The Numerical Analysis of Beta Decay S*timulation* by the High Thermal Spike of Photon Incidence to Valence Nucleons

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Abstract

Since, at RHIC and LHC heavy-ion colliders the classical color field play an important role to study production of quark-gluon plasma, we propose a theory to describe strong-field inside the nucleons based on Dual Ginzburg-Landau-Pitaevski (DGL) theory .

We provide a detailed analysis of physically important quantities as regarding the nucleons substructure as: the uniform chromoelectric field (vortex) strength inside a nucleon, the mass of monopole viewed as gluons which are the propagators of the QCD and carry colour and anti-colour, with an hedgehog-like configuration, or as a results of interaction of spin-orbit of the monopole current , or of Rashba field interaction, all giving the same result; the quantification of the interaction energies of one vortex ($W^{\pm}$) and of a giant vortex (GV), $(Z, Higgs)$ as to be *encapsulated* by the Abrikosov triangular lattice generated by the coalescence of the flux lines. Therefore, it is proved for the first time, that in the nucleon exist sufficiently high electromagnetic fields that permit to continue extract (with a rate of $\cong 1$ pair) from vacuum of pairs $e^{+}-e^{-}$ (virtual) of high energy electrons, of $W^{\pm}$, Higgs bosons, quarks, by a Schwinger effect, etc, to transform its into real one of very short time life, just like in a veritable laboratory. Thus, it was discovered for the first time that $v.e.v.$ is in fact the Schwinger critical field E_{cr} for the pair $W^{\pm}$ creation from vacuum. These pairs decay or annihilate into electrons, which passes the monopole condensate barrier as beta-electrons by *quantum tunneling* due of the *phase slip* with $2\pi-\varphi$ and of a Φ_0 energy release, the entire model is proved for a free neutron decay life-time.
Equally, the same Schwinger pairs-production rates are enhanced by a thermal Boltzmann factor in place of quantum tunneling, when this thermalization due of the incidence of an high thermal spike of a photon with nucleons destroys the superconductivity.

This effect is proved in the case of ^{26}Al , through its β-decay to 1.809 MeV γ-ray, when at high temperatures ($T_9 = 0.42GK$) equilibrium is reached between ^{26gs}Al and ^{26m}Al which is relevant to some high temperature astrophysical events such as novae.
In the applications, as based on these data, there are calculated: the Higgs boson energy release due of two gluons fusion during the pp collision at LHC, gluon pair production from space-time dependent chromofield due of the collision of pp and of heavy nuclei; the $e^+ - e^-$ pairs creation due of the thermally-induced vacuum instability as induced by a laser pulse in a crossed field of a single plane wave generated by a single high energy photon.
A proposal to use a laser pulse to reduce the half life of beta decay nuclides is discussed.

Keywords-beta decay; photonuclear reactions; high energy lasers; W,Z,H bosons; G-L theory; Schwinger effect pairs creation; ELI; gluons; monopoles condensate.
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1.Introduction

The nuclear power in order to be considered sustainable needs firstly, the elimination of radioactive waste, but not by storage in definitive repositories, as is now in the intention of the world.
Secondary, also, it needs to eliminate nuclear insecurity by making nuclear power plants that can *not* physically melt down following sever accidents, by *design* [1], and not probabilistically as is now considered.
The Photonuclear reactions in range of Giant Dipole Resonance (GDR) are proven to be of potential interest for nuclear transmutation[2]. The dominant mechanism for nuclear photoabsorption at intermediate energies (>20MeV) is is based on the *quasideuteron* model . Accelerated radioactive decay has been proposed by bombarding spent fuel with electromagnetic (photon) rays.
Due to the interaction of physics and astrophysics we are witnessing in these years a splendid synthesis of theoretical, experimental and observational results originating from three fundamental physical processes. They were originally proposed by Dirac, by Breit and Wheeler and by Sauter, Heisenberg, Euler and Schwinger. For almost seventy years they have all three been followed by a continued effort of experimental verification on Earth-based experiments. The Dirac process, $ee^- \rightarrow 2\gamma$, has been by far the most successful. It has obtained extremely accurate experimental verification and has led as well to an enormous number of new physics in possibly one of the most fruitful experimental avenues by introduction of storage rings in Frascati and followed by the largest accelerators worldwide: DESY, SLAC etc.
The Breit–Wheeler process, $2\gamma \rightarrow ee^-$, although conceptually simple, being the inverse process of the Dirac one, has been by far one of the most difficult to be verified experimentally. Only recently, through the technology based on free electron X-ray laser and its numerous applications in Earth-based experiments, some first indications of its possible verification have been reached.

The vacuum polarization process in strong electromagnetic field, pioneered by Sauter, Heisenberg, Euler and Schwinger, introduced the concept of critical electric field $E_c = m_e^2 c^3 / e\hbar$, m_e-electron mass. It has been searched without success for more than forty years by heavy-ion collisions in many of the leading particle accelerators worldwide.
The QCD-monopole has an intrinsic structure relating to a large amount of off-diagonal gluons around its center, similar to the 't Hooft-Polyakov monopole [3]. At a large scale where this structure becomes invisible, QCD-monopoles can be regarded as point-like Dirac monopoles.
In the Maximally Abelian (MA) gauge, the off-diagonal gluon contribution can be neglected and monopole condensation occurs at the infrared scale of QCD. Therefore, the QCD vacuum in the MA gauge can be regarded as the dual superconductor described by the DGL theory, and quark confinement can be understood with the dual Meissner effect. Therefore, in the first part we proceed to a review of our analytical model based on the Dual Ginzburg-Landau theory, already presented in [4], and where we insist more on the equivalence of our model with that described in the works from RCNP-Japan [5-7], and where, also, is proved the connection between QCD and the dual superconductor scenario. In the next parts, as based on these data obtained, are calculated: the Higgs boson energy release due of two gluons fusion during the pp collision at LHC, the gluon pairs and quarks pairs production from space-time dependent chromofield, in high energy collisions where jets are the signatures of quark and gluon production.
It will be demonstrated based on the results of DGLP theory, respectively: the value of the maximum chromoelectrical field of the Giant Vortex (GV) of $E_0 = 2.18 \times 10^{28}\ N/C)$, and of magnetic field of $B = 1.28 \times 10^{17}\ J/Am^2$ due of the monopole condensate current inside the nucleon or of spin-orbit interaction. These values are shown as being near of Schwinger critical electric field and of parallel magnetic field for *heavy* $e^+ - e^-$ pairs creation by Schwinger effect, all that making possible of *one pair per nucleon* to be obtained. This pair supplies the charges balance (now, not very clear) making possible the quarks conversion ($u \to d$).
Thus, a new understanding of beta decay process it will be proposed, when, also, a pair of boson $W^- - W^+$ is simultaneously created due of the Schwinger effect in the giant vortex (GV) where the electrical field is $E \cong 6.58 \times 10^{29}\ N/C$, and near equally with $E_0 \geq v.e.v. = E_{cr}^{W^\pm} = 3.5 \times 10^{28}\ N/C \leftrightarrow 247 GeV$.
Also, it is given for the first time the demonstration of the discovery, that $v.e.v.$ is in fact the Schwinger critical field E_{cr} for a pair of $W^\pm$ creation from vacuum.
This pair decays in beta-electrons during *quantum tunneling* due of the *phase slip* with $2\pi - \varphi$ and of a Φ_0 energy release, and this ad-hoc *bias current* produces a spontaneous suppression of the superconducting order parameter, all the model is proved for a free neutron decay.
Also, it is shown that, equally, the same Schwinger pair-production rate is enhanced by a thermal Boltzmann factor, when the quantum tunneling is substituted by a thermalization which destroy the superconductivity due of the incidence of an high thermal spike of a photon with valence nucleons.

For that, is given a numerical application, when is considered the case of ^{26}Al, through its β-decay to 1.809 MeV γ-ray, when at high temperatures ($T_9 = 0.42 GK$) equilibrium is reached between ^{26gs}Al and ^{26m}Al which is relevant to some high temperature astrophysical events such as novae, this being proved by our model.

2. The DGL model for nucleon substructure (review)

About 20 years ago, Y. Nambu proposed an interesting picture for the color confinement based on the analogy between the superconductor and the QCD vacuum. In the superconductor, magnetic field is excluded due to the Meissner effect, which is caused by Cooper-pair condensation. As the result, the magnetic flux is squeezed like the Abrikosov vortex in the type II superconductor [5]. On the other hand, the color-electric flux is excluded in the QCD vacuum, and therefore the squeezed color-flux tube is formed between color sources. In this analogy, the color confinement is brought by the dual Meissner effect originated from color-magnetic monopole condensation, which corresponds to Cooper-pair condensation in the superconductivity. As for the appearance of color-magnetic monopoles in QCD, 't Hooft [3] proposed an interesting idea of the abelian gauge fixing, which is defined by the diagonalization of a suitable gauge-dependent variable.

In this gauge, QCD is reduced into an abelian gauge theory with magnetic monopoles, which will be called as QCD-monopoles in order to distinguish from GUT-monopoles. The QCD-monopoles appear from the hedgehog-like configuration corresponding to the nontrivial homotopy class on the nonabelian manifold. Then, the abelian gauge fixing is expected to provide the basis of the analogy between the superconductor and the QCD vacuum. We compare the dual Higgs mechanism in the QCD vacuum with the ordinary Higgs mechanism in the superconductor.. In the superconductor, there are two kinds of degrees of freedom, the gauge field (photon) and the matter field corresponding to the electron and the metallic lattice, whose interaction provides the Higgs mechanism through Cooper-pair condensation. On the other hand, there is only the gauge field in the pure gauge QCD, and therefore it seems difficult to find the analogous point between these two systems. However, in the abelian gauge, the diagonal part and the off-diagonal part of gluons play different roles. While the diagonal gluon behaves as the gauge field, the off-diagonal gluon behaves as the charged matter and provides QCD-monopoles. Condensation of QCD-monopoles leads to mass generation of the dual gauge field through the dual Higgs mechanism [5-7], which is the dual version of the Higgs mechanism. Thus, QCD can be regarded as the dual superconductor in the abelian gauge.

From, the abelian monopoles arise from non-abelian gauge fields as a result of the abelian projection suggested by 't Hooft. The abelian projection is a partial gauge fixing under which the abelian degrees of freedom remain unfixed. For example, the abelian projection of a theory with SU(N) gauge symmetry leads to a theory with [U(1)]N−1 gauge symmetry.

Since the original SU(N) gauge symmetry group is compact, the remaining abelian gauge group is also compact. But the abelian gauge theories with compact gauge symmetry group possess abelian monopoles. Therefore SU(N) gauge theory in the abelian gauge has abelian monopoles.

In our model is adopted a basic *dual* form of Ginzburg-Landau (G-L) theory, [4], which generalizes the London theory to allow the magnitude of the condensate density to vary in space . As before, the superconducting *order parameter* is a complex function $\psi(\vec{x})$, where $|\psi(\vec{x})|^2$ is the condensate density n_s. Also is defined the wave function $\psi(\vec{x}) = \sqrt{n_s}\exp(i\varphi(\vec{x}))$, or QCD monopole field $\chi_\alpha\ (\alpha = 1,2,3)$, [5-7], and where n_s is the London (bulk) condensate density, and φ are real functions describing the spatial variation of the condensate.

The characteristic scale over which the condensate density varies is ξ, the G-L coherence length or the vortex core dimension . The x denote the radial distance of points from the z-axis, the superconductor occupying the half space $x \rhd 0$. Outside of the superconductor in the half space $x \lhd 0$, one has $B = E = H = H_0$, where, "the external" vector H_0 is parallel to the surface and correspond to E the external color-electric field inside the hadron flux tube assumed as $E = (E_3T_3 + E_8T_8)$ which is formed between valence quarks for the $q\overline{q}$ pair creation rate [5-7],. The Ψ theory of superconductivity [8] is an application of the Landau theory of phase transitions to superconductivity. In this case, some scalar complex ψ function fulfils the role of the order parameter.

First of all, we write the magnetic induction the $B = curlA = \nabla \times A$, where A is the electromagnetic field potential , or the diagonal gluon $\overline{A}_\mu = (A_3^\mu, A_8^\mu)$ and the dual gauge field $\overline{B}_\mu \equiv (B_3^\mu, B_8^\mu)$, as in [5,6,7], see also the appendix A. To obtain the full system of equations we must incorporate the Maxwell equation

$$\nabla \times B = \frac{4\pi}{c} j_{CGS} = \frac{1}{c^2\varepsilon_0} j \qquad (1)$$

and the divergence

$$\nabla \cdot B = 0 \qquad (2)$$

The extended Maxwell's equations (in cgs) which allow for the possibility of "magnetic charges" analog with electric charges (monopoles condensate), the Gauss' law for magnetism is $divB \neq 0 = 4\pi\rho_m$,and the Faraday's law of induction contains a new term $\frac{4\pi}{c}$ or, in SI, $\mu_0 j_m, where \rightarrow \mu_0 = \frac{1}{c^2\varepsilon_0}$; $-\nabla \times E = \frac{1}{c}\frac{\partial B}{\partial t} + \frac{4\pi}{c} j_m$, also, the Ampere law is identical to the one without monopoles: $\nabla \times B = \frac{1}{c}\frac{\partial E}{\partial t} + \frac{4\pi}{c} j_e$

The Ampere's law, expressed as the integral over any arbitrary loop, where J_s is the current enclosed by this loop, is:

$$\oint B \cdot dl = \mu_0 J_s \qquad (3)$$

A charged particle moving in a $B-$ field experiences a *sideways* force that is proportional to the strength of the magnetic field , the component of velocity that is perpendicular to the magnetic field and the charge of the particle. This force is known as Lorentz' force and is given by (in $(A.m)$ convention) :

$$F_L = q(E + vB) + q_m(B - v(E/c^2)) \qquad (4)$$

, where, $q_m = \frac{2\pi\varepsilon_0\hbar c^2}{q_e}$ -the magnetic charge, B in $[Teslas]$, F_L in $[N]$)

In absence of a magnetic field. one gets for free energy of the superconductor, J.Pitaevski [8]:

$$f = f_n + \int\left(\frac{\hbar^2}{4m}|\nabla\psi|^2 + a|\psi|^2 + \frac{b}{2}|\psi|^4\right)dV \qquad (5)$$

Here, f_n is the free energy at $\psi = 0$, i.e. f_n is the free energy of the normal state.

Let us consider the behavior in presence of a magnetic field. The density of the magnetic field is $B^2/8\pi$ must be added to the integrand (5). But this is insufficient in the gradient term in (5) is not invariant with respect of gauge transformations:

$$A \to A + \nabla\gamma \qquad (6)$$

And for phase transformation

$$\varphi \to \varphi + 2e\gamma/\hbar c \qquad (7)$$

The gradient of phase φ defines the velocity of the superconductive pairs (in our case of the monopoles condensate!)

$$v_s = \frac{\hbar}{2m}\nabla\varphi \qquad (8)$$

Equation (8) is not invariant under a such transformation. To restore the required invariance , one must include a further term containing the vector potential

$$v_s = \frac{\hbar}{2m}\left(\nabla\varphi - \frac{2e}{\hbar}A\right) \qquad (9)$$

Finally, one gets for the superconducting current density

$$j_s = n_s v_s e = \frac{e\hbar}{2m}n_s\left(\nabla\varphi - \frac{2e}{\hbar}A\right) \text{ or,}$$

$$A = \frac{m}{e^2 n_s}j_s - \frac{\hbar}{2e}\nabla\varphi = \frac{\lambda^2}{2c^2\varepsilon_0}j_s - \frac{\hbar}{2e}\nabla\varphi, \quad g = 1/2e = \frac{\alpha}{2}e \qquad (10)$$

Or from [16, see appendix A, eq. (A.7), it is $B_\mu = \frac{1}{2\hat{g}^2}\frac{k_\mu}{\phi^2} - \frac{1}{\hat{g}}\partial_\mu f = \frac{1}{2\lambda_L^{-2}}k_\mu - \frac{1}{\tilde{g}}\partial_\mu f$,

that resulting from the following correspondence: $B_\mu \to A$; $n_s \to \phi^2 \to v^2$, $m = \hat{g}v = \lambda_L^{-1}$; $\partial_\mu f \to \nabla\varphi$

$\hat{g} \to e$. $m_{W^\pm} = \hat{g}v = \lambda_L^{-1}$;

$\partial_\mu f \to \nabla\varphi$

$\hat{g}^2 = 4\pi\alpha$; $e^2 = \hat{g}^2\sin^2\theta_W$.

For $v = 247GeV \to \lambda_L^2 = \left(4\pi\alpha v^2/(\hbar c)^2\right)^{-1} \cong 5.5e-36m^2$, or $\lambda_L \cong 2.3e-18[m]$, which is the Compton length for $W^{\pm}$ bosons, see the section 4.2, below. Therefore, a perfect equivalence exists between both models.
The magnetic induction is

$$B = \nabla \times A \qquad (11)$$

Applying the curl operator to both sides of (10) and using (11) , we obtain the London equation

$$\nabla \times j_s = \frac{e^2 n_s}{mc}\frac{\varepsilon_0 c^2}{\varepsilon_0 c}B = \frac{\varepsilon_0 c^2}{\lambda^2}B \qquad (12)$$

Therefore, to restore the invariance in (5), one substitute for $|\nabla\psi|^2$ the combination $\left|[\nabla - i(2e/\hbar)A]\right|^2$, which is obviously gauge invariant. The final expression for the free energy then takes the form

$$f = f_n + \int\left\{\frac{\hbar^2}{2m}\left|\left(\nabla - i\frac{2e}{\hbar}A\right)\psi\right|^2 + a|\psi|^2 + \frac{b}{2}|\psi|^4 + \frac{B^2}{8\pi}\right\}dV \quad (13)$$

Here, the magnetic induction must be expressed as in (11). One can obtain the basic equations of Ginzburg-*Landau* theory by varying this functional with respect to A and ψ^*. Carrying first variation with respect to A, we find after a simple calculation:

$$\delta f = \int\left[c\frac{ie\hbar}{2m}(\psi^*\nabla\varphi - \psi\nabla\psi^*) + \frac{2e^2}{m}|\psi|^2 A + \frac{curlB}{4\pi}\right]\delta A dV + \\ + \int div(\delta A \times B)\frac{dV}{4\pi} = 0 \qquad (14)$$

The second integral can be transformed into an integral over remote surface and disappears. To minimize the free energy, the expression in the brackets must be equal to zero. This results in the Maxwell equation

$$curlB = \frac{4\pi}{c}j_s = \frac{1}{c^2\varepsilon_0}j_s(in\_SI) \qquad (14.1)$$

,or

$$\nabla \times \nabla \times A = \frac{4\pi}{c}.j_s = \frac{1}{c^2\varepsilon_0}j_s \qquad (15)$$

, provided that the current density is given by

$$j_s = \frac{ie\hbar}{2m}\left(\psi^*\nabla\psi - \psi\nabla\psi^*\right) + \frac{2e^{*2}}{m.}|\psi|^2.A \qquad (15.1)$$

In appendix A, eq. (23), the equivalent construction being:

$$\partial^{\nu} {}^{*}F_{\mu\nu} \equiv k_{\mu} = -i\hat{g}(\chi^{*}\partial_{\mu}\chi - \chi\partial_{\mu}\chi^{*}) + 2\hat{g}^{*2}B_{\mu}\chi^{*}\chi \quad (23)$$

According to the definition of n_s we can substitute $\psi = \sqrt{n_s}\exp(i\varphi)$. Then (15.1) becomes

$$j_s = \frac{\hbar e}{2m}|\Psi|^2\left(\nabla\varphi - \frac{2e}{\hbar}A\right) \quad (16)$$

Equation (16) coincides with (10). This justifies our identification of $|\psi|^2$ with n_s .Variation of (13) with respect ψ^{*} gives, after simple integration by parts,

$$\delta f = \int\left[-\frac{\hbar^2}{4m}\left(\nabla - i\frac{2e}{\hbar}A\right)^2\psi + a\psi + b|\psi|^2\psi\right]\delta\psi^{*}dV + \\ + \frac{\hbar^2}{4m}\oint\left(\nabla\varphi - i\frac{2e}{\hbar}A\psi\right)\delta\psi^{*}\cdot dS = 0 \quad (17)$$

The second integral is over the surface of the sample. The volume integral vanishes when

$$-\frac{\hbar^2}{4m}\left(\nabla - i\frac{2e}{\hbar}A\right)^2\psi + a\psi + b|\psi|^2\psi = 0 \quad (18)$$

Equations (15) and (18) form the complete system of the Ginzburg-Landau(G-L) theory.

