*)

To build the Hubble diagram in new values z^* a sample of SNe1a observational data is produced from the papers [8], [9] (these data were used in [6]). A small portion of available data (113 points) was taken only to demonstrate that simply transforming the observed redshifts z into the newly-defined values z^* we can get the data points to be lying on a straight line.
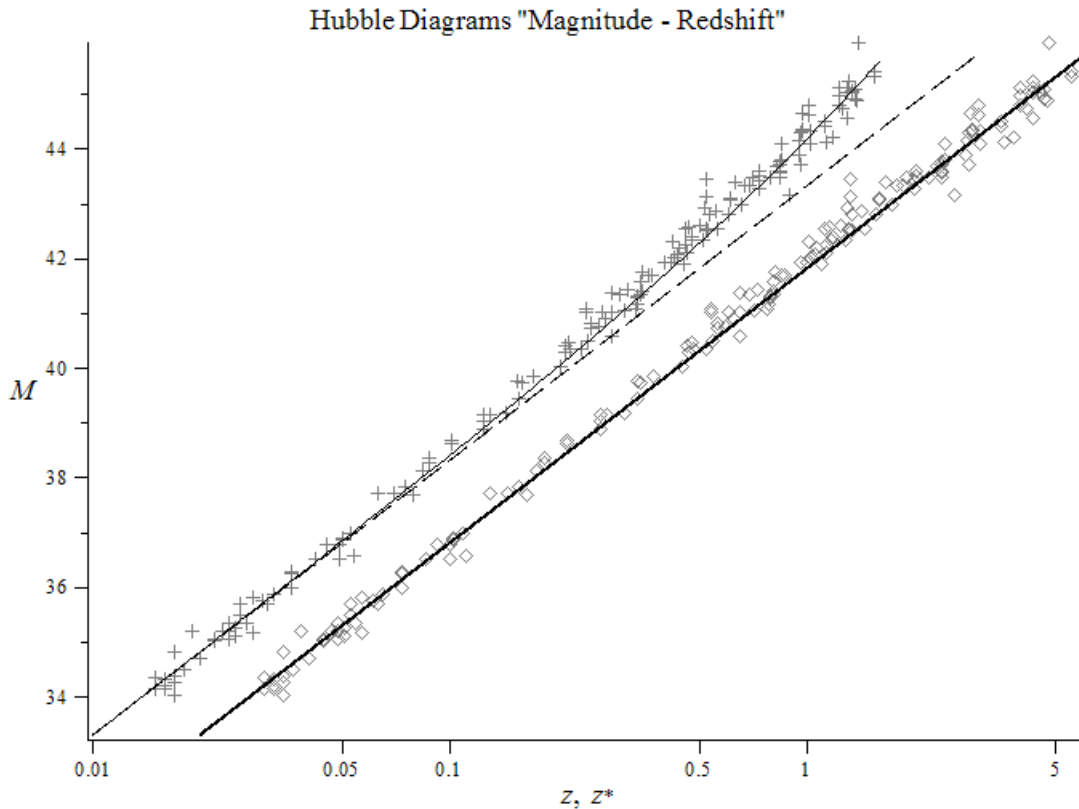

Fig.1. Two theoretical Hubble diagrams "magnitude – redshift" compared with the observational data on SNe1a: 1) M(z) (upper solid line, crosses) and 2) M(z*) (lower solid line, diamonds). The redshifts z and z* are in logarithmic coordinates.

Fig. 1 shows two SNe1a Hubble diagrams "magnitude - redshift": $M(z)$ (crosses) and $M(z^*)$ (diamonds). Both are constructed on the basis of the same sample (113 points) from the SNe1a observational data.

Upper solid line $M(z)$ is given by formula (20). It approximates the observational data in which the original values of redshift (z) are stored (as in the paper [6]). Dashed straight line is optional and serves to demonstrate a deviation of observational data (with conventional redshifts z) from the linear Hubble law. It should be mentioned that similar deviations were recognized in 1998 and served as a basis for the hypothesis of the Universe expansion rate acceleration (assuming the existence of dark energy).

Lower solid straight line defined by formula (19) is the expression of the Hubble law under the new definition of redshift parameter (red square). The observational data containing the recalculated values of redshift $z*$ are shown as diamonds.

It should be recognized that in this paper we does not aim to assess how well the resulting formula (19) approximates observational data. This, we believe, is a separate task that requires a serious statistical analysis and use of the entire body of observational data on supernovae. In this regard, we only refer to the second paper [10] by F.Farley in which a numerical estimation of the quality of his approximation (20) is given (in a somewhat different representation: instead of the argument $2z + z^2$ the expression $z + z^2/2$ is used).

3.3. Linearity of the Hubble diagram M(z*)

We emphasize that the graph $M(z^*)$ built on the SNe observational data is a straight line. As mentioned above the Hubble diagram magnitude redshift being a straight line should be regarded as evidence of the lack of acceleration in the Universe. Such a diagram does not require any type of dark energy hypothesis for its explanation.

4. Conclusions

The problem of dark energy has been solved by a simple redefinition of the redshift parameter z without resorting to assumptions of cosmic acceleration and dark energy itself. The newly-defined parameter $z*$ has been named squared redshift or red-square.

The solution presented is a model-independent one since:

1) It involves only making changes in the experimentally obtained values of redshift parameter and does not need to attract a specific physical mechanism of redshifting (i.e. a specific theoretical model).

2) This new definition of redshift parameter is substantiated in alternative models such as the Standard cosmological model and the Milnes Kinematic cosmology and lead to the same theoretical Hubble diagram "magnitude M squared redshift $z*$".

Some inaccuracies have been recognized in the definition of the conventional redshift z. They are associated with incorrect assessment of the value of the initial (emitted) wavelength as it appears to the observer in the expanding Universe. Correction of these metrological inaccuracies by involving the new redshift parameter $z*$ (red-square) ultimately led to the solution of the dark energy problem. Nevertheless it does not mean that the earlier redshift z is not suitable for use in cosmology. The parameter z, in our opinion, can be reasonably used keeping in mind that it expresses the quantitative increase in the wavelength of light measured in the expanding Universe to be compared with the corresponding wavelength in another universe being steady-state.

The linearity of Hubble diagram in terms of red-squares $z*$ gives reason to abandon the hypothesis of the accelerating Universe with a mysterious dark energy. The solution of the dark energy problem reported in this work restores, to a certain extent, the

situation in cosmology before 1998 when the current question was: is the Universe expansion slowing down under the influence of gravity?

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