ON ANOTHER ERDÖS' OPEN PROBLEM

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Paul Erdös has proposed the following problem:

(1) "Is it true that $\lim\limits_{n\to\infty} \max\limits_{m<n}(m+d(m))-n=\infty$?, where $d(m)$ represents the number of all positive divisors of m."

We clearly have :

Lemma 1. $(\forall)n \in N \setminus \ 0,1,2\ $, $(\exists)!s \in N^*$, $(\exists)!\alpha_1,...,\alpha_s \in N$, $\alpha_s \neq 0$, such that $n = p_1^{\alpha_1} \cdots p_s^{\alpha_s} + 1$, where $p_1, p_2,...$ constitute the increasing sequence of all positive primes.

Lemma 2. Let $s \in N^*$. We define the subsequence $n_s(i) = p_1^{\alpha_1} \cdots p_s^{\alpha_s} + 1$, where $\alpha_1,...,\alpha_s$ are arbitrary elements of N, such that $\alpha_s \neq 0$ and $\alpha_1 + ... + \alpha_s \to \infty$ and we order it such that $n_s(1) < n_s(2) < ...$ (increasing sequence).

We find an infinite number of subsequences $n_s(i)$, when s traverses N^*, with the properties:

a) $\lim\limits_{i\to\infty} n_s(i) = \infty$ for all $s \in N^*$.

b) $n_{s_1}(i),\ i \in N^* \ \cap \ n_{s_2}(j),\ j \in N^* \ = \Phi$, for $s_1 \neq s_2$ (distinct subsequences).

c) $N \setminus \ 0,1,2 \ = \bigcup\limits_{s \in N^*} \ n_s(i),\ i \in N^*$

Then:

Lemma 3. If in (1) we calculate the limit for each subsequence $n_s(i)$ we obtain:

$$\lim_{n\to\infty}\left(\max_{m<p_1^{\alpha_1}\cdots p_s^{\alpha_s}}(m+d(m)) - p_1^{\alpha_1}\cdots p_s^{\alpha_s} - 1\right) \geq \lim_{n\to\infty} p_1^{\alpha_1}\cdots p_s^{\alpha_s} + (\alpha_1+1)...(\alpha_s+1) - p_1^{\alpha_1}\cdots p_s^{\alpha_s} - 1 =$$

$$= \lim_{n\to\infty}\big((\alpha_1+1)...(\alpha_s+1)-1\big) > \lim_{n\to\infty}\big(\alpha_1+...+\alpha_s\big) = \infty$$

From these lemmas it results the following:

Theorem: We have $\overline{\lim\limits_{n\to\infty}} \max\limits_{m<n}(m+d(m))-n=\infty$.

REFERENCES

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[2] P. Erdös - Letter to the Author - 1986: 01: 12.

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