In equation (16), to emphasize: $\lambda = \left(\frac{\varepsilon_0 .m.c^2}{n_s .e^2}\right)^{1/2} = 1.17e-16[m]$, I did a lot of multiplications , and I used the quantized flux: $\Phi_0 = \frac{\pi\hbar}{e}$, and $|\Psi|^2 = n_s$; $n_s = 3\_monopoles/V * 1.e-45m^3$, $V = 4/3\pi r^3 = 1.45e-45[m^3]$, $r = 0.7[fm]$ $\varepsilon_0 = 8.8e-12[C^2.N^{-1}.m^{-2}]$.

Since, the magnetic charge of monopole being [17] $g_d = 4\pi\varepsilon_0\frac{\hbar c}{2e} = \frac{4\pi\varepsilon_0\hbar c}{e^2}e = \frac{137}{2}e = 68.5e$, and assuming that the classical electron radius be equal to "the classical monopole radius" from which one has the monopole mass $m_M = g_d^2 m_e/e^2 = 4700m_e$, the value of λ remains unmodified. In appendix B is presented a fully calculation modality for monopole mass.
Thus, we obtain

$$j_s = \frac{\hbar e.n_s}{2m}\nabla\varphi - \frac{2e^2 n_s}{m}A \quad (19)$$

$$j_s = \frac{\hbar c}{2e}\frac{e}{c}\frac{en_s}{\varepsilon_0 mc^2}c^2\varepsilon_0\nabla\varphi - \frac{2e^2 n_s}{\varepsilon_0 mc^2}\varepsilon_0 c^2 A$$

,or

$$j_s = \frac{\pi}{2\pi}\frac{\hbar c}{e}\frac{1}{\lambda^2}c^2\varepsilon_0\nabla\varphi - \frac{2}{\lambda^2}\varepsilon_0 c^2 A$$

,or

$$j_s = \frac{1}{2\pi}\Phi_0\frac{1}{\lambda^2}c^2\varepsilon_0\nabla\varphi - \frac{2}{\lambda^2}\varepsilon_0 c^2 A \tag{20}$$

We can assume that the induction vector B is directed along the z-axis. Then the vector potential A can be chosen along the y-axis and

$$B = \frac{dA}{dx} \tag{21}$$

We must solve the G-L equations (15) and (18) for this one-dimensional problem subject to the $s - n$ boundary conditions:

$$x \to -\infty, \psi \to 0, B \to H_c, \tag{22}$$

$$x \to \infty, \psi \to (a/b)^{1/2}, B \to 0$$

The quantity $\left[\nabla - i\frac{e}{\hbar}\right]\psi|^2$ is gauge invariant, J.Pitaevski [8], when $A \to A + \nabla\varphi$.

If we transforming the equation dimensionless by:

$$\bar{x} = \frac{x}{\lambda}, \bar{A} = \frac{A}{H_c\lambda}, \bar{B} = \bar{\nabla}\times\bar{A} = \frac{B}{H_c} \tag{23}$$

Substituting these variables into G-L equations (18) and (15). The G-L equations for our one-dimensionally problem take the form (here are omitted the hats):

$$\psi'' - \kappa^2\left[\left(\frac{1}{2}A^2 - 1\right)\psi + \psi^3\right] = 0 \tag{24}$$

, and

$$A'' - A\psi^3 = 0 \tag{25}$$

The boundary conditions (22) are:

$\psi = 0, E = A' = 1$ for $x \to -\infty$

$\psi = 1, A' = 0$ for $x \to \infty$

Note that the boundary condition $A = 0$

Equations (24) and (25) give

$$\frac{2}{\kappa^2}\psi'^2 + A'^2 - (A^2 - 2)\psi^2 - \psi^4 = constnt \tag{26}$$

This expression is an "energy", and as follows from boundary conditions that this energy must equal unity.

$$\frac{2}{\kappa^2}\psi'^2 + A'^2 - (A^2 - 2)\psi^2 - \psi^4 = 1 \tag{27}$$

For $\kappa = \lambda/\xi \lhd 1$ when $\lambda \lhd \xi$ the electrical field penetrates only slightly into superconducting phase, and the penetration is of order $1/\sqrt{\kappa}$, the wave function is small in this region and gives only a small contribution. Let us consider the distance $x \lhd\lhd \frac{1}{\kappa}$ and $\kappa^2 A^2 \lhd\lhd 1$. Then one can neglect the right-hand side (r.h.s) of (24) and the solution matched to (29) bellow is $\psi = \kappa x/\sqrt{2}$. Substituting this in (25), we find $A'' = \kappa^2 x^2/\sqrt{2}$.

The main contribution arises from the region where ψ changes rapidly, which is of the order of $\frac{1}{\kappa}$.

There is not electric field in this region and one can put $A = 0$ in (27). Solving this equation for ψ', we have

$$\psi' = \frac{\kappa}{\sqrt{2}}(1 - \psi^2) \qquad (28)$$

This equation have a simple solution

$$\psi = \tanh(\kappa x/\sqrt{2}) \qquad (29)$$

The superconductors of second kind are those with $\kappa \rhd 1/\sqrt{2}$, and $\lambda \rhd \xi$.

We now consider the phase transition in superconductors of the second kind.

For this we can omit the non-linear ($|\psi|^2\psi$) term in (18), we have

$$-\frac{\hbar^2}{4m}\left(\nabla - i\frac{2e}{\hbar}A\right)^2\psi + a\psi + b|\psi|^2\psi = 0$$

$$\frac{1}{4m}\left(-i\hbar - \frac{2e}{1}A\right)^2\psi = |a|\psi \qquad (30)$$

This equation coincides with the Schrodinger equation for a particle of mass $2m$ and charge $2e$ (in the case of dual, the factor 2 for the charge, which is specific to the "pairs", it is actually 1) in a magnetic field H_0 (in our case the chromo-electrical flux $E(0)$). The quantity $|a|$ plays the role of energy ($E\psi$) of that equation. The minimum energy for a such particle in a uniform electro-magnetic field is

$$\varepsilon_{(0)} = \frac{1}{2}\hbar\omega_B = \frac{1}{2}\hbar\frac{2eH_0}{8mc}\left[J.s\frac{C * Kgm}{Cs^2\, Kg * m/s}\right] \Rightarrow [J]$$, H_0-an "external" electro-magnetic field of a dipole created by the pair $u\bar{u}$ (the chromoelectrical colors field)

$$H_0 = E_0 = \frac{de}{4\pi\varepsilon_0 r^3} = 8.33e24\left[\frac{N}{C}\right] \quad (30.1)$$

,where $r \cong 0.05[fm]$-is the electrical flux tube radius, $d = 0.48[fm]$-the distance between the two quarks charges, $m = m_e \cong 9.e-31[Kg]$, usually $H[A/m]$, but here is used as $B = \mu_0 H\left[\frac{J}{Am^2}\right]$

Hence, equation (30) has a solution only if $|a| \triangleright 2\hbar * eH_0/8mc$, when following power-law conformal map is applied for complex number of the r.h.s of (30), or equivalently if the electro-magnetic field is less than an upper critical field, see figures.1a;1b.

$$H_0 \leq \frac{4mc|a|}{\hbar e} \leq H_{c2} = \frac{\Phi_0}{2\pi\xi^2} = \frac{\pi\hbar c}{2\pi e\xi^2} = 8.33e24\left[\frac{N}{C}\right] \qquad (31)$$

, and in terms of

$$B_{c2} = H_{c2} = \frac{\pi\hbar c}{2\pi e\xi^2 \cdot c} = 2.7e16\left[\frac{J}{Am^2}\right]$$

The particle energy is

$$\varepsilon_{(0)} = \frac{1}{2}\frac{\hbar^2}{8m\xi^2} = 6.28e-08[J] = 6.28e-25[Kg] \Rightarrow 392[GeV]$$, with $\lambda \triangleright \xi$;

$\xi = \frac{\lambda}{\kappa} = \frac{0.117}{1.05} = 0.1114[fm]$, or $\kappa \triangleright 1/\sqrt{2} \triangleright 1 = 1.05$ (of type II-superconductor).

One of the characteristic lengths for the description of superconductors is called the coherence length. It is related to the Fermi velocity for the material and the energy gap ($k_B T_c$) associated with the condensation to the superconducting state. It has to do with the fact that the superconducting electron density cannot change quickly-there is a minimum length over which a given change can be made, lest it destroy the superconducting state. For example, a transition from the superconducting state to a normal state will have a transition layer of finite thickness which is related to the coherence length.
However, superfluids possess some properties that do not appear in ordinary matter. For instance, they can flow at low velocities without dissipating any energy—i.e. zero viscosity. At higher velocities, energy is dissipated by the formation of quantized vortices, which act as "holes" in the medium where superfluidity breaks down.

More exactly, this quantity is called the correlation or healing length [8] , and is defined as $\xi(T) \cong \xi_0 \left(\frac{T_c}{T_c - T}\right)^{1/2} \triangleright\triangleright \xi_0$

, where $\xi_0 = a\hbar\upsilon_F / E_g$ (31.1)

is for $T \to 0$, $a = 2/\pi$ from [20], E_g-gap energy, k_B Boltzmann constant; at confinement $T_c = 175[MeV] \to 2e12[K]$, and the Fermi velocity of electrons (monopoles) is

$$\upsilon_F = \frac{\sqrt{2*4700m_e E_F}}{4700m_e} \qquad (31.2)$$

, where as the Fermi energy we have for monopoles condensate viewed as boson condensate $E_{bosonCond} = 3.31\frac{\hbar^2 n_s^{2/3}}{4700m_e} \cong T_c k_B = 0.7E_F$

, where $E_F = \frac{\hbar^2}{2*4700m_e}\left(3\pi^2 n_s\right)^{2/3}$ (31.3)

, numerically, we have:

$E_F = 9.32e-12[J] \to 55[MeV]$

,where

$n_s = 3\_monopoles/V*1.e-45m^3$, $V = 4/3\pi r^3 = 2$, $r = 0.48[fm]$

$\varepsilon_0 = 8.8e-12[C^2.N^{-1}.m^2]$.

$V = 1.5[fm]^3$, and the velocity of monopole is

$\upsilon_F = 0.55e8 \triangleleft c = 2.997e8[m/s]$

,and $\xi_0 = 1.02e-16[m]$ at $T \to 0$ (31.4)

The best choice for coherence length [8] is to consider $\xi_0 \approx \lambda$, when $\upsilon_F < c$, but strictly $\lambda \geq \xi$, as $0.111 \leq \xi \leq 0.112$

Note that, if we use only the mass of electrons (as in the case of superconductors), the velocity obtained is greater than the speed of light, so this strengthens the use monopole condensate.

In the following we will consider the structure of the mixed state. The main problem is to understand how the electric field penetrate in the bulk of the superconductor. Let us again consider a superconducting cylinder in the electric field. It is natural to expect that the normal regions , with their accompanying electric field, are cylindrical tubes parallel to the field. The electrical flux inside such tube must be integral multiple n of the flux quantum $\Phi_0 = \pi\hbar c/e \to usually \frac{\pi\hbar}{e} = 2.07e-15[Tm^2] \to Js/C$

$1T = Wb/m^2 = \frac{Ns}{Cm}$ (32)

The electrical field is concentrated inside the tube. At large distances from the tube it is shielded by annular superconducting flowing around the tube. This current is analog of the superfluid velocity field surrounding the vortex lines in the superfluid liquid. We can then picture the mixed state as an array of quantized vortex lines. Such vortex lines were predicted by A.A. Abrikosov in 1957. Their existence is crucial for explaining the proprieties of type II superconductors (dual in our case).

The presence of a vortex line in the center of the tube increases the free energy of the superconducting media. The G-L equations are solved analytically only for $\lambda \rhd\rhd \xi$ (near T_c this means $\kappa \rhd\rhd 1$), *since, in the MA gauge, the charged gluon(M_{ch}) effects become negligible and the system can be described only by the diagonal gluon component at the long distance as* $r >> M_{ch}^{-1} \cong 0.2fm$. For the short distance as $r \leq M_{ch}^{-1} \cong 0.2fm$, the effect of charged gluons appears, and hence all the gluon components have to be considered even in the MA gauge, see appendix B. Thus, when the electrical flux is applied parallel to the superconducting cylinder, the first flux penetrating should be located along the axis of the cylinder.

Substituting j_s from Maxwell equation, we can rewrite (10) as:

$$\frac{1}{\lambda^2} c^2 \varepsilon_0 \left(\nabla \varphi \frac{2\Phi_0}{2\pi} - A \right) = j_s$$

From Maxwell equation (in SI):

$$curlB = \frac{1}{c^2 \varepsilon_0} j_s$$

$$\left(\nabla \varphi \frac{2\Phi_0}{2\pi} - A \right) = \frac{\lambda^2}{c^2 \varepsilon_0} j_s = \lambda^2 curlB$$

, or

$$A + \lambda^2 \nabla \times B = 2\Phi_0 \nabla \varphi / 2\pi \qquad (33)$$

The phase φ in presence of vortex line is not a single-valued function of the coordinates. For a vortex line with minimum flux Φ_0, the phase increase by 2π on traversing a closed contour that enclose the line. Thus the integral along such a contour is

$$\oint \nabla \varphi \cdot dl = 2\pi \qquad (34)$$

Integrating (33) we find

$$\oint (A + \lambda^2 \nabla \times B) \cdot dl = 2\Phi_0 \qquad (35)$$

It is not difficult to check that in the range

$$\lambda \gg x \gg \xi \qquad (36)$$

The second term from l.h.s of (35) gives the main contribution. We take the contour of integration in (35) a circle of radius x. For this geometry the vector $(\nabla \times B)$ has only one component $(\nabla \times B)_\varphi$ along the contour.

The integration is then simple and we have

$$(\nabla \times B)_\varphi = -\frac{dB}{dx} = \frac{2\Phi_0}{2\pi x \lambda^2} \qquad (37)$$

To note (in cgs):

$$(\nabla \times B)_\varphi = \frac{4\pi}{c} e n_s v_{s\varphi}$$

Equation (37) then gives $v_{s\varphi} = \hbar/2mx$ for the superfluid velocity as it must be for a vortex line in a superfluid of particles with mass $2m$.

Integrating of (37) for B gives

$$B(x) = \frac{2\Phi_0}{2\pi\lambda^2}\log\left(\frac{\lambda}{x}\right) \qquad (38)$$

This equation is valid in the interval (36) with logarithmic accuracy.

Notice also that every vortex carries the flux Φ_0 and hence the mean value of B over the cross-section of the cylinder is

$$\overline{B} = 2\nu\Phi_0 \qquad (39)$$

, where ν is the number of lines per unit area. This result is invalid near the upper critical flux H_{c2} where the cores of the vortex lines begin to overlap. To calculate this number we have to take into account the interaction between vortex lines. As the first step we have to find the electrical field trough a loop of arbitrary radius surrounding the line without the restriction (36) . Let us calculate the curl of the both sides of (33) .
Note that

$$curl\nabla\varphi = 2\pi \cdot n_z \cdot \delta(x) \qquad (40)$$

, and $curlA = B$
where

$\delta(x)$ -the Dirac function

Where r is the two-dimensional radius-vector in the $x-y$ plane and n_z is a unit vector along axis z (We assume that the axis of the vortex line coincides with z). Indeed, integrating $\nabla\varphi$ along the contour encircling the line and transforming the integral by Stokes' theorem into an integral over a surface spanning the contour, we have according to (34)

$$\oint \nabla\varphi \cdot dl = \int curl\nabla\varphi \cdot dS = 2\pi \qquad (41)$$

Since this equation must be satisfied for any such contour of integration , we have (40).

