A METHOD OF RESOLVING IN INTEGER NUMBERS OF CERTAIN NONLINEAR EQUATIONS

Florentin Smarandache, Ph D
Associate Professor
Chair of Department of Math & Sciences
University of New Mexico
200 College Road
Gallup, NM 87301, USA
E-mail:smarand@unm.edu

Let's consider a polynomial with integer coefficients, of degree m

$$P(X_1,...,X_n) = \sum_{\substack{0 \le i_1+...+i_n \le m \\ 0 \le i_j \le m, j=\overline{1,n}}} a_{i_1...i_n} X_1^{i_1}...X_n^{i_n}$$

which can be decomposed in linear factors (which can eventually be established through the undetermined coefficients method):

$$P(X_1,...,X_n) = \left(A_1^{(1)}X_1 + ... + A_n^{(1)}X_n + A_{n+1}^{(1)}\right)\cdots\left(A_1^{(m)}X_1 + ... + A_n^{(m)}X_n + A_{n+1}^{(m)}\right) + B$$

with all $A_j^{(k)}$, B in Θ, but which by bringing to the same common denominator and by eliminating it from the equation $P(X_1,...,X_n) = 0$ they can be considered integers.. Thus the equation transforms in the following system:

$$\begin{cases} A_1^{(1)}X_1 + ... + A_n^{(1)}X_n + A_{n+1}^{(1)} = D_1 \\ \text{..} \\ A_1^{(m)}X_1 + ... + A_n^{(m)}X_n + A_{n+1}^{(m)} = D_m \end{cases}$$

where $D_1,...,D_m$ are the divisors for B and $D_1 \cdots D_m = B$.

We resolve separately each linear Diophantine equation and then we intersect the equations.

Example. Resolve in integer numbers the equation:

$$-2x^3 + 5x^2y + 4xy^2 - 3y^3 - 3 = 0.$$

We'll write the equation in another format

$$(x+y)(2x-y)(-x+3y) = 3.$$

Let m, n and p be the divisors of $3, m \cdot n \cdot p = 3$. Thus

$$\begin{cases} x + y = m \\ 2x - y = n \\ -x + 3y = p \end{cases}$$

For this system to be compatible it is necessary that

$$\begin{pmatrix} 1 & 1 & m \\ 2 & -1 & n \\ -1 & 3 & p \end{pmatrix} = 0,$$

or

$$5m-4n-3p=0 \qquad (1)$$

In this case

$$x=\frac{m+n}{3} \text{ and } y=\frac{2m-n}{3} \qquad (2)$$

Because $m,n,p\in\mathbf{Z}$, from (1) it results – by resolving in integer numbers – that:

$$\begin{cases} m=3k_1-k_2 \\ n=\qquad k_2 \\ p=5k_1-3k_2 \end{cases} \qquad k_1,k_2\in\mathbf{Z}$$

which substituted in (2) will give us $x=k_1$ and $y=2k_1-k_2$. But $k_2\in D(3)=\{\pm1,\pm3\}$; thus the only solution is obtained for $k_2=1$, $k_1=0$ from where $x=0$ and $y=-1$.

Analogue it can be shown that, for example the equation:

$$-2x^3+5x^2y+4xy^2-3y^3=6$$

does not have solutions in integer numbers.

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