Finally, we obtain

$$B + \lambda^2 curlcurlB = 2n_z\Phi_0\delta(x) \qquad (42)$$

Using the vector identity $curlcurlB = \nabla divB - \Delta B = -\Delta B$, we obtain

$$B - \lambda^2 \cdot \Delta B = 2\Phi_0\delta(x) \qquad (43)$$

This equation is valid only at all distances

$$x \triangleright\triangleright \xi \qquad (44)$$

Throughout all the space except the line $x = 0$ equation (43) coincides with the London equation (12)

The $\delta(x)$ function on r.h.s defines the character of the solution at $x \to 0$. Actualy this singularity has already been defined in (38), which is valid at small x.
The solution of this equation at $x \to \infty$ is $B(r) = const \cdot K_0(x/\lambda)$, where K_0 is the Hankel function of imaginary argument. The coefficient must be defined by matching with the solution of (38). Using the asymptotic formula $K_0(x) \approx \log(2/\gamma x)$ for $x \lhd\lhd 1$, where $\gamma = e^C \cong 1.78$ (C is Euler's constant), we finally have

$$B(x) = \frac{2\Phi_0}{2\pi\lambda^2} K_0(x/\lambda) \qquad (45)$$

Exactly, the same solution is obtained by G.Bali et al [8], respectively the equation (2.6). Using equation (45) we can rewrite (38) as:

$$B(x) = \frac{2\Phi_0}{2\pi\lambda^2} \log\frac{2\lambda}{\gamma x}, \quad x \lhd\lhd \lambda \qquad (46)$$

In opposite limit of large distances one can use the asymptotic expression $K_0(x) \approx (\pi/2x)^{1/2} e^{-x}$ for $x \rhd\rhd 1$. Thus, at large distances from the axis of the vortex line the field decreases according to

$$B(x) = \frac{2\Phi_0}{(8\pi x\lambda^3)^{1/2}} e^{-x/\lambda}, \quad x \rhd\rhd \lambda \qquad (47)$$

Accordingly the superconductive current density decreases (in SI):

$$j_\varphi = -\frac{c}{4\pi}\frac{dB}{dx}\left(4\pi c\varepsilon_0\right) = \frac{2c^2\varepsilon_0\Phi_0}{8(2\pi^3 x\lambda^5)^{1/2}} e^{-x/\lambda} \qquad (48)$$

We can now calculate the energy ε of the vortex line. The magnetic part of free energy corresponding to London equation is given by the integral.

$$F_B = \frac{1}{8\pi}\int\left[B^2 + \lambda^2(curlB)^2\right]dV \qquad (48.1)$$

Indeed, by varying the expression with respect to B, we immediately obtain the London equation (12). The main contribution to the integral is due to the second term, which contains a logarithmic divergence. Substituting (37) in (48.1), and integrating in the range (36), we obtain for the energy per unit length of vortex line.

$$\varepsilon = \left(\frac{2\Phi_0}{4\pi\lambda}\right)^2 \log\left(\frac{\lambda}{\xi}\right) \qquad (49)$$

Equation (49), explains why only vortex lines with the minimum flux Φ_0 are the most favorable. The energy of a line is proportional to the square of its magnetic flux. Thus, the fragmentation of one line with the flux $n\Phi_0$ into n lines with flux Φ_0 results in an n-fold gain in energy.
A discussion of the physical background of this energy can be found, e.g. in the books [25-27] , as related to Dirichlet's energy and harmonic maps.

Thus, in [26], when is induced a magnetic stray field h which has a certain energy, according to the static Maxwell equation, the stray field satisfies $curl(h) = 0$; $div(u + h) = 0$, where u, is extended by 0 outside Ω . The first equation implies that $-h$ can be written as the gradient of function U. By the second equation, this U is a solution of $\Delta U = div(u)$ in the distribution sense (since, $curl(\nabla U) = 0$, and $div(\nabla U) = \Delta U$). There exists exactly one solution such that the integral

$$\frac{1}{2}\int_{R^3} |\nabla U|^2 dx = \frac{1}{2}\int_{\Omega} u \cdot \nabla U dx \tag{49.1}$$

is finite, and for this choice of U, this integral gives the main contribution to the micromagnetic energy. It is called the magnetostaic energy [26].

In our terms, $B = u = const$,

$$\nabla U = curlB = \frac{\partial B}{\partial x},$$

since $\Delta B = \Delta U = 0 \rightarrow div(u) = 0 \rightarrow u = const$.

Substituting $u = B|_{x \lhd \lhd \lambda} = \frac{2\Phi_0}{2\pi\lambda^2}$ from (46) with $x \approx \lambda/10 \rightarrow \log(...) \Rightarrow \cong 1$ on the boundary, or the dual gauge component of the total electrical field

, when $B^{monopoles} = 4.65e16\left(\frac{J}{Am^2}\right)$ (49.2)

,and ∇U from (37) , one have

$$\varepsilon = \frac{1}{2}\int_{\Omega} \lambda^2 \frac{1}{8\pi}\frac{2\Phi_0}{2\pi\lambda^2} 4\pi c^2 \varepsilon_0 \frac{2\Phi_0}{2\pi\lambda^2 x} dx =$$

$$c^2\varepsilon_0 \int_{\Omega} \left(\frac{2\Phi_0}{4\pi\lambda}\right)^2 \frac{1}{x} dx = c^2\varepsilon_0 \left(\frac{2\Phi_0}{4\pi\lambda}\right)^2 \log(x)\Big|_{\xi}^{\lambda} = \tag{49.3}$$

$$c^2\varepsilon_0 \left(\frac{2\Phi_0}{4\pi\lambda}\right)^2 \log\left(\frac{\lambda}{\xi}\right)$$

Here, the factor $4\pi c\varepsilon_0$ is used to convert from $(cgs) \rightarrow (SI)$.

Because the magnetic induction of the monopoles current which is powered by electric field given by a pair of quarks (H_0), $B^{monopoles} \geq 2 \cdot H_0 \cong H_{c2}$, as resulting from the comparison (49.2) with (30.1) and (31), it has the raw flow consequences squeezing this cromoelectrical flux into a vortex line, followed by forcing an organization into a triangular Abrikosov latticesee figures 1a,1b

The core of every vortex can be considered to contain a vortex line, and every particle in the vortex can be considered to be circulating around the vortex line. Vortex lines can start and end at the boundary of the fluid or form closed loops.

The presence of vortex line which increases the free energy of the superconducting media with εL, it is thermodynamically favorable if the contribution is negative; i.e. if $\varepsilon L - \Phi_0/c \cdot H_0 L/4\pi\mu \lhd 0$, and $B_0 = \frac{H_0}{\mu_0}$, $\mu_0 = 1/c^2\varepsilon_0$,or

$$H_0 \triangleright H_{c1} = \frac{4\pi\varepsilon\mu_0}{\Phi_0} \qquad (49.4)$$

Substituting (49.3) in (49.4) , we find the lower critical field

$$B_0 = H_{c1} = \frac{2\Phi_0}{2\pi\lambda^2}\log\left(\frac{\lambda}{\xi}\right) = \frac{\pi\hbar c}{\pi\lambda^2}\log(\kappa) = 1.e15\left[\frac{J}{Am^2}\right] \qquad (49.5)$$

, where $\xi = 0.1114$, and when near the axis, for $x = 0.116 \cong \xi$, when the induction is

$$B(\xi) \cong 2.e15 \cong 2H_{c1} \qquad (49.6)$$

Let us the results obtained to the calculation of the energy of interaction of vortex lines. It is important that equation (43), which defines the distribution of the field, is linear one. It means that under condition (44) the field produced by different vortex lines is additive. Let us consider two vortex lines placed at x_1 and x_2 separated by a distance d from each other. Then, $B = B_1 + B_2$. The energy of the lines is given by (48.1). Let us transform the first term in integrand by means of (42) (to multiply with B), which gives

$$\begin{aligned} &B^2 + \lambda^2(curlB)^2 = \\ &\lambda^2\left[-B\cdot curlcurlB + (curlB)^2\right] + \\ &2\Phi_0 B_z(x)\left[\delta(x - x1) + \delta(x - x_2)\right] \end{aligned} \qquad (50)$$

The first term in the r.h.s can be transformed into the form

$$\begin{aligned} &-B\cdot curlcurlB + (curlB)^2 = \\ &div(B\times curlB) \end{aligned} \qquad (51)$$

The volume integration of this term in (48.1) can be reduced to an integration over a remote surface. This integral disappears, because of the fast decrease of the field. Because we are interested here in the energy of interaction of the lines, we must takes into account only "the mixed terms" of the type $B_{2z}(x)\delta(x - x_1)$. (Terms likes $B_{1z}(x)\delta(x - x_1)$ contribute to the self-energy of the vortex lines (49). Now the integration in (48.1) is trivial. We have for the interaction energy

$$L\varepsilon_{\text{int}} = \frac{2L\Phi_0}{8\pi}(B_2(x1) + B_1(x2)) \qquad (52)$$

Both terms on the right contribute equally and using (45) we have

$$\varepsilon_{\text{int}}(d) = \frac{2\Phi_0}{4\pi}B_d(x) = \frac{4\Phi_0^2}{8\pi^2\lambda^2}K_0\left(\frac{d}{\lambda}\right) \qquad (53)$$

One can also use the asymptotic expression for ε_{int} (see (47))

$$\varepsilon_{\text{int}} = c^2\varepsilon_0\frac{4\Phi_0^2}{2^{7/2}\pi^{3/2}\lambda^2}\left(\frac{\lambda}{d}\right)^{1/2}e^{-x/\lambda}\ ,\ x \triangleright\triangleright \lambda \qquad (54)$$

When the distance $d \approx \lambda \triangleright\triangleright \xi$ the cores of vortex lines overlap [8]. The equation (42) is no longer valid. However, (39) is still valid.

Let us consider a closed contour near the surface of the cylinder. The change of wave function on passing round the contour is $2\pi \nu S$, where S is the cross-section area of the cylinder and ν -the number of vortex lines. One obtain from (16) that the electric flux is

$$\Phi = 2\Phi_0 \nu S - \frac{2m}{e\hbar}\oint \frac{j_s}{n_s} \cdot dl \qquad (55)$$

Let us recall that a similarly relationship [8], [28], it was introduced for the first time by London, called fluxoid equation.

Each fluixoid, or vortex, is associated with a single quantum of flux represented as Φ_0 ,and is surrounded by a circulating suppercurrent , j_s, of spatial extent, λ . As the applied field increases, the fluxoids begin to interact and as the consequence ensembles themselves into a lattice. A simple geometrical argument for the spacing, d of a triangular lattice then gives the flux quantization condition [29],

$$Bd^2 = \frac{2}{\sqrt{3}}\Phi_0 \qquad (56)$$

, where B , is the induction.

The solution of Ginzburg-Landau phenomenological free energy (13) is useful for understanding the Abrikosov flux lattice. The coordinate-dependent order parameter φ describes the flux vortices of periodicity of a triangular lattice. Fluctuations from φ change the state to ψ , the minimization of free energy with respect to ψ , gives the ground state $\varphi(r/0)$.

The free energy is given by,

$$f = f_n + \int\left\{\frac{\hbar^2}{2m}\left|\left(\nabla - i\frac{2e}{\hbar}A\right)\psi\right|^2 + a|\psi|^2 + \frac{b}{2}|\psi|^4 + \frac{(B - H_0)^2}{8\pi}\right\}dV \qquad (57)$$

, the average magnetic induction is $\vec{B}(-y,0,0)$. The free energy has solutions of vortices of triangular form. The coordinates of the three vertices of a triangular vortex are given by $(0,0),(l,0)$, and $\left(\frac{1}{2},\frac{\sqrt{3}}{2}\right)l$. The fluctuation from ground state corresponding to that of triangular lattice is that for small fluctuations. The deviation of the free energy from the mean-field value $F - F_{FM}$ with respect to the thermal energy, $k_B T$, can be used to obtain the physical properties of the fluctuations which are useful for understanding the melted vortex lattices. The deviation from the triangular Abrikosov lattice is defined as

$$D = \langle|\psi - a_1\varphi(r/0)|^2\rangle / a_1^2 \qquad (58)$$

which uses the spatial and thermal averages calculated with the probability $\exp(-F/k_B T)$. Classically,

$$D = \frac{k_B T}{F - F_{MF}} \qquad (59)$$

measures the fluctuations from the triangular vortex state. The fluctuation in the distance between vortexes becomes [29]:

-case **1,** $\left(1 - T_{FM}/T_c\right) \cong 10^{-5} c^{-4/3} B^{2/3}$ (60)

-case **2**, $1 - T_{FM}/T_c \cong c^{-1} B^{5/4}$; (61)

-case **3,** a vortex transition below the transition temperature see [29]

, where, T_{FM} -the flux-lattice melting temperature, and $c = 0.1$ from Lindemann criterion of lattice melting when $d^2 = c^2 l^2$, and the flux quantization condition $l^2 = \Phi_0/B$, $B = 2\pi n/\kappa$., where $\kappa = \frac{e\sqrt{2}}{\hbar c}\lambda^2 H_c(T)$, and $H_c = \kappa/\ln\kappa \cdot H_{c1}$.

For numerical values $T_c = 175[MeV]$, in case of symmetry breaking, the case 1, results $T_{FM} \approx T_c$, and in case 2, results $T_{FM} \cong 100[keV]$ or $1.15 \times 10^9\,{}^\circ K$, by using (56) in place of $\Phi_0/B \cong d^2$ with $d = 0.3982[fm]$ (a very precisely value), and $\kappa \cong 1$, which is the temperature of fusion (melting!) of two nucleons.

This triangular lattice corresponds to the arrangement of the quarks pairs $u\bar{u}, u\bar{u}, d\bar{d}$ in the frame of a nucleon, see fig.1a, fig.1b.

A direct numerical analysis allows to obtain the following values for the current, force and energy. Thus, from (48) the current for $x \cong \lambda$ is given by:

$$j_\varphi(\lambda) = 1.15e7[A/fm^2] \qquad (62)$$

For $x = 2\cdot\lambda$, the current density decreases at $j_F \cong 3.e5[A/fm^2]$ (62.1)

Note that velocity υ_F , moreover, if one considers the monopole current given by equation (10), as $j_\varphi = n_s v_F g_D$

, where the magnetic charge is:

$$g_d = 4\pi\varepsilon_0 \frac{\hbar c}{2e} = \frac{4\pi\varepsilon_0 \hbar c}{e^2} e = \frac{137}{2} e = 68.5e \qquad (63)$$

If we use the range $x \cong 0.1 \le \lambda$, then the current is obtained by derivation of (46):

$$j_\varphi = \frac{1}{4}\frac{1}{x}\frac{1}{\lambda^2}\frac{4\pi\hbar c}{e}\frac{c^2\varepsilon_0}{c} = \frac{1}{4}\frac{1}{x}\frac{1}{\lambda^2} g_D c = \frac{n_s}{V}\upsilon_{Fi} g_D \to$$

$$\frac{\upsilon_{Fi}}{c} = \frac{1}{9.89} = 0.302e8[m/s] \qquad (64)$$

The fusion temperature of deuteron

With numerical values, in case 2 : $T_c = 175[MeV] \to 2\times 10^{12} K$, at confinement, results $T_{FM} \cong 100[keV] \to 1.15\times 10^{9} K$, by using $\Phi_0 / B \cong d^2$ with $d = 0.3982[fm]$ (a very precisely value-the distance between neutron-proton into a deuteron), and $\kappa \cong 1$, that is the temperature of *fusion* (melting!) of two nucleons.

The Higgs boson release condition at CERN-LHC

With the value of $d = 0.04 fm$ as will be used bellow to Higgs boson mass calculation, results, in the case 2 as described above , $T_{FM} = 8.3\times 10^{16} K \to 7.2TeV$ as the *melting* (fusion) temperature of two vortices (two gluons-monopoles) which enclose the "Higgs" zone in each one of protons during the $p-p$ impact. That value corresponds to the necessary energy at CERN to the "release" of the Higgs energy as the $2\gamma\gamma$. Therefore, again a strong confirmation of our model (two gluons-monopoles melting with $2\gamma\gamma$ release), all that "thanking" to the CERN p-p collision test. To note that, below in section 4.2, the signification of d it could be that of Compton length! of gluons (heavy electrons pair as created by Schwinger effect.

Lorentz' force of the flux tube

From (4) and (47), the Lorentz' force is:

$$F_L = qv_{Fi}B \cong 2.25.e4[N] \qquad (65)$$

,when B is given by (46) and $x \cong \lambda$, for the upper limit:

$$B(\lambda) \cong 4.7e15[J/Am^2] \qquad (66)$$

With B from (47) and for $x \rhd\rhd \lambda$, we have

$$B(72\lambda) = 3.8e-16\left[\frac{J}{Am^2}\right] \qquad (67)$$

, then, the force becomes $F_L \cong 1.8e-26[N]$, or in terms of energy

$$\varepsilon_{barrier} = F_L * x = 1.8e-26 * 72\lambda / 1.e-15 = 945[MeV] \qquad (68)$$

,or the nucleon overall.

In case of $x \cong \xi \to (0)$, with (38)

$$B(0) = 1.03e15\left[\frac{J}{Am^2}\right] \qquad (69)$$

, which respect (49.6).

The energy of nucleon:

The magnetic energy results from (49), and (49.3), and for ($\lambda \rhd\rhd x \rhd\rhd \xi$) from (36):

……….

$$\varepsilon = c^2\varepsilon_0\left(\frac{2\Phi_0}{4\pi\lambda}\right)^2 \log\left(\frac{\lambda}{\xi}\right) =$$

$$\frac{1.e17 * 8.82e-12 * 2.06e-15^2 * 2^2}{(4\pi)^2 0.117e-15 * 0.117} ALOG(0.117/0.1114) = \qquad (70)$$

$$1.32e-10[J/fm] \Rightarrow 0.82[GeV/fm]$$

the force on the flux tube (string tension).

$$\Phi_0 = \frac{\pi\hbar}{e} = 2.06e-15$$

Now, from (54) and $d \approx (4 \div 8)\lambda \rhd\rhd \xi$, we have *The energy of pion* π^+

$$\varepsilon_{\text{int}} = c^2\varepsilon_0 \frac{4\Phi_0^2}{2^{7/2}\pi^{3/2}\lambda^2}\left(\frac{\lambda}{d}\right)^{1/2} e^{-x/\lambda} =$$

$$\frac{1.e17*8.82e-12*4*2.06e-15^2}{2^{3.5}3.14^{1.5}0.117e-15*0.117}\left(\frac{\lambda}{5.65\lambda}\right)^{1/2} EXP\left(-\frac{5.65\lambda}{\lambda}\right) = \tag{71}$$

$$2.3e-11[J/fm] \Rightarrow 144[MeV]$$

Now, that it would be the value of the mass of the pion π^+, composed of a pair of quarks $u\bar{d}$ interacting at a distance $d \approx 5.65*\lambda \cong 0.66[fm]$ of the radius of the nucleus.

The fields energies to create pairs of $e^+ - e^-$, W, Z, Higgs bosons and bias current for beta decay are the following:

Now, others important values of energy:

$$\varepsilon_0(0) = \varepsilon_{\text{int}-pair}(d = x-\lambda; x = 0.14)*0.117[fm] \cong 1.e-09[J] \tag{71.1}$$

, and from (69) with $x \cong \xi = 0.107[fm]$; (71.2)

$$\varepsilon_{0h} = Vc^2\varepsilon_0(2H_{c1})^2/8\pi = 5.e-11[J] \tag{71.3}$$

The vortex energy (W-boson) is:

$$\varepsilon_{vortex} = Vc^2\varepsilon_0 H_{c2}^2/8\pi = 1.16e-08[J] \tag{71.4}$$

, where V-is the volume, accordingly, the corresponding equivalently masses are $M = \varepsilon_{vortex}/c^2 \Rightarrow 73[GeV]$, which seems to be equal to the mass of W boson.

The interaction between vortexes at distance x and of separation $d = x-\lambda$ is given as from (54), see figure 1a; 1b.

$$\varepsilon_{\text{int}-pair} = c^2\varepsilon_0 \frac{4\Phi_0^2}{2^{7/2}\pi^{3/2}\lambda^2}\left(\frac{\lambda}{d}\right)^{1/2} e^{-x/\lambda} =$$

$$\frac{1.e17*8.82e-12*4*2.06e-15^2}{2^{3.5}3.14^{1.5}0.117e-15*0.117}\left(\frac{\lambda}{0.36*\lambda}\right)^{1/2} EXP\left(-\frac{1.36*\lambda}{\lambda}\right) = \tag{72.1}$$

$$6.65e-09[J/fm] \Rightarrow 41.6[GeV]$$

The energy of the neutral *boson* Z is assimilated with the vortex-vortex two pairs of quarks spins $(1/2+1/2=1)$ interaction energy $\varepsilon_Z = 2*\varepsilon_{\text{int}-pair} = 84[GeV]$, with above $\varepsilon_{\text{int}-pair}$. When the three pairs of vortexes with the outermost $(d \cong 0.04 fm)$ vortices lines which interacting (repel) at the center of the triangle, that will generate a neutral current in this zone called *Higgs boson* (H). Its spin and charge are both zero due of the vortices coalescence, here into a giant vortex (GV), all that happens during the triangular arrangement of the lattice, see figures.1a;1b Thus, it results another energy state-maximum possible ($d \cong 0$), probable that of *Higgs boson (H):*

$$\varepsilon_H = 3 * \varepsilon_{\text{int}-pai} = 3 \times 41.6 \rightarrow 125[GeV].$$

In other words, in order to equilibrate these energies are necessarly to admit the existence of two particles Z, H of exactly these energies values.

3. The model of the $\beta - decay$ mechanism based on transverse(bias) current (review)

Atomic nuclei are known to exhibit changes of their energy levels and electromagnetic transition rates among them when the number of protons and/or neutrons is modified, resulting in shape phase transitions from one kind of collective behavior to another. These transitions are not phase transitions of the usual thermodynamic type. They are quantum phase transitions (QPT)[30] (initially called ground state phase transitions).
However, since the proton-neutron quadrupole interaction dominates over the proton-proton and neutron-neutron ones for medium-heavy and heavy deformed nuclei, the axial deformation parameters β are related by a constant of proportionality determined by equating the corresponding intrinsic quadrupole moments. The geometrical variable β is obtained by multiplying the boson.
The nuclear $\beta - decay$ provides a severe test of the nuclear model because the decay rates are very sensitive to the wave functions of both the initial and the final nuclei.
The description of β decay of odd-mass nuclei in the interacting boson-fermion model (IBFM) was formulated in [30].
Thus, here are presented the results of an investigation of the effect of a fermion (in our case the electron $e^{\pm}$) on QPTs in bosonic systems. That is done in atomic nuclei by making use of the Interacting Boson Fermion Model ($IBFM$), a model of odd-even nuclei in terms of correlated pairs with angular momentum $J = 0,2(s,d)$ and unpaired particles with angular momentum $J = j$ (j fermions). To note, however, that the method of analysis from [30] can also be used for systems with other values of the fermion, j, and boson, J, angular momenta, for example the spin-boson systems, the simplest case of which is a fermion with $j = 1/2$ (*i.e.*, a single spin) in a bath of harmonic oscillator one-dimensional bosons of interest in dissipation and light phenomena. Here the focus is on the effect of a fermionic impurity on $QPTs$ in bosonic systems. The main results are that, (a) the presence of a single fermion greatly influences the location and nature of the phase transition, the fermion acting either as a catalyst or a retarder of the QPT, and (b) there is experimental evidence for quantum phase transitions in odd-even nuclei (bosonic systems plus a single fermion).

Also, a main conclusion from [30] is that the effect of the fermionic impurity is to *wash out* the phase transition for states with quantum numbers: $K = 1/2, 3/2, 5/2$ and to enhance it for states with $K = 7/2, 9/2, 11/2$. In other words, the fermion acts as a catalyst for some states and as a retarder for others. Also, when the coupling strength becomes very large, the minima for some large K like $K = 11/2$, shift to negative values (oblate deformation).
An important property of atomic nuclei is that they provide experimental evidence for shape $QPTs$, in particular, of the spherical to axially-deformed transition $U(95) - SU(3)$. One of three signatures have been used to experimentally verify the occurrence of shape phase transitions in nuclei, namely: the behavior of the gap between the ground state and the first excited 0^+ state [31].
The nucleus ^{152}Sm, with $N = 90$ and $Z = 62$, lies intermediate between nuclei of known spherical shape and well-deformed axially-symmetric rotor structure. A sudden change in deformation occurs at $N \approx 88 - 90$ for the Sm and neighboring isotopic chains. New data obtained [31] constrain the description of this nucleus within the IBM to parameter values near the critical point of the transition from oscillator to rotor structure. The performed IBA calculations are also given in [31], for the entire region $N \approx 90$ with the simple IBA Hamiltonian which involves only one parameter, ξ, which depends only on the neutron number N, for all the isotopic chains.
In [4] a new approach for $\beta - decay$ is done, where, it was established a logarithmic equation of the β decay rate that resulting in a straight line as a function of the transverse barrier width of $e^{\pm}$ $(w = r\lambda)$ for every nuclide, it decreasing in case of long lived nuclides, like ^{60}Fe, see fig.5.
The same it happens in the unrelativistic Fermi model of beta decay, when the final states of the electron [32] as to be coupled with the initial state (parent) by a coupling matrix, the Kurie plot or Fermi plot show that beta ray spectrum may be plotted as a straight-line graph of energy difference multiplied by a constant $const \times (E_0 - E)$, E_0-the end point energy of beta ray spectrum, or for allowed transition the matrix element H'_{if} which enters in the transition probability per unit time (λ) for the transition from the initial state (i) to final state (f) of a quantum mechanical system, and which is independent of the electron momentum (p). This matrix in relativistic approach is given as : $H'_{if} = (\Psi^*_f \Psi^*_e | H' | \Psi_i \Psi_\nu)$, where Ψ are the wavefunctions for initial and final state of neutron and neutrino, respectively.
This matrix have five elements as quantities of interaction: scalar, vector, tensor, axial vector and pseudoscalar. Any one of these quantities is a beta decay interaction, but not only one.
This matrix H' must also contain a factor containing the dependence of the strength of interaction on the distance between nucleon and lepton. The transmitter of the weak interaction to be an intermediate boson denoted by W with spin 1 and large mass about

80 GeV [32]. Thus, the weak interaction would have a short range ~10^{-14} cm. Because of this short range, it will be assumed that the weak interaction is a contact interaction expressed by introducing the factor $\delta(r - r_L)$, where r and r_L are the nucleon and the lepton (our $e^{\pm}$) position vectors, respectively. Besides these factors H' might contain other operators acting on the nucleon and lepton wavefunctions, so $H' = (operators)\delta(r - r_L)$.The model developed in [4] confirm all that conclusions, together with a lot of $\beta - decay$ experimental results.

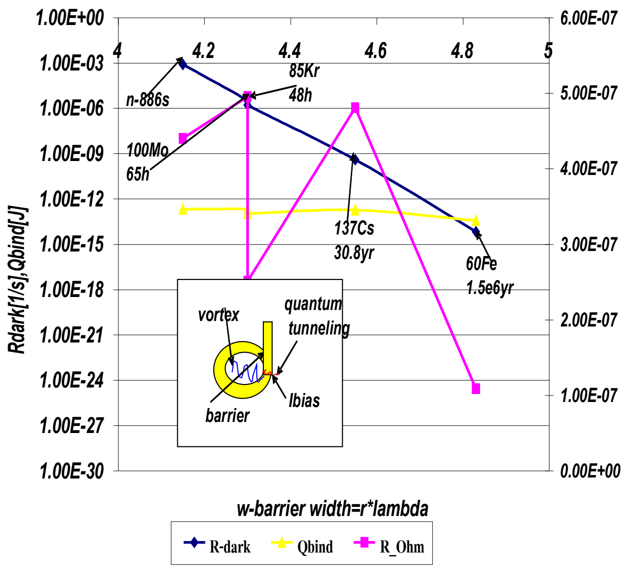

Fig.2. The evolution of dark count (β decay) rate as function of barrier width.

3.1. The review of the bias current model of $\beta - decay$ stimulation by a thermal spike of a photon

In order to accelerate the $\beta - decay$ by a single photon reaction, a new model it was proposed in [4] to calculate a *direct reaction of single photon with one of nucleon of the valence n-n; p-p; n-p pairs (see IBM model [30,31]) of the nucleus, that being in the unstable state (a* $\beta - decay$ *nuclide), they are the most susceptible to react with the photon, see some of model's results from [4], figures* $1 \div 4$, *respectively.*
In the following, in order to strength the idea of a $\beta - decay$ acceleration, we would summarize and update some of mainly results of this *companion author's article* , but more one can find in [4] :

The interaction between a photon of high energy and of low band width $\Delta E/E \leq 10^{-3}$ and of nucleon into state of excitation has been characterized by the beta decay energy Q_β from the nuclei, that is viewed as a *direct reaction,* without the formation of a compound nucleus, as it was mentioned before [33].

Essentially, the general picture of this model described in details in [4], is that the vortex (boson W) crossing may trigger the $s \to n$ transition. A photon makes this process much more probable by creating a spot (melt) with suppressed order parameter and thus with lower energy barrier for vortex crossing. A sketch of the strip and of the belt across are shown in Figure 2., the induced vortex crossing together with an electron (*travel current* $e^{\pm}$), which turns superconducting hot belt into the normal state resulting in a vortex assisted photon beta decay.

Therefore, by using the same nucleon model we can account for a vortex (W boson) assisted photon count rate, as in [4]:

$$R_{pc} = R_h[1 - \exp(-\Re)]$$

,where:

$$R_v(I,\nu_h) = \frac{4k_B Tc^2 R}{\pi \Phi_0^2} \frac{\xi}{w} \left(\frac{\nu_h}{2\pi}\right)^{1/2} \left(\frac{I}{I_{ch}}\right)^{\nu_h + 1}$$

$$\Re \approx \frac{\tau_{GL} R_v}{\nu_0 - \nu_h} \frac{1 - (I_{ch}/I)^{\nu_h - \nu_1 - 1}}{\ln(I_{ch}/I)}$$

The current being

$$I_{ch} = I_{c0}(\nu_h/\nu_0)^{3/2}$$

We can suppose than along the hot belt induced by the incident photon , the charge $e^- + W^-$ creates a *bias current*($I > 2/3e[(1/(\nu_h/\nu_0)^{3/2}) \cong 3] \Rightarrow 2e$ who circulates due of the potential difference between the vortex and the rest of isotope.

At the first sight, the ohmic resistance of this ad-hoc electrical circuit created by the *bias current* is given as:

$$R = \frac{U_\beta}{\tau_{GL}} \frac{1}{V_{vortex}^2}$$

,where the vortex potential is $V_{vortex} = H_0 \xi$,

H_0-an "external" electro-magnetic field of a dipole created by the pair $u\bar{u}$ (the chromoelectrical field)

$$H_0 = E_0 = \frac{de}{4\pi \varepsilon_0 r^3} = 8.33e24\left[\frac{N}{C}\right]$$

,where, $r \cong 0.05[fm]$-is the electrical flux tube radius, $d = 0.7[fm]$-the distance between the two quarks charges, usually $H[A/m]$, but here is used as $B = \mu_0 H\left[\frac{J}{Am^2}\right]$

, and the characteristic distance $\xi \le \lambda$, the coherence length,

and the power is $U_\beta/\tau \cong \varepsilon_{vortex(W^\pm)}/\tau$, with $\tau_{GL} = \pi\hbar/(8k_B T_c) = 1.5e-24[s]$-the Ginzburg-Landau life time of $W^\pm$ bosons.

Numerically, with $T_c = 0.4e9K$, $E_{prag} = k_B T$, result $\nu_0 = \varepsilon_W / k_B T_c = 1.e-09/(1.38e-23*2.e12) = 36.2$, where ε_W results from eq. (2) as $\varepsilon_W = \varepsilon_{int}(d = x - \lambda; x = 0.14)*0.117[fm] \cong 1.e-09[J]$;
$\nu_h = \varepsilon_{0h}/E_{ph} = 5.e-11/E_{prag} = 6.6$,
where ε_{0h} is obtained by using the lower critical field

$$B_0 = H_{c1} = \frac{2\Phi_0}{2\pi\lambda^2}\log\left(\frac{\lambda}{\xi}\right) = \frac{\pi\hbar c}{\pi\lambda^2 c}\log(\kappa) = 1.e15\left[\frac{J}{Am^2}\right]$$

, and with $x \cong \xi = 0.107[fm]$;

respectively: $\varepsilon_{0h} = Vc^2\varepsilon_0(2H_{c1})^2/8\pi = 5.e-11[J]$
The value of E_{prag} is determined by trials in order to have $R_{pc}/R_h \cong 1$.

The model results show that in order to have *instant rates(100% decay*), or a beta decay rate of $R_{pc} = 1.e13\,counts/s$, with the incident of single photons (non laser) rate of $R_h = 1.e13\,photons/s$, $R_{pc}/R_h \cong 1$, for all beta-decay isotopes, i.e. these rates are not dependent of the nuclide type, the photons energy needs to be above a threshold energy value of very precise value $0.4e9K \to 33.5keV$, figure.3.

This vortex-assisted mechanism may be verified by application of magnetic fields, which effectively enhance I_{ch} along with the vortex crossing rates but do not affect the creation of hot spots by photons.

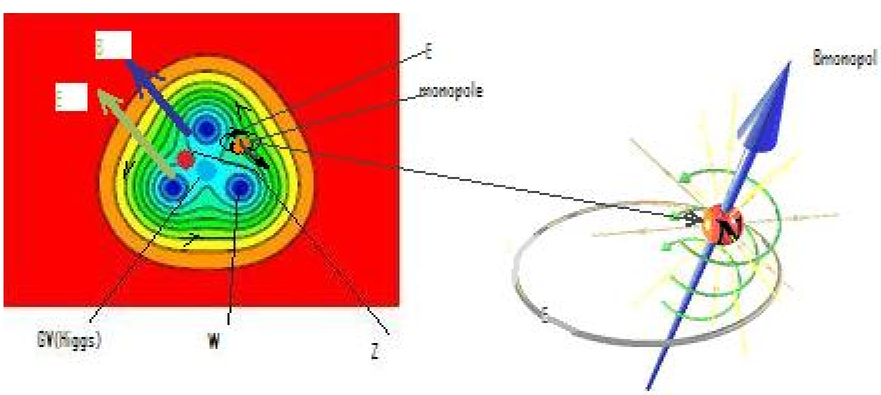

Fig.1a. The Giant-Vortex (after an idea of Ref.[18]) that could be also the arrangement for the nucleon (only illustration).. A spin-orbit nonabelian field is shown (after a idea of the ref [19]).

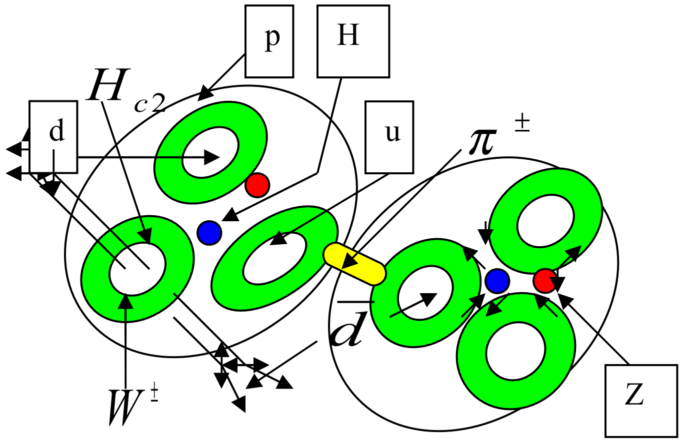

Fig.1b. Abrikosov's triangular lattice for a nucleon (proposal [18])

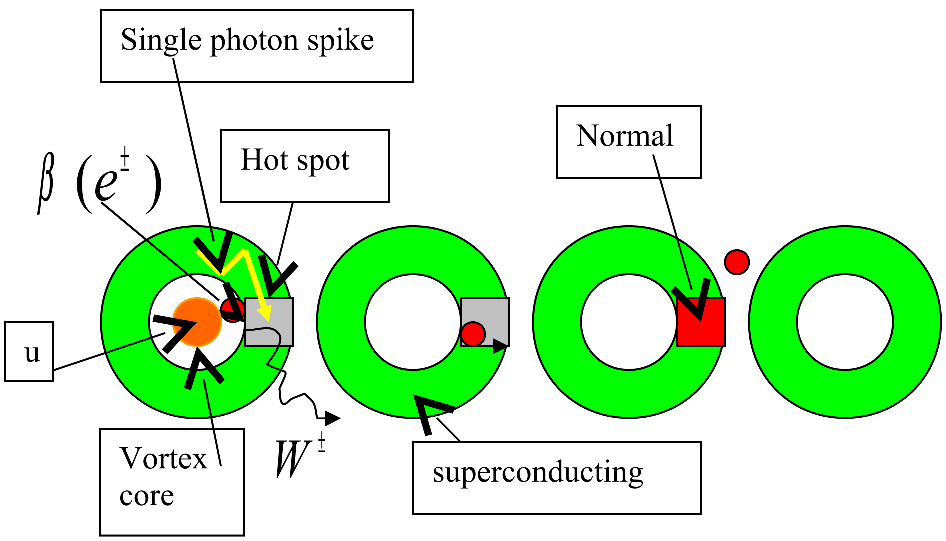

Fig.3. The photonuclear mechanism. From left to right, illustration of incident photon creating superconducting hot spot (hot belt) across nucleon, followed by a thermally induced vortex crossing together with an electron (*bias current*), which turns superconducting hot belt into the normal state resulting in a vortex assisted photon beta decay.

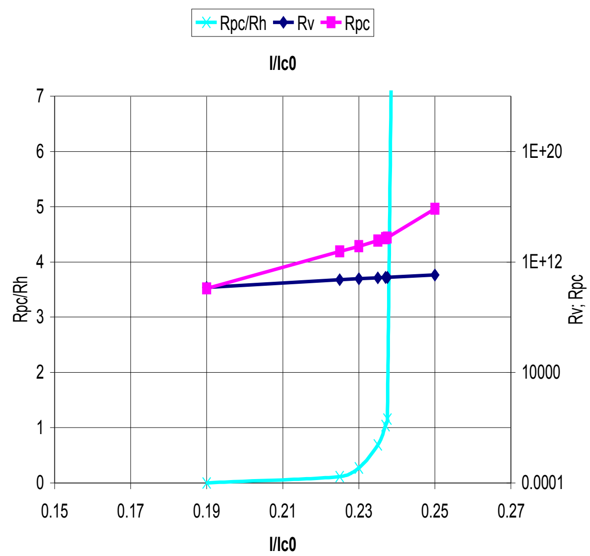

FIG.4. The vortex-assisted photon count rate R_{pc}/R_h vs. *bias current.*

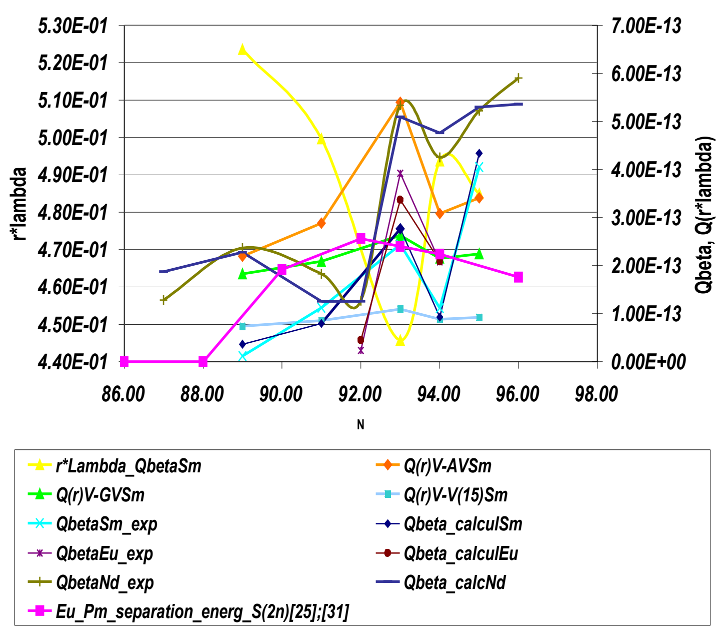

Fig.5. The evolution of experimental values of β decay of $Nd - Sm - Eu$ isotopic chains with $N > 82$ compared with the IBA(Interacting Boson Fermion Model (IBM) [30,31] results, and of Q_β $(w = r\lambda)$ as calculated by the model.

.4. The pair creation inside the nucleon by Schwinger effect

4.1. electron-positron pair

An interesting aspect of virtual particles (in vacuum) both theoretically and experimentally is the possibility that they can become real by the effect of external fields. In this case, real particles are excited out of the vacuum. In the framework of quantum mechanics by Klein, Sauter, Euler and Heisenberg who studied the behavior of the Dirac vacuum in a strong external electric field. If the field is sufficiently strong, the energy of the vacuum can be lowered by creating an electron-positron pair. This makes the vacuum unstable.
A particle with charge q and mass m in a constant magnetic field undergoes circular motions with the Larmor frequency $\omega_c = qB/m$. In quantum theory charged particles occupy the Landau levels with energy $E = \hbar\omega_c \cdot n$ and in a strong magnetic field of $B_c \cong 10^{16}[T]$, the energy difference of Landau levels $\delta E = \hbar\omega_c$ can be comparable to the rest mass of the particle. The number of Landau levels is $N \le \frac{m\omega_c L^2}{2\pi\hbar} = \frac{qBL^2}{2\pi\hbar}$, where L is the length. In the transverse direction of a magnetic field above the critical strength electrons of an atom are strongly bounded and fill the lowest Landau levels but in the parallel direction are attracted by the Coulomb force. As classically the charged particle moves along a spiral of circular motion in the transverse direction and linear.
In the case analyzed in [34,35], in which $E^2 - B^2$ and $E \cdot B$ are not both zero, one can go to a frame in which E and B are parallel with magnitude E and B, and is obtained the imaginary part of the one-loop effective action per four-volume for spinor QED.
In a pure magnetic field $B = Be_z$ along the z-direction, $A = (0, Bx, 0)$, and for the electric and magnetic fields parallel to each other along the z-direction, the 4-potential is given by $A_\mu = (-Ez, 0, Bx, 0)$, that copy very well with our new understanding, when the electric field E is that of the quark-anti-quark pairs, and B is induced either by the spin-orbit interaction of the monopole (nonabelian field), or by the monopole current as Rashba effect, see appendix C, and figures 1a,1b.
In [34] is obtained the pair-production rate for fermions (eq. (67)) as

$$\frac{\Gamma_{JWKB}}{V} = \frac{\alpha}{\pi\hbar}\left(\sqrt{4\pi\varepsilon_0}\right)^2 (Bc) \cdot E \cdot \coth\left(\frac{\pi Bc}{E}\right) \exp\left(-\frac{\pi E_c}{E}\right) \quad (73)$$

for spin $-1/2$ particle,
where $\alpha = 1/137$, $V[m^3 s]$ and $\sqrt{4\pi\varepsilon_0}$ to convert $cgs \to SI$ (to note that the authors of these articles do not have used this factor, that affecting *enormous* the numerical results with $\cong 10^{-11}$), JWKB meaning (Jeffreys-Wentzel-Kramers-Brillouin) model [34].
The discrete spectrum due to the magnetic field is the Landau levels for charged particles. Note that all the Landau levels are non-degenerate for the scalar particles, whereas all the states of the Dirac spinor are doubly degenerate, $j, \sigma_+ = 1/2$, and $j-1, \sigma_- = -1/2$, except for the unique lowest Landau level, $j = 0, \sigma_+ = 1/2$.

The effect of magnetic field is the same as shifting the effective mass $m_*^2c^4 = m_e^2c^4 + qB\hbar c^2(2j+1-\sigma)$ for fermions for each Landau level.

Case 1-The use of nonabelian field

The Compton space-time volume of an electron has the size $V_{Compton} = \lambda_C^3 \times (\lambda_C/c) = 1.59e-70m^3s$,

Where $\lambda_C = \hbar/m_*c = 4.6e-16[m]$, the effective mass is $m_* = \sqrt{m_e^2c^4 + qB\hbar c^2}/c^2$, the *critical field* $E_c = \frac{m_*^2c^3}{e\hbar} \cong 8.58e23 < E = 8.24e24[N/C]$; $B = 2.86e15[T]$ as from eq. (C.40), that results $m_* = 7.14e-28kg$

The electron energy is $\varepsilon = m_*c^2 \cong 0.4GeV$ - the same and sufficiently to broken the quark-anti-quark string strength $\sigma \cong 0.4GeV$ during beta-decay process followed by the release of a beta-decay electron from $W^{\pm}$ decay, and which gets the final beta energy as equally to that of the out of barrier turning point after the tunneling and accounting for the valence nucleons interactions (shell-energy levels). The number of *assaults* of the barrier, like in Gamow theory [37,38] is $n_a = v_b/R$; where the velocity is $v_b \cong (2\varepsilon/m_*)^{1/2}$, and the radius of the barrier is $R \cong \lambda_C$; $\varepsilon = \hbar eB/m^* \cong 0.4GeV$ the energy of the particle for the first Landau level (as above), and we can see that it results to be equally with the rest mass of the electron, that resulting $n_a \cong 9.e23s^{-1}$. In case of WKB [37] , the transmission coefficient $T = 2\frac{\sqrt{2m|V-Q|}}{\hbar}\Delta r$, and the decay constant $R = n_a e^{-T}$.

For the thick barrier the transmission coefficient is $T = 2\pi\frac{Qb}{\hbar v} = 2\pi\frac{\sqrt{2mQ}}{\hbar}b$; where, the kinetic energy of the particle after the barrier at b is $Q \equiv \frac{1}{2}mv^2$

To "materialize" a virtual $e^+ - e^-$ pair in a constant electric field E the separation d must be sufficiently large $eEd = 2mc^2$

Probability for separation d as quantum fluctuation

$$P \propto \exp\left(-\frac{d}{\lambda_{Compton}}\right) = \exp\left(-\frac{2m^2c^3}{e\hbar E}\right) = \exp\left(-\frac{2E_{cr}}{E}\right)$$

The emission (transmission through barrier) sufficient for observation when $E \approx E_{cr}$, with $Q = eE_{cr}d$, results $T = 2\pi\frac{mcb}{\hbar} \cong \frac{2\pi b}{\lambda_C}$, or $b \cong \lambda_C/2\pi$ (73.1)

With these values it result : the number of pairs $\Gamma_{JWKB}/s = 4.65e23s^{-1} \approx n_a$, for a volume $V \cong (\lambda_b)^3 \cong 1.0e-46[m^3] \geq V_b$, the penetration length $\lambda = 0.117fm$, and for a four-volume of $\lambda_b^4/c \cong 1.6e-70[m^3s]$, results $\Gamma_{JWKB} \cong 0.72 \approx 1pair$, or in other words, all the time inside the nucleon is *available* one real pair of electron-positron which combine with quarks pairs, as was shown before, resulting an e^+, or e^- which help the quarks transformation $(u \to d)$ for beta-decay. In our model, $W^\pm$ is also created (see bellow) there as a *vortex* which decay into an electron which takes the energy at the *turning point* out of the barrier equally with the binding energy of nucleon in isotope nucleus, and it *passes* the barrier of monopole condensate characterized by an *quantum tunneling* suppression given as: $\exp(-\Delta E\tau/\hbar) \cong 3.96e-27$, where, as $\tau \cong \hbar/m_* c^2 \cong 1.56e-24[s]$ near $\tau_{GL} = \pi\hbar/(8k_B T_c) = 1.5e-24[s]$ is the Ginzburg-Landau life time of $W^\pm$ bosons, since these decay in beta electrons, in the same time with pair generation, their lifetime need to be near equally with the *lifetime* of the pair $\tau_{e^+-e^-} = \hbar/m_* c^2 \cong 1.56e-24[s]$, that is happen, and ΔE, which corresponds to the height of monopole condensate barrier, due of the *phase slip* with $2\pi - \varphi$ and of a Φ_0 energy release: $\Delta E = c^2\Phi_0^2\varepsilon_0/d_b$; $d_b \cong 7.15\lambda$, and $\Delta E = 3.898e-09[J]$; , and the quantized flux is :

$\Phi_0 = \pi\hbar c/e \to usually \dfrac{\pi\hbar}{e} = 2.07e-15[Tm^2]$. To note that ΔE

Thus, the probability (rate), into a more simple way- without the external interactions of the neutron (free-not bounded) , is s given as:

$\Gamma_{JWKB} V \exp(-\Delta E\tau/\hbar) \cong 1.84e-03s^{-1} \to \tau_{1/2} \cong 544[s]$, that corresponds for *free neutrons decay* by emission of an electron and an electron antineutrino to become a proton, $n^0 \to p^+ + e^- + \bar{\nu}_e$, with half-life of $611s$.

Case 2-The use of Rashba effect

The Compton space-time volume of an *heavy* electron (gluon!) has the size $V_{Compton} = \lambda_b^3 \times (\lambda_b/c) = 8.e-74m^3s$,

Where $\lambda_b = \hbar/m_* c = 7.e-17[m]$, the effective mass is

$$m_* = \sqrt{m_e^2c^4 + qB\hbar c^2}\Big/c^2, \qquad m^* = 4.77e-27[kg] \qquad (74)$$

, and the *critical field* $E_c = \dfrac{m_*^2c^3}{e\hbar} \cong 3.84e25 < E = 2.18e28[N/C]$

The "heavy electron" energy is $\varepsilon = m_* c^2 \cong 2.68GeV$ - the same and sufficiently to either broken or to create an quark-anti-quark string, that fact it will be established experimentally !.

Therefore, if we consider the situation of the GV, in eq. (73) is introduced the magnetic field induced by the entire monopole condensate as being

$3\times\varepsilon_{vortex} = \dfrac{V\varepsilon_0 c^2 B_{monop}^2}{8\pi} \to 7.9e-07[J]$, where $B_{monop} \cong E_{monopol}/c = 1.28e17[J/Am^2]$;

$$E_{monop}^2 = \frac{(\varepsilon_{monop})^2}{\varepsilon_0(\lambda_C^{e*})^2\hbar c} \;; \qquad (74.1)$$

$\varepsilon_{monop} \cong H_R\_or\_\cong 3monopoles\_mass$, the Rashba energy being $H_R = 1.2e-09[J]$ from eq. (C.32), and $3\times\varepsilon_{vortex}$ from (71.4), resulting $E_{monopole} \cong E_{cr}^{e^+-e^-} \cong 3.835e25[N/C]$, and the electrical field of the Giant Vortex (GV), (see figures.1a,1b) is

$E^2 = \dfrac{(3\times \varepsilon_{vortex})^2}{\varepsilon_0 (\lambda_C^{e*})^2 \hbar c} \to E = 2.18e28[N/C]$, where $V \cong 1.45e-45m^3$ for the Giant Vortex, near the value of $v.e.v = 3.5e28[N/C]$. To mention that, independently we can obtain the critical value of magnetic field as: $B^{cr}_{monop} \cong E^{e^\pm}_{cr} / c \cong \dfrac{3.84e25}{2.997e8} \equiv 1.28e17[J/Am^2]$

With these values it results : the number of pairs created $\Gamma_{JWKB}/s = 4.12e24[s^{-1}] \approx n_a$, and finally with Compton volume it results $\Gamma_{JWKB} \cong 1pair$, or in other words, all the time inside the nucleon is *available* one real pair of electron-positron which combine with quarks pairs as was described above, or creates a new quarks pair-for a new hadron type (with four quarks as a new state of mater!).

In our model, in the same time is created a pair $W^\pm$ by a Schwinger effect (see the next section).

This "heavy electron" itself can not *pass* the barrier of monopole condensate as characterized by an *quantum tunneling* suppression given as: $\exp(-\Delta E \tau/\hbar) \cong 1.2e-04$, where, as $\tau \cong \hbar/m_* c^2 \cong 2.33e-25[s]$ small that $\tau_{e^\pm} = 1.5e-24[s]$, or the Compton length being too small by the barrier width $\lambda_C \cong 7.e-17[m] << b \cong 7\lambda = 0.83 fm$. The same conclusion is obtained if we apply eq. (73.1),. Therefore, this passes only when a external energy (by collisions!) is applied in order to maintain "open" the barrier by "melting" it.

Therefore, it look like of two permanent pairs of $e^+ - e^-$ and of $W^\pm$ to be present during the beta decay, or an energy of a vortex $W^\pm$ exists here, as it was found in the previously work [4]. In the same way, many others particles could be created here, like $s;c;t$ quarks ; muons etc. , but anyone can not penetrates the barrier as itself, these can only decay, or annihilate into electrons of low energy and neutrinos.

To remember that in [4], we have described the process as a spontaneous nucleation of a normal-state belt across the strip with $2\pi - \varphi$ *phase slip* with Φ_0 release, and a *bias currents* which may produce a spontaneous suppression of the superconducting order parameter (ψ), and when a vortex $(W^\pm)$ (after it is created by the Schwinger effect, as the main finding of this new work, see section 4.2.), crossing from one strip edge (just the monopole condensate barrier) to the opposite one induces a *phase slip* without creating a normal (vacuum) region across the strip (one of three vortexes of nucleus) width. Then, is treating the vortex as a particle moving in the energy potential formed by the superconducting currents around vortex center inside the strip and by the Lorentz force induced by the bias current. For a free neutron the *dark count rate* in [4] is obtained as $R_{dark} = 7e-04s^{-1}$, which is comparable with the above result.

To see the order of magnitude we extract from [4] some results on the decay of a free neutron, thus, the bias current is :

$$I = \frac{2w}{\pi\xi} I_0 \kappa (1-\kappa^2)$$

, and the main superconducting current is:

$$I_0 = \varepsilon_0 \frac{c\Phi_0}{8\pi\Lambda}; \Lambda = \frac{2\lambda^2}{h_z}$$

Here, $h_z \cong \lambda$ -the axial (z) height of the monopole condensate.

Also, here, the critical current at which the energy barrier vanishes for a single vortex crossing:

$I_c = \dfrac{2\mu^2 w I_0}{2.72\pi\xi}$; respectively:

$I = 5.197e3$; $I_c = 1.6e4$; $I_0 = 3.e5$ from eq.(62.1); where $\mu^2 = 1 - \kappa^2$; $w/\xi = 4.15$; $\kappa = \lambda/\xi \cong 0.117/0.1114$; and $I/I_0 \cong 0.017 \rightarrow 1e/68e \cong 0.014$

4.2. The Bosons pair production

Case 1-$W^{\pm}$ bosons creation

It has long been known that an inevitable consequence of Dirac's theory of the electron is that in regions of sufficiently high energy density, the quantum vacuum can break down in a spontaneous generation of electron-positron pairs. Following the initial results of Sauter, Heisenberg and Euler and Weisskopf , in a seminal work, Schwinger derived a central result of strong-field quantum electrodynamics, the rate per unit volume of pair creation R in a constant and uniform electric field of strength E, of leading order behavior ,

$R = (E/E_{cr})^2 (c/\lambda^4)(8\pi^3)^{-1} * \exp(-\pi E_{cr}/E)$

for $E/E_{cr} << 1$, positron charge e, mass m, Compton wave-length $\lambda = \hbar/mc$ and so-called "critical" electric field $E_{cr} = m^2c^3/e\hbar$.

Now, the energy corresponding to E_{cr} is given as: $v^2 = E_{cr}^2 \varepsilon_0 \left(\lambda^{W^{\pm}}_{Compton}\right)^2 \hbar c$, or

$$v^2 = \frac{m_W^4 c^6}{e^2\hbar^2}\frac{\hbar^2}{m_W^2 c^2}\frac{4\pi\hbar c\varepsilon_0}{4\pi} = \frac{m_W^2 c^4 \varepsilon_0 \hbar c 4\pi}{4\pi e^2} = \frac{M_W^2}{4\pi\alpha} \cong \frac{0.25 M_W^2}{\pi\alpha} \cong (267GeV)^2$$

, which in fact is the vacuum expectation value $(v.e.v.)$, here the fine-structure constant is

$$\alpha = \frac{e^2}{4\pi\varepsilon_0\hbar c} = \frac{1}{137}$$

If we will remember the Fermi disintegration constant

$$G_F = \frac{1}{4\sqrt{2}}\frac{g^2}{M_W^2} = 1.15 \times 10^{-5} GeV^{-2}$$

,where the mass of boson $M_W = m_W c^2 = 81GeV$, and $e = g\sin\theta_W$; the Weinberg angle $\sin^2\theta_W \cong 1/4$, and the Higgs vacuum expectation value $v.e.v$ or

$v = \left(\sqrt{2}G_F\right)^{-1/2} = 247GeV$, if we substitute $M_W = \dfrac{gv}{2}$ from Standard Model.

$$M_W^2 = \frac{e^2}{4s_W^2}v^2 = \frac{\pi\alpha}{s_W^2}v^2 = \left(\frac{37.2}{s_W}GeV\right)^2 \cong (80GeV)^2$$

, where $s_W^2 = \sin^2\theta_W \approx 0.22$

If in eq. above in place of $0.25 \to 0.22$, it will result $v = 247GeV$
Therefore, it is obtained for the *first time in literature* the derivation of the *Higgs vacuum expectation value field* $(v.e.v)$ as being *Schwinger critical field* for creation of pair of bosons $W^- - W^+$ as needed in beta-decay process. To note that, the Standard Model is centered, also in $W^\pm$, the difference being that, in my model this pair is created by the Schwinger effect, and the field of the nucleon substructure having values which support this effect.

Numerically, $m_W = 81GeV \to 1.442e-25kg$, $r \cong \lambda_{Compton}^{W^\pm} = \hbar/m_W c = 2.31e-18[m]$ for (GV), results $E^2 = \dfrac{(3\times \varepsilon_{vortex})^2}{\varepsilon_0(\lambda_C^W)^2\hbar c} \to E = 6.58e29[N/C]$, and

$3\times \varepsilon_{vortex} = \dfrac{V\varepsilon_0 c^2 B^2}{8\pi} \to 7.9e-07[J] \to \cong 5TeV$, and $B \cong E_{monopol}/c = 1.28e17[J/Am^2]$

with E_{monop} from eq. (74.1), and Schwinger critical field

$E_{cr} = m_W^2 c^3/e\hbar = 3.5e+28[N/C] \leftrightarrow v.e.v = 267GeV$;

The rate of $W^- - W^+$ pair production is again $R \cong 1$, with the Compton volume $V_C^W = (\lambda_C^W)^4/c = 9.54e-80[m^3 s^{-1}]$, and $R/s = 1.56e26s^{-1}$. Again, due of very small Compton length by comparison with the barrier width, $W^\pm$ itself can not penetrate the barrier, as describe above.

To note, that in order to obtain the pair rates of $\approx$ *one* for $W^\pm$, only then when are used the values of GV energy $(3\times e_{vortex} \cong 7.9e-07[J] \to 5TeV)$, which can means that these particles are obtained by the *melting* of two gluons (gg), as is find in paragraph 2 , and at LHC-see section 6. Therefore, a collision of a such value can maintain *"open"* the barrier till the release of decay products!.

For the first time in [39] is studied the entire dynamics of energy conversion from initial overcritical electric field, ending up with thermalized electron-positron-photon plasma. Such conversion occurs in a complicated sequence of processes starting with Schwinger pair production which is followed by oscillations of created pairs due to back-reaction on initial electric field, then production of photons due to annihilation of pairs and finally isotropization of created electron-positron-photon plasma. Evolution of electric field E and pairs bulk parallel momentum for $E > 30E_{cr}$, shows that following oscillations E tend asymptotically to E_{cr} after $\cong 1000\tau_c$. After some time, the photons energy density becomes equal and then overcomes the pairs energy density. This growth continues until the equilibrium between pairs annihilation and creation processes is established

$e^- - e^+ \leftrightarrow \gamma\gamma$.

4.3 A new understanding of beta decay

In the classic understanding of β^- disintegration $n \to p + e^- + \overline{\nu}_e$, or this occurs when one of the down quarks in the neutron (udd) decays into an up quark by emitting a virtual W^- boson, transforming the neutron into a proton (uud). The W^- boson then decays into an electron and an electron antineutrino: $udd \to uud + e^- + \nu_e$.

In our *new understanding* of beta-decay process, it results that, in order to make possible this transformation (balance of charges) it needs to have supplementary an $e^- + e^+$ pair as created by a Schwinger mechanism, as will we show above, and which is necessarily to *always* exist here, and consequently, for the beta decay process, we have in terms of quarks for the neutral mesons: $u\bar{u}; d\bar{d}$:

$d(-1/3e) + e^+(+3/3e) = u(+2/3e)$, (75)

$\bar{d}(1/3e) + e^-(-3/3e) = \bar{u}(-2/3e)$

,and for β^+ decay can only happen inside nuclei when the daughter nucleus has a greater *binding energy* (and therefore a lower total energy) than the mother nucleus. The difference between these energies goes into the reaction of converting a proton into a neutron, a positron and a neutrino and into the kinetic energy of these particles.
In an opposite process to the above negative beta decay, the weak interaction converts a proton into a neutron by converting an up quark into a down quark by having it emit a W^+ or absorb a W^-.

β^+ decay of nuclei (only bounded proton) when: $p \to n + e^+ + \nu_e$, or $energy + uud \to udd + e^+ + \nu_e$

, or, $u(2/3e) + e^-(-3/3e) = d(-1/3e)$ (76)

$\bar{u}(-2/3e) + e^+(3/3e) = \bar{d}(1/3e)$

In the process of electron capture, one of the orbital electrons, usually from the K or L electron shell, is captured by a proton in the nucleus, forming a neutron and an electron neutrino.

$p + e^- \to n + \nu_e$

In our *new understanding*, when the vortex equilibrium is disturbed by the transformation of one quark ($d \to u$) due of the interaction with the new created $e^- + e^+$ pair (coincidently of the same energy as that of the strings (neutral mesons) $u\bar{u}; d\bar{d}$), then, this is accompanied by a creation of a $W^\pm$ boson which decay into $e^\pm$ and neutrino which pass the barrier as *crossing vortex* , that being the new idea of this model.

5. Schwinger pair-production thermally stimulated by a laser pulse

From [40] results that the Schwinger pair-production rate by a time-dependent electric field is enhanced by a thermal factor of the initial Bose-Einstein distribution known as the Boltzmann factor.
Thus, it is found that the Schwinger pair production rate at finite temperature is enhanced by the thermal Boltzmann factor: $f_{nk}(T) = \frac{1}{e^{\omega_{nk}^{(-)}/T} - 1} \cong \frac{k_B T}{\omega_c \hbar}$.

Where the cyclotron frequency induced by the monopole B is ω_c.

Thus, with $B = 2.8e15[T]$ from eq.(C.40), $T = 0.4e9K$ for decay of $^{26}Al \to ^{26}Mg$, results $\omega_c = eB/m_* = 6.4e23s^{-1}$, finally the enhancement factor $f_{Boltzmann} \cong 8.6e-05$ or the

lifetime is reduced with $1.e7$, and the number of pairs per a thermal spike it could be $\Gamma_{JWKB} \cdot V \cdot \tau \cdot f_{Boltzmann} \cong 1 pair$, if the volume affected by laser is $V\tau \cong (\lambda_C^{e*})^3 * 30zs[m^3 s]$; represent the turning point at which the decay energy $\cong 1.8MeV$ for ^{26g}Al.
To note that the duration of the *single attosecond spikes* in the APT amounts to $300\,as/2$ means $30zs$ in the projectile frame [41,42,43]. This value approaches the natural QED time scale of $1/m \approx 1zs = 10^{-21}[s]$. Near the same result is obtained in case 2, or when a barrier is "open" by the thermal spike, the heavy electrons passing more easy.
An important thing to be verified experimentally is that by using a thermal spike of only $T = 0.4e9K$, when it could be obtained one $W^{\pm}$ pair, and one of H boson.
We may thus conclude that the Schwinger pair-production rate is indeed enhanced by the thermal effect given by the Bose-Einstein distribution expressed by the Boltzmann factor. In this case, in place of the *quantum tunneling* suppression factor we introduce the *thermal stimulation* with Boltzman factor. The model it needs to be experimentally tested, for example on radioisotope $^{26}Al \rightarrow {}^{26}Mg$, as it was described bellow as the proposal to test the model, also suggested in [44].
To mention that from the model based on *bias current* (described above) it results a necessary photon flux of $R_h \cong 5.18e13\, photons/s$, and from this new model results a duration $\tau \cong 30zs$; both parameters can be obtained only by the laser from ELI (Extreme Light Infrastructure) [45,46].

6.The gluon pair production from arbitrary time dependent chromo-electric field via Schwinger mechanism

The subject of quark/anti-quark and gluon pair production from the non-abelian field is relatively new and is not fully solved [47]. It might be important for the production of the quark-gluon plasma (QGP) in the laboratory by high energy heavy-ion collisions. Lattice QCD predicts the existence of such a state of matter at high temperatures (~ 200 MeV) and densities. In high energy heavy-ion collisions at RHIC and LHC [45] the receding nuclei might produce a strong chromofield which would then polarize the QCD vacuum and produce quark/anti-quark pairs and gluons. These produced quarks and gluons collide with each other to form a thermalized quark-gluon plasma. The space-time evolution of the quarkgluon plasma in the presence of a background chromofield is studied by solving relativistic non-abelian transport equation of quarks and gluons with all the dynamical effects taken into account. Quark and gluon production from a space-time dependent chromofield is needed to study the production and equilibration of a quark-gluon plasma in ultra relativistic heavy-ion collisions at RHIC and LHC.
We again mention here that the fermion pair production from the space-time dependent field is studied in the literature but gluon production from space-time dependent chromofield is not studied so far. This is because a consistent theory involving the interaction of the gluons with the classical chromofield is not available in the conventional theory of QCD. The production of $q\bar{q}$ pairs from a non-abelian field via vacuum polarization is simillar to that of the production of $e^- e^+$ pairs from the abelian field. This is because the interaction lagrangian of the quantized Dirac field with the

classical gauge potential is similar in both the cases.Gluons are the propagators of the QCD and carry colour and anti-colour, described by 8 Gell-Mann matrices, λ , see appendix A.
In high energy collisions, jets are the signatures of quark and gluon production.
For $m_H < 140GeV$, the most promising discovery mode for the Higgs boson at the LHC has involved the production via gluon fusion, gg → H, followed by the rare decay into two photons, H → γγ.
The transverse distribution of particle production from strong constant chromo-electric fields has been explicitly calculated in Ref. 1 for soft-gluon production and in Ref. 2 both cited in ref. [48] for quark (antiquark) production. At high energy large hadron colliders, such as RHIC (Au-Au collisions at $\sqrt{s} = 200GeV$ [49]), and LHC (Pb-Pb collisions at $\sqrt{s} = 5.5TeV$ [50]), about half the total center-of-mass energy, E_{cm}, goes into the production of a semi-classical gluon field, which can be thought to be initially in a Lorentz contracted disc. The gluon field in SU(3) is described by two Casimir invariants, the first one, $C_1 = E^a E^a$, being related to the energy density of the initial field, whereas the second one, $C_2 = \left[d_{abc} E^a E^b E^c\right]$, is related to the SU(3) color hypercharge left behind by the leading particles.
This was already evident from the Schwinger calculation of the production of a fermion-antifermion pair by an electric field E that is constant in space and time, namely,

$$\frac{dW}{d^4x} = \frac{(qeE)^2}{4\pi^2} \sum_{n=1}^{\infty} n^{-2} e^{-n\lambda m^2/(|q|eE)}$$

where q denotes the charge of the fermion.

This calculation was generalized recently to the nonabelian color group SU(3)c in the special E^a case in which (i) there is only a chromoelectric field, E^a , i.e., the chromomagnetic field $B^a = 0$, (ii) E^a is a constant in space and time, and (iii) all of the group components of E^a point along the same spatial direction.
For example, for $q\bar{q}$ it was found that

$$\frac{dW_{q\bar{q}}}{d^4x d^2 p_T} = -\frac{1}{4\pi^3} \sum_{j=1}^{3} \left|g\lambda_{q,j}\right| \ln\left[1 - e^{-\pi (p_T^2 + m^2)/\left|g\lambda_{q,j}\right|}\right]$$

where p_T denotes the momentum of the quark transverse to the direction of the chromoelectric field $\mathrm{E}^a = E^a \hat{z}$ and where the sums of $SU(3)_c$ group indices a,b,c are from 1 to 8. Integration over p_T yields

$$\frac{dW_{q\bar{q}}}{d^4x} = \frac{1}{4\pi^2} \sum_{j=1}^{3} (g\lambda_{q,j})^2 \sum_{n=1}^{\infty} n^{-2} e^{-n\pi m_q^2/\left|g\lambda_{q,j}\right|}$$

where the $\lambda_{q,j}$,depends on three independent gauge and Lorentz invariant eigenvalues λ_j of $f^{abc}E^c$ in SU(3)

$$\lambda_1 = \sqrt{\frac{C_1}{3}} \cos\theta \ , \ \lambda_{2,3} = \sqrt{\frac{C_1}{3}} \cos\left(\frac{2\pi}{3} \pm \theta\right)$$

, where
$C_1(t) = \left|E^a(t)E^a(t)\right|$

The following result was obtained for the probability of gluon pair production from arbitrary time dependent chromo-electric field $E^a(t)$ in $\alpha = 1$ gauge via Schwinger mechanism [], [48,51,52]:

$$f(p_T, \theta, C_1) = \frac{dW_{g(\bar{g})}}{dtd^3xd^2p_T} = -\frac{1}{4\pi^3}\sum_{j=1}^{3}\left|g\lambda_j(t)\right|\ln\left[1 + e^{-\pi (p_T^2)/\left|g\lambda_j(t)\right|}\right]$$

$$\lambda_1^2 = \frac{C_1(t)}{2}\left[1 - \cos\theta(t)\right] \qquad \lambda_{2,3}^2 = \frac{C_1(t)}{2}\left[1 + \cos\left(\frac{\pi}{3} \pm \theta(t)\right)\right]$$

At RHIC and LHC heavy-ion colliders the classical color field play an important role to study production of quark-gluon plasma.
In these situations it is necessary to know how QCD coupling constant depends on SU(3) color field. In this paper we solve the renormalization group equation in QCD in the presence of SU(3) constant chromo-electric field E^a with arbitrary color index a=1,2,...8. Using background field method in QCD we derive β function from the one loop effective action of quark and gluon in the presence of constant chromo-electric field E^a. Using these two facts we determine the exact dependence of the QCD coupling constant α_s on chromo-electric field E^a in SU(3). We find from eq. (17) the QCD coupling constant results from fig.1 [51]: in the presence of SU(3) chromo-electric field as function θ for fixed values of first Casimir invariant C. The λ_j's used are from the above eq. Thus, for $\theta = 0 \rightarrow \alpha_s(\lambda_1) \approx 1; \alpha_s(\lambda_2) \approx 0.15; \alpha_s(\lambda_3) = 0.1$, and for $C_1 = 100;1000,7200 GeV^4$, respectively.

(19)

$$\alpha_s(\lambda_j) = \frac{g^2(t_j)}{4\pi} = \frac{\alpha_s}{\left[1 + 4\pi\bar{\beta}_0\alpha_s\log\left(\frac{g^2\lambda_j^2}{\mu^4}\right)\right]} = \frac{1}{4\pi\bar{\beta}_0\log\left(\frac{g^2\lambda_j^2}{\Lambda^4}\right)}$$

$\Lambda = \mu e^{(-1/(4\bar{\beta}_0 g^2))} \cong 200MeV$; and $0 \le \theta \le 2\pi/3$; $\alpha_s = \frac{g^2}{4\pi}$

$g = 3$

$\bar{\beta}_0^g = -\frac{11}{32\pi^2}$

The value of C1 can be estimated from the initial centerof-mass energy of the colliding ions, and the volume of the Lorentz contracted Nuclei. For example for gold, $R \approx 10 fm$ and at RHIC the center-of-mass energy is $\approx 200GeV$ per nucleon. The initial density is then of the order

$\rho \cong E_{cm}/(\gamma V_0)$;
with $V_0 = 4/3\pi R^3$, and $\gamma = M_{ion}/E_{cm}$. For the above RHIC case $\rho \cong 100GeV^4$.
The results are plotted in fig.6

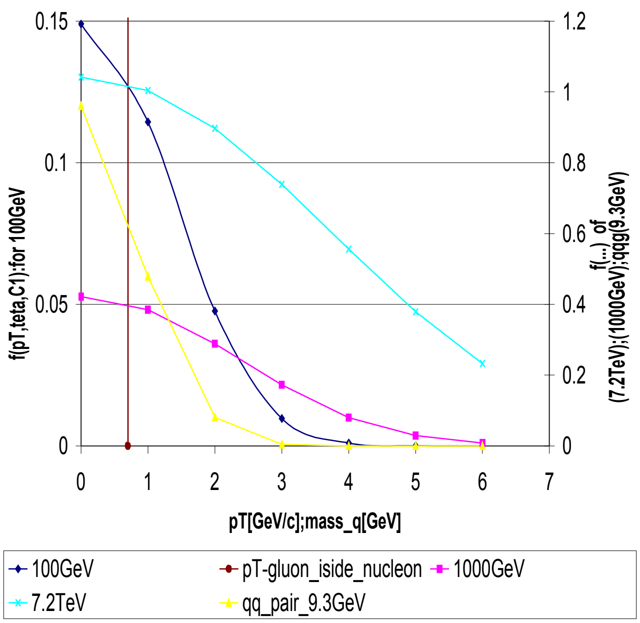

Fig. 6 The evolution of probability of gluon/quarks pairs production for different collision energies

If gluons fragment similar to quarks certainly the fastest gluon jet can be singled out and identified safely like quark jets which start to show up clearly at jet energies of ≈ 3 GeV as had been already shown at SPEAR (Stanford) in 1975 [53].

7. The calculation of the gluon transverse momentum

In order to point out the value of probability is necessarily to have the value of the gluon transverse momentum, for this purpose a model is proposed.

The vector potential of magnetic field produced by magnetic moment of the gluon m_{Mo} is

$$A(r) = \frac{\mu_0}{4\pi}\frac{m_{Mo} \times \vec{r}}{r^3}[N/Ampere]$$

Since, the spin angular momentum, for a monopole $\mu_{Mo} = \frac{gQe\hbar}{2m}$, the color magnetic charge is $Q = 68.5$, $\vec{s} = \hbar$, $g \cong 2$

, and $m_{Mo} = -\frac{\mu_{Mo} S}{\hbar} \cong \mu_{Mo}$, $S = \hbar/2$, or $m_{Mo} = 2.7e-25[J/Tesla^{-1}]$

The energy being $T_{P_T} = p_T^2/2m[J]$, and from the equality of Lorenz force $F_L = q_m(B - vE_g/c^2)[N]$ with the centripetal force $F_c = mv^2/r$ and with $p_T = mv$, results $p_T = r/v \cdot q_m(B - vE_g/c^2)[N \cdot s]$. By using $\tau = \hbar/m_g c^2 = 2.8e-25[s]$, $m_g \cong 2.2[GeV]$, result $E_g = A_g/\tau = 2.1e24[N/C]$, $B = 1.17e15[J/Am^2]$,and with color magnetic charge $q_m = \frac{2\pi \varepsilon_0 \hbar c^2}{q_e}[A \cdot m]$, and $r = 0.2[fm]$, results the transverse momentum for the gluon $p_T \cdot c = 0.7[GeV]$.

With these, as in fig.6, in the case of pp collision at LHC, the probability of gluon pair production from arbitrary time $f(p_T, \theta = \pi/3, C_1 = 7.2GTeV^4) \cong 1$, which in fact that it was happen at LHC.

In the lowest approximation, the Drell-Yan lepton pair of invariant mass M > 1GeV is produced by annihilation of two quarks from the colliding hadrons:

$q_f \bar{q}_f \to \gamma^* \to l^+ l^-$

The creation of e+e− pairs in intense laser fields is encountering a growing interest in recent years. It has been stimulated by a pioneering experiment at SLAC (Stanford, USA) where e+e− pair creation was observed in the collision of a 30 GeV γ -photon with an optical laser pulse of 10^{18} w/cm^2 . The high-energy photon was first produced by Compton backscattering of the same laser beam off a 46 GeV electron beam.

Due to the high photon density in the intense laser pulse, the simultaneous absorption of more than one laser photon is possible with a non-negligibly small probability. In the experiment, $n = 5$ m laser photons of $\hbar\omega_0 \approx 2[eV]$ combined their energies with the γ - photon upon the collision to overcome the pair creation threshold: $\gamma + n\omega_0 \to ee^-$ (*nonlinear Breit-Wheeler process*).

The production rate and kinematic distributions of isolated photon pairs produced in hadron interactions are studied [54]. The effects of the initial–state multiple soft–gluon emission to the scattering subprocesses $q\bar{q}$, qg , and $gg \to \gamma\gamma X$ are resummed with the Collins–Soper–Sterman soft gluon resummation formalism.

The effects of fragmentation photons from $qg \to \gamma q$, followed by $q \to \gamma X$ q → X, are also studied. The results are compared with data from the Fermilab Tevatron collider. A prediction of the production rate and kinematic distributions of the diphoton pair in proton–nucleon reactions is also presented. In this work, the Collins–Soper–Sterman (CSS) soft gluon resummation formalism, developed for Drell–Yan pair (including W and Z boson) production, is extended to describe the production of photon pairs.

In [55] is given a proof of factorization using background field method of QCD.

8. Proposal for an experiment of verification

In [44] is done a proposal to use a laser pulse to reduce the half life of beta decay nuclides.
Thus, it is known that, ^{26}Al, through its β-decay to the 2^+ excited state ^{26}Mg and the subsequent decay into the ground state (0^+) via a 1.809 MeV γ-ray, see fig.7, is an important observable for many astrophysical events, and many efforts have been put forth to map the Galaxy by means of this γ-ray.
As the half life $^{26gs}Al(5^+) \overset{\beta^+,EC}{\rightarrow} 7.2\times 10^5\, y \rightarrow 2^+ \rightarrow {}^{26}Mg(0^+)$, the presence of this nucleus provides evidence of ongoing galactic nucleosynthesis.
The nucleosynthesis of ^{26}Al is complicated by the presence of a low-lying (228.3 keV) 0^+ isomeric state. This isomeric state is very strongly inhibited from decaying by γ-ray emission to the ground state (5^+) of ^{26}Al due to the large spin difference. Its lifetime is much shorter (6.345 sec) and it β-decays directly to the ground state of ^{26}Mg through a super-allowed 0^+ to 0^+ transition, thus avoiding the observable 1.809 MeV γ-ray of interest.
However, at high temperatures ($T_9 = 0.42$) equilibrium is reached between ^{26gs}Al and ^{26m}Al which is relevant to some high temperature astrophysical events such as novae. ^{26m}Al decays via β^+ emission with $T_{1/2} = 6.35s$ directly to $^{26gs}Mg(0^+)$.
Theoretical work [56,57], based on shell-model calculations predicts the a dramatic reduction of the effective life time τ_{eff} (^{26gs}Al) by a factor of 10^9 within the temperature range from 0.15 to 0.4 GK, superseding previous estimates by Ward and Fowler [57] by orders of magnitude. This significant decrement of τ_{eff} is due to a variety of physical processes triggered and influenced by hot plasma environments which will gradually become accessible with the emerging ELI project. At high densities the increasing Fermi energy of the electron opens up electron capture channels otherwise energetically forbidden. Moreover, hot bremsstrahlung radiation will lead to an enhancement of the coupling of ground and isomeric state via the manifold of known as well as hitherto unresolved intermediate state (IS) at several MeV where the nuclear level density is high.
In a first instance, we want to expose a miniature 26Al target specimen to an isochorically heated environment with ELI. Work by Patel et al. shows that isochorical heating by laser induced thermally distributed proton beams with end-energies of only a few MeV can be used to create very localized ($\Phi = 50\mu m$) high energy-density plasma states [56]. The ELI system, even in the first phase, will be able to surpass this values by several orders of magnitude, especially once the onset of the pressure dominant acceleration regime is established as predicted by Esirkepov [56]. For increasing laser intensity the electromagnetic field will eventually start to directly interact with the nucleus, thus presumably contributing further to an enhancement of the decay probability. In all instances the spatial confinement of particles and radiation emerging from laser acceleration will help this particular investigation tremendously. The isotope ^{26}Al is only available in minute quantities, which will just allow the production of miniature pellet targets or thin layers on backing or radiator materials. The onset of an

enhanced transition rate and the coupling of ground and isomeric state via IS can be deciphered via the 511 keV annihilation radiation following the β^+ of ^{26}Al.

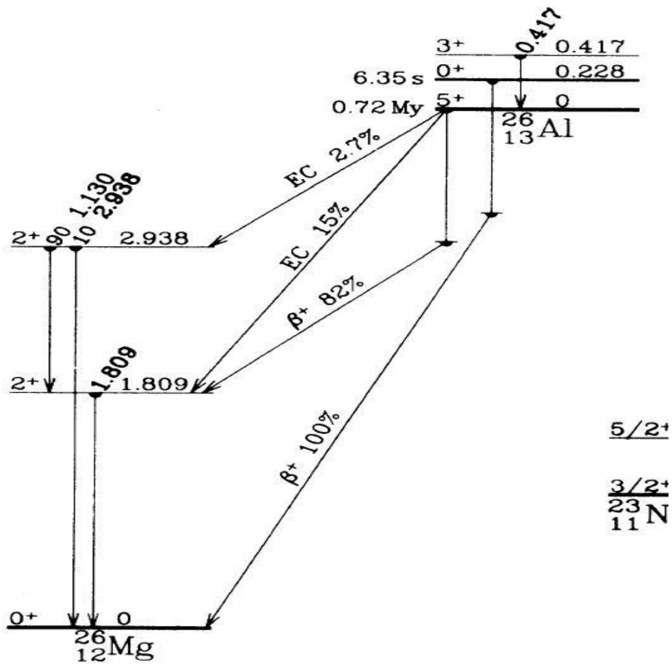

Fig. 7. Level scheme from [58]:

9. Conclusions

We have presented a new analytic approach based on the Dual Ginzburg-Landau theory in order to calculate the strong-field inside nucleons, thus, deriving the values of the monopoles current , the induction, the electromagnetic field, the interaction energies in/between the electric flux tubes as an energy encapsulated by the monopoles circulation, or of the vortex, and between these (giant vortex), respectively.
In the first part we proceed to a review of our analytical model based on the Dual Ginzburg-Landau theory, founding an equivalence with that described in the works from RCNP-Japan, and where, also, is proved the connection between QCD and the dual superconductor scenario.
We provide a detailed analysis of physically important quantities as regarding the nucleons substructure as: the uniform chromoelectric field (vortex) strength inside a nucleon, the mass of monopole viewed as gluons which are the propagators of the QCD and carry colour and anti-colour, with an hedgehog-like configuration, or as a results of interaction of spin-orbit of the monopole current , or of Rashba field interaction, all giving the same result; the quantification of the interaction energies of one vortex ($W^{\pm}$) and of a giant vortex (GV), as to be *encapsulated* by the Abrikosov triangular lattice generated by the coalescence of the flux lines.

In the applications, as based on these data, there are calculated: the Higgs boson energy release due of two gluons fusion during the pp collision at LHC, gluon pair production from space-time dependent chromofield due of the collision of pp and of heavy nuclei; Due of very promising results in these applications, but mainly of the result of the value of the chromoelectrical field ($8.3\times 10^{24}\,V/m$) inside the nucleon, as greater than of Schwinger critical electric field and of parallel magnetic field around the monopoles for $e^+ - e^-$ pairs creation by Schwinger effect, makes possible of *one pair per nucleon* to be obtained. This pair supplies the charges balance (missing) making possible the quarks conversion ($u \to d$).

Thus, a new understanding of beta decay process is proposed, when a pair of boson $W^- - W^+$ is simultaneously created due of the Schwinger effect when the electrical field (nonabelian) is of maximum value, and near equally with $E_0 \le v.e.v. = E_{cr}^{W^\pm}$.

It was discovered that $v.e.v.$ is in fact the Schwinger critical field E_{cr} for a pair of $W^\pm$ production. This pair decays in beta-electrons which penetrate the condensate barrier by *quantum tunneling* due of the *phase slip* with $2\pi - \varphi$ and of a Φ_0 energy release.

Also, an ad-hoc *bias current* produces a spontaneous suppression of the superconducting order parameter, the model is proved for a free neutron decay. Equally, the same Schwinger pair-production rate is enhanced by a thermal Boltzmann factor in place of quantum tunneling, when this thermalization due of the incidence of an high thermal spike of a photon with valence nucleons destroys the superconductivity.

As a numerical application, is considered the case of ^{26}Al, through its β-decay to 1.809 MeV γ-ray, when at high temperatures ($T_9 = 0.42GK$) equilibrium is reached between ^{26gs}Al and ^{26m}Al which is relevant to some high temperature astrophysical events such as novae, this being proved by our model.

Thus, from the model based on *bias current* it results a necessary photon flux , and from the Schwinger model results the necessary temperature and the duration of the thermal spike; all these parameters can be obtained only by the laser from ELI project (Extreme Light Infrastructure) .

APPENDIX A

Various Abelian Projections

There is an infinite number of abelian projections. In the previous section we have considered the $\hat{F}_{12}$ abelian gauge. Instead of the diagonalization of the tensor component $\hat{F}_{12}$ by the gauge transformation, we can diagonalize any operator X which transforms under the gauge rotation as follows: $X \to \Omega^+ X \Omega$. Each operator X defines an abelian projection. At finite temperature one can consider the so-called Polyakov abelian gauge which is defined by the diagonalization of the Polyakov line.

The most interesting numerical results are those obtained in the Maximal Abelian (MaA) gauge. This gauge is defined by the maximization of the functional

$$\max_{\Omega} R[\hat{A}^{\Omega}], \quad R[\hat{A}] = -\int d^4x \left[\left(A_\mu^1\right)^2 + \left(A_\mu^2\right)^2 \right],$$

The condition of a local extremum of the functional R is

$$\left(\partial_\mu \pm igA_\mu^3\right)A_\mu^{\pm} = 0$$

Clearly, this condition (as well as the functional $R[A]$ is invariant under the $U(1)$ gauge transformations (7). The meaning of the MaA gauge is simple: by gauge transformations we make the field $\hat{A}_\mu$ as diagonal as possible.

From [9], which expresses the total amount of the off-diagonal gluon component, here, we have sed the Cartan decomposition,

$$A_\mu = A_\mu^a T^a = \hat{A}_\mu \cdot \vec{H} + \sum_{\alpha=1}^{N_c(N_c-1)} \left|A_\mu^\alpha\right|^2 ; \quad \vec{H} \equiv \left(T_3, T_8, \cdots, T_{N_c^2-1}\right)$$

is the Cartan subalgebra, and $E^\alpha \left(\alpha = 1,2,\cdots, N_c^2 - N_c\right)$ denotes the raising or lowering operator.

$(T^a = 1/2\sigma^a, a = 1,2,3)$, and again $A_\mu = \vec{A}_\mu(x) \cdot \vec{H} + A_\mu^\alpha E^\alpha$. Usually indicated by the Greek letter *sigma* (σ), they are occasionally denoted with a *tau* (τ) when used in connection with isospin symmetries. They are:

$$\sigma_1 = \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma_2 = \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \sigma_3 = \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

In the abelian gauge, the diagonal and the off-diagonal gluons play different roles in terms of the residual abelian gauge symmetry: the diagonal gluon behaves as the abelian gauge field, while off-diagonal gluons behave as charged matter fields. Under the $U(1)_3$ gauge transformation by $\omega = \exp(-i\varphi \frac{\tau_3}{2}) \in U(1)_3$, one finds

$$A_\mu^3 \to (A_\mu^\omega)^3 = A_\mu^3 + \frac{1}{e}\partial_\mu \varphi \qquad (15)$$

$$A_\mu^{\pm} \to (A_\mu^{\varpi})^{\pm} = A_\mu^{\pm} e^{\pm i\varphi} \qquad (16)$$

with $A_\mu^{\pm} = \frac{1}{\sqrt{2}}\left(A_\mu^1 \pm iA_\mu^2\right)$

The abelian projection is simply defined as the replacement of the gluon field $A_\mu = A_\mu^a \frac{\tau^a}{2} \in su(2)$ by the diagonal part $\mathrm{A}_\mu = A_\mu^3 \frac{\tau^3}{2} \in u(1)_3 \subset su(2)$.

The dual Ginzburg-Landau theory (DGL) theory [6,7], [10-11],,[12,13],,[14,15], is considered as an infrared effective theory of QCD in the abelian gauge, and is described by the diagonal gluon $\vec{A}_\mu = (A_\mu^3, A_\mu^8)$, $\vec{B}_\mu$ and χ_α denote the dual gauge field with two components (B_μ^3, B_μ^8) and the complex scalar monopole field, respectively. The label

$\alpha = 1,2,3$ corresponds to the color-electric charge, red(R), blue(B) and green(G). In the DGL theory, the self-interaction of the QCD-monopole field χ_α is introduced. At the quenched level, the color sources are given as the c-number current, and the heavy $q - \bar{q}$ system provides

$$\vec{j}_\mu^\alpha(x) = \vec{Q}_\alpha g^{\mu 0}\left[\delta^3(x - a) - \delta^3(x - b)\right],$$

where $\vec{Q}_\alpha \equiv e\vec{w}_\alpha$ is the abelian color-electric charge of the quark. Here, a and b are position vectors of the quark and the antiquark, respectively, and w_α is the weight vector of $SU(3)$ algebra, $w_1 = (1/2, \sqrt{3}/6), w_2 = (-1/2, \sqrt{3}/6), w_3 = (0, -1/\sqrt{3})$.

Hence the weights are

$$\left|w_1\right\rangle \equiv \begin{pmatrix}1\\0\\0\end{pmatrix} = \left|\left(\frac{1}{2}, \frac{1}{2\sqrt{3}}\right)\right\rangle, \quad \left|w_2\right\rangle \equiv \begin{pmatrix}0\\1\\0\end{pmatrix} = \left|\left(-\frac{1}{2}, \frac{1}{2\sqrt{3}}\right)\right\rangle,$$

$$\left|w_3\right\rangle \equiv \begin{pmatrix}0\\0\\1\end{pmatrix} = \left|\left(0, -\frac{1}{\sqrt{3}}\right)\right\rangle$$

where $\vec{B}_\mu$ and χ_α denote the dual gauge field with two components (B_μ^3, B_μ^8) and the complex scalar monopole field, respectively. The label $\alpha = 1,2,3$ corresponds to the color-electric charge, red(R), blue(B) and green(G). According to the Gauss law, one finds the color-electric field and then the dual gauge field $\vec{B}_\mu$ is proportional to the quark charge $\vec{Q}_\alpha$. For instance, when we consider the $R - \bar{R}$ system, the dual gauge field can be defined by using the weight vector as $\vec{B}_\mu = w_1 B_\mu^R$.

Finally, from [6], in $R - \bar{R}$ system, by introducing the QCD-monopole field $\chi_\alpha (\alpha = 1,2,3)$ and its coupling with the dual gauge fields $\vec{B}_\mu$, the effective dual Ginzburg-Landau Lagrangian :

$$L_{DGL} = -\frac{1}{4}(\partial_\mu B_\nu - \partial_\nu B_\mu)^2 + \left|(\partial_\mu + i\hat{g}B_\mu)\chi\right|^2 - \hat{\lambda}(\left|\chi\right|^2 - \hat{v}^2)^2 \qquad (A.1)$$

Now, it is known that the Abelian Higgs model is the Mexican-hat model coupled to electromagnetism

$$S(\phi, A) = \int \frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \left|(\partial - iqA)\phi\right|^2 + \lambda \cdot \left(\left|\phi\right|^2 - \Phi^2\right)^2 \qquad (A.2)$$

If $\phi(x) = \Phi\, e^{i\theta(x)}$ (A.3) with a constant prefactor, then the action for the field $\theta(x)$, i.e., the "phase" of the Higgs field $\Phi(x)$, has only derivative terms.

For the other two color-singlet cases such as the $B-\overline{B}$ and the $G-\overline{G}$ system, one obtains the same expression owing to the *Weyl* symmetry among three color charges, R, B and G. The lagrangian (A.1) has the $U(1)$ gauge symmetry and its form coincides with the Ginzburg-Landau theory for superconductivity. This type of *lagrangian* has the flux-tube solution such as the *Abrikosov vortex*.

The classical vacuum is again at the minimum of the potential, where the magnitude of the complex field φ is equal to Φ. But now the phase of the field is arbitrary, because gauge transformations change it. This means that the field $\theta(x)$ can be set to zero by a gauge transformation, and does not represent any actual degrees of freedom at all.

Furthermore, choosing a gauge where the phase of the vacuum is fixed, the potential energy for fluctuations of the vector field is nonzero. So, in the abelian Higgs model, the gauge field acquires a mass. To calculate the magnitude of the mass, consider a constant value of the vector potential A in the x direction in the gauge where the condensate has constant phase.
To see this solution, we consider the field equations,

We investigate the solution of the coupled equation of the abelian monopole field $|\chi_\alpha(x)|$ and the $\vec{B}_\mu(x)$ field at the tree level, which is analogous to the vortex solution in the superconductivity. The solutions for the color-electric field and the QCD-monopole field are given by functions of ρ, the distance from the cylindrical axis, and take forms as $E_{diag}(\rho) = E_3(\rho) \equiv E_8(\rho)$ and $|\chi(\rho)| \equiv |\chi_1(\rho)| \equiv |\chi_2(\rho)| \equiv |\chi_3(\rho)|$ in the flux tube.
To see this solution, from [9] result the field equations,

$$(\partial_\mu + i\hat{g}B_\mu)^2\chi = 2\hat{\lambda}_\chi(\hat{v}^2 - \chi * \chi), \quad \text{(A.4)}$$

$$\partial^\nu {}^*F_{\mu\nu} \equiv k_\mu = -i\hat{g}(\chi^*\partial_\mu\chi - \chi\partial_\mu\chi^*) + 2\hat{g}^{*2}B_\mu\chi^*\chi, \quad \text{(A.5)}$$

$${}^*F_{\mu\nu} \equiv \partial_\mu B_\nu - \partial_\nu B_\mu,$$

with the proper boundary conditions that quantize the color-electric flux. The flux is given by

$$\Phi = \int {}^*F_{\mu\nu}\, d\sigma^{\mu\nu} = \oint B_\mu(x)dx^\mu, \quad \text{(A.6)}$$

where $\sigma^{\mu\nu}$ is a two-dimensional surface element in the Minkowski space. By the polar decomposition of the monopole field using two scalar variables, ϕ and f as,

$$\chi(x) = \phi(x)e^{if(x)}$$

We obtain from Eq. (A.5)

$$B_\mu = \frac{1}{2\hat{g}^2}\frac{k_\mu}{\phi^2} - \frac{1}{\hat{g}}\partial_\mu f \quad \text{(A.7)}$$

In the Euclidean QCD, the MA gauge is defined so as to minimize the total amount of off-diagonal gluons, $R_{off} = \int d^4x \sum_{\mu,\alpha} \left| A_\mu^\alpha(x) \right|^2$, by the $SU(N_c)$ gauge transformation. Here, $A_\mu^\alpha(x)$ denotes the off-diagonal gluon in the Cartan decomposition, $A_\mu = \vec{A}_\mu(x)\cdot\vec{H} + A_\mu^\alpha E^\alpha$. In the MA gauge, by removing the off-diagonal gluons, QCD can be well approximated as an abelian gauge theory like the electro-magnetism keeping the essence of the infrared QCD properties. This approximation is called as abelian projection.

Owing to this remarkable feature of MA gauge fixing, the gluon field can be approximated to be abelian as $A_\mu^a(x)T^a \cong \vec{A}_\mu(x)\cdot\vec{H}$ for the argument on long-distance physics. Accordingly, the field equation of the abelian projected QCD becomes linear like the Maxwell equation,

$$\partial_\mu F^{\mu\nu} = j^\nu, \quad \partial_\mu \tilde{F}^{\mu\nu} = k^\nu, \qquad \text{(A.8)}$$

with the color-electric current j^μ and the color-magnetic current k^μ. Thus, the additivity on color-electromagnetic fields $F^{\mu\nu}$ works in the abelian projected QCD in the MA gauge. This is the most attractive point of the MA gauge.

APPENDIX B

The calculation of the gluon (monopole) mass

Before we construct these projection operators, let us consider the general momentum dependence of the Green's functions $G_{ab}(k)$. They are defined as inverse of the corresponding differential operator, here the Klein-Gordon operator $\Diamond + m^2$. Thus

$$-(\Diamond + m^2)G_{ab}(x,x') = \delta(x - x'). \qquad \text{(B.1)}$$

Because of translation invariance, the Green functions depend only on $x - x'$. Therefore it is convenient to perform a Fourier transformation F and to go to momentum space,

$$(k^2 - m^2)G_{ab}(k) = 1. \qquad \text{(B.2)}$$

(Remember $F(fg) = F(f)\cdot F(g)$ and $F(\delta) = 1$. Thus, the momentum dependence $G_{ab}(k)$ is independently of the spin given by

$$G_{ab}(k) = \frac{1}{k^2 - m^2 + i\varepsilon} \quad a,b = 1,\cdots,n, \qquad \text{(B.3)}$$

where we added Feynman's $i\varepsilon$ prescription for causal propagators

A vector field A_μ has four components in $D = 4$ space-time dimensions, while it has only $2s + 1 = 3$ independent spin components. Thus we have add one constraint equation to the four Klein-Gordon equations; the only linear, Lorentz invariant choice is

$$(\Diamond + m^2)A_\mu(x) = 0 \text{ and } \partial_\mu A^\mu = 0. \qquad \text{(B.4)}$$

In momentum space, $(k^2 - m^2)A^\mu(k) = 0$ and $k_\mu A^\mu(k) = 0$. In the rest frame of the particle,

$k_\mu = (m,0)$, the constraint becomes $A^0 = 0$. Hence there clearly only 3 components and we can choose the three polarization vectors e.g. as $\varepsilon_i \propto e_i$.

The two equations can be combined in

$$(g^{\mu\nu}\Diamond - \partial^{\mu}\partial^{\nu})A_{\nu} + m^2 A^{\mu} = 0. \qquad \text{(B.5)}$$

To show the equivalence of (5) and (4), act first with ∂_{μ} on it,

$$(\partial^{\nu}\Diamond - \Diamond\partial^{\nu})A_{\nu} + m^2\partial_{\mu}A^{\mu} = 0. \qquad \text{(B.6)}$$

Hence, (B.5) fulfils automatically the constraint $\partial_{\mu}A^{\mu} = 0$ for $m^2 > 0$. On the other hand, we can neglect the second term in (5) for $\partial_{\nu}A^{\nu} = 0$ and obtain the Klein-Gordon equation. The derivatives $\partial^{\mu} = (\partial^0;-\nabla)$, $\partial_{\mu} = (\partial^0;\nabla)\partial_{\mu}\partial^{\mu} = \partial_0^2 - \nabla^2 = \partial^{\mu}\partial_{\mu} = \Diamond$, ∇^2 is the Laplacian and $\Diamond = \left(\frac{\partial^2}{\partial t^2} - \nabla^2\right)$ is the d'Alembertian. Scalar, φ, and vector, $\vec{A}$, potentials are introduced via

$$\vec{E} = -\nabla\varphi - \partial\vec{A}/\partial t \quad , \vec{B} = \nabla\times\vec{A}$$

The four vector potential $A^{\mu} = (\varphi, \vec{A})$; $g_{\mu\nu}A^{\mu}A^{\nu} = \varphi^2 - \vec{A}^2$.The antisymmetric field-strength tensor $F^{\mu\nu} = \partial^{\mu}A^{\nu} - \partial^{\nu}A^{\mu}$ with components $F^{0i} = \partial^0 A^i - \partial^i A^0 = -E_i$ and $F^{ij} = \partial^i A^j - \partial^j A^i = -\varepsilon^{ijk}B^k$. The Levi-Civita symbol ε^{ijk} is antisymmetric under exchange of any two indices.

For a massive vector boson (spin 1) field the Proca equation

$$\Diamond A^{\nu} - \partial^{\nu}(\partial_{\mu}A^{\mu}) + m^2 A^{\nu} = j^{\nu} \qquad \text{(B.7)}$$

is obtained as a Euler-Lagrange eq. emerging from the Lagrangian

$$L = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m^2 A_{\mu}A^{\mu} - j_{\mu}A^{\mu} \qquad \text{(B.8)}$$

after expressing the field-strength tensor, $F^{\mu\nu}$, in terms of the four potential A^{μ}.
The Maxwell field is a massless $(m = 0)$ Proca field.
If the source current is conserved $(\partial_{\nu}j^{\nu} = 0)$ or if there are no surces $(j^{\nu} = 0)$ follows that $\partial_{\nu}A^{\nu} = 0$.
The field eq. gets simplified $(\Diamond + m^2)A^{\nu} = 0$ for free particles, leading to four Klein-Gordon eqs. for projections.

Propagator for spin-1 fields

The propagator for a spin-1 field is determined by

$$\left[g^{\mu\nu}(\Diamond + m^2) - \partial^{\mu}\partial^{\nu}\right]D_{\nu\lambda}(x) = \delta^{\mu}_{\lambda}\delta(x). \qquad \text{(B.9)}$$

Inserting the Fourier transformation of the propagator gives

$$\left[(-k^2 + m^2)g^{\mu\nu} + k^{\mu}k^{\nu}\right]D_{\nu\lambda}(k) = \delta^{\mu}_{\lambda} \qquad \text{(B.10)}$$

The tensor structure of $D_{\mu\nu}(k)$ has to be of the form

$$D_{\mu\nu}(k) = Ag_{\mu\nu} + Bk_{\mu}k_{\nu} \qquad \text{(B.11)}$$

with two unknown scalar functions A and B.

Factoring out $-k^2$ and inserting the above ansatz for $D_{\nu\lambda}(k)$, we obtain the propagator for a massive spin-1 field by adding the two terms produced by the $m^2 g_{\mu\nu}$ term to the case of Klein-Gordon equation $(m = 0)$ and introducing (B.11) in (B.10).

$$\left[(-k^2 + m^2)g^{\mu\nu} + k^\mu k^\nu / k^2\right]\left[Ag_{\nu\lambda} + Bk_\nu k_\lambda\right] = \delta^\mu_\lambda$$

$$-Ak^2\delta^\mu_\lambda + Ak^\mu k_\lambda + Am^2\delta^\mu_\lambda + Bm^2 k^\mu k_\lambda = \delta^\mu_\lambda$$

$$-A(k^2 - m^2)\delta^\mu_\lambda + (A + Bm^2)k^\mu k_\lambda = \delta^\mu_\lambda \qquad \text{(B.12)}$$

or $A = -1/(k^2 - m^2)$ and $B = -1A/m^2 = 1/\left[m^2(k^2 - m^2)\right]$. (B.13)

Thus the massive spin-1 propagator follows as

$$D_F^{\mu\nu}(k) = \frac{-g^{\mu\nu} + k^\mu k^\nu / m^2}{k^2 - m^2 + i\varepsilon}. \qquad \text{(B.14)}$$

Massive Vector Boson Propagator

In this section, we investigate the propagator of the massive vector boson in the Euclidean metric for the preparation of the analysis on the efective gluon mass in the MA gauge in the Euclidean lattice QCD. We start from the Lagrangian of the free massive vector boson with mass *M* in the Proca formalism following the works [21-24].

$$L = \frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)^2 + \frac{1}{2}M^2 A_\mu A_\mu \quad \text{(B.15)}$$

in the Euclidean metric. For the massive case of $M \neq 0$, the propagator $\tilde{G}_{\mu\nu}(k;M)$ of the massive vector boson is derived from eq.(B.14) , changing $m = M$, as:

$$\tilde{G}_{\mu\nu}(k;M) \equiv \frac{1}{k^2 + M^2}\left[\delta_{\mu\nu} + \frac{k_\mu k_\nu}{M^2}\right] \text{(B.16)}$$

in the momentum representation . The propagator $\tilde{G}_{\mu\nu}(x;M)$ in the coordinate space is obtained by performing the Fourier transformation as

$$G_{\mu\nu}(x;M) \equiv \langle A_\mu(x) A_\nu(0)\rangle =$$

$$\int \frac{d^4k}{(2\pi)^4} e^{ik\cdot x}\tilde{G}_{\mu\nu}(k;M) = \int \frac{d^4k}{(2\pi)^4} e^{ik\cdot x}\frac{1}{k^2 + M^2}\left[\delta_{\mu\nu} + \frac{k_\mu k_\nu}{M^2}\right] \quad \text{(B.17)}$$

Since $G_{\mu\nu}(r;M)$ depends only on the four-dimensional distance $r \equiv (x_\mu x_\mu)^{\frac{1}{2}}$, itis convenient to examine the scalar-type propagator,

$$G_{\mu\nu}(r;M) = \langle A_\mu(x) A_\mu(0)\rangle = \int \frac{d^4k}{(2\pi)^4} e^{ik\cdot x}\frac{1}{k^2 + M^2}\left[4 + \frac{k^2}{M^2}\right]$$

$$= \int \frac{d^4k}{(2\pi)^4} e^{ik\cdot x}\left[\frac{3}{k^2 + M^2} + \frac{1}{M^2}\right] = 3\int \frac{d^4k}{(2\pi)^4} e^{ik\cdot x}\frac{1}{k^2 + M^2} + \frac{1}{M^2}\delta^4(x) \qquad \text{(B.18)}$$

for the study of the interaction range. To carry out the calculation of $G_{\mu\nu}(r;M)$, we take $x_\mu = (r,0,0,0)$ without loss of generality, and then the integration in Eq.(B.18) is found to be expressed with $K_1(z)$

$$I(r) = \int \frac{d^4k}{(2\pi)^4} e^{ik\cdot x} \frac{1}{k^2+M^2} = \int \frac{d^3k}{(2\pi)^3}\left(\int_{-\infty}^{+\infty} \frac{dk_0}{2\pi} e^{ik_0 r} \frac{1}{k_0^2+k^2+M^2}\right) =$$
$$\int \frac{d^3k}{(2\pi)^3} \frac{1}{2\sqrt{k^2+M^2}} e^{-\sqrt{k^2+M^2}r} = \frac{1}{4\pi^2}\int_0^\infty dk \frac{k^2}{\sqrt{k^2+M^2}} e^{-\sqrt{k^2+M^2}r} = \qquad \text{(B.19)}$$
$$\frac{1}{4\pi^2}\int_M^\infty dE e^{-Er}\sqrt{E^2-M^2} = \frac{1}{4\pi^2}M^2\int_1^\infty d\varepsilon\, e^{-\varepsilon Mr}\sqrt{\varepsilon^2-1} =$$
$$\frac{1}{4\pi^2}\frac{M}{r}K_1(Mr)$$

using the integration formula for the modified Bessel function,

$$K_1(z) \equiv z\int_1^\infty dte^{-zt}\sqrt{t^2-1}, \ (\Re z > 0) \qquad \text{(B.20)}$$

Thus, the scalar-type propagator $G_{\mu\mu}(r;M)$ can be expressed as

$$G_{\mu\mu}(r;M) = \frac{1}{4\pi^2}\frac{M}{r}K_1(Mr) + \frac{1}{M^2}\delta^4(x) \qquad \text{(B.21)}$$

In the infrared region, the asymptotic expansion

$$K_1(Mr) \cong \sqrt{\frac{\pi}{2Mr}}e^{-Mr}\sum_{n=0}^{\infty}\frac{\Gamma(3/2+n)}{n!\Gamma(3/2-n)}\frac{1}{(2Mr)^n}$$

is applicable for large Mr,

$$G_{\mu\mu}^{ch}(r) \equiv \left\langle A_\mu^+(x)A_\mu^-(y)\right\rangle = \frac{1}{6}\sum_{a\neq 3,8}\langle A_\mu^a(x)A_\mu^a(y)\rangle$$

$$G_{\mu\mu}^{ch}(r) \equiv \left\langle A_\mu^+(x)A_\mu^-(y)\right\rangle \cong A_{poloidal}^2 = \frac{1}{6}\left\{\left\langle A_\mu^1(x)A_\mu^1(y)\right\rangle + \left\langle A_\mu^2(x) + A_\mu^2(y)\right\rangle\right\}$$

Here, in the Euclidean QCD, the MA gauge is defined by minimizing the global amount of the off-diagonal gluon, $R_{off} \equiv \int dx^4 tr\left\{\left[\hat{D}_\mu, \vec{H}\right]\left[\hat{D}_\mu, \vec{H}\right]^\perp\right\} = \frac{e^2}{2}\int d^4x \sum_{\alpha=1}^{N_c^2-N_c}\left|A_\mu^\alpha\right|^2$

Here, we have used the Cartan decomposition, $A_\mu(x) \equiv \vec{A}_\mu(x)\cdot\vec{H} + \sum_{\alpha=1}^{N_c^2-N_c} A_\mu^\alpha(x)E^\alpha$ and the covariant derivative operator $\hat{D}_\mu = \hat{\partial}_\mu + ieA_\mu$. In the MA gauge, off-diagonal gluon components are forced to be as small as possible, and then the gluon fieeld $A_\mu(x)$ mostly approaches to the abelian gauge field $\vec{A}_\mu(x)\cdot\vec{H}$.

For the infinitesimal gauge transformation $\Omega(x) \equiv e^{i\varepsilon(x)} \cong 1 + i\varepsilon(x)$ with $\varepsilon(x) \in su(N_c)$

From the extremum condition $\delta R_{off} \equiv R_{off} - R_{off}^\Omega = 0$ for the arbitrary infinitesimal variation $\varepsilon(x) \in su(N_c)$, we derive the local gauge-fixing condition in the MA gauge as

$\left[\vec{H},\left[\hat{D}_\mu,\left[\hat{D}_\mu,\vec{H}\right]\right]\right] = 0$,

which leads to

$\left(i\partial_\mu \pm eA^3_\mu\right)A^\pm_\mu = 0$ in $SU(2)$ QCD for the $N_c = 2$ case. Thus, the operator Φ to be diagonalized in the MA gauge is found to be

$\Phi\left[A_\mu\right] = \left[\hat{D}_\mu,\left[\hat{D}_\mu,\vec{H}\right]\right]$ (B.26)

in the continuum theory.

Next, we calculate the scalar-type propagator $G_{\mu\mu}(r)$ as the function of the fourdimensional distance $r = \sqrt{(x_\mu - y_\mu)^2}$ in the MA gauge with the U(1)*3*-Landau gauge.

In this gauge, $U_\mu(s)$ is determined without the ambiguity on local gauge transformation. Here, we study the scalar-type propagator of the diagonal (neutral) gluon as

$G^{Abel}_\mu(r) = \left\langle A^3_\mu(x)A^3_\mu(y)\right\rangle$ and that of the off-diagonal (charged) gluon as

$G^{ch}_{\mu\mu}(r) \equiv \left\langle A^+_\mu A^-_\mu(y)\right\rangle = \frac{1}{2}\left\{\left\langle A^1_\mu(x)A^1_\mu(y)\right\rangle + \left\langle A^2_\mu(x) + A^2_\mu(y)\right\rangle\right\}$

with the charged gluons $A^\pm_\mu = \frac{1}{\sqrt{2}}\left\{A^1_\mu(x) \pm iA^2_\mu(x)\right\}$. In $G^{ch}_{\mu\mu}(r)$, the imaginary part, $-\frac{i}{2}\left\{A^1_\mu(x)A^2_\mu(y) - A^2_\mu(x)A^1_\mu(y)\right\}$, disappears automatically due to the symmetry

$\left\langle A^1_\mu(x)A^2_\mu(y)\right\rangle = \left\langle A^2_\mu(x)A^1_\mu(y)\right\rangle$

and $\left(i\partial_\mu \pm eA^3_\mu\right)A^\pm_\mu = 0$

Therefore, the eq.(B.21) reduces to

$G^{ch}_{\mu\mu}(r;M) \cong \frac{3M}{2r(2\pi)^{3/2}}\frac{e^{-Mr}}{(Mr)^{1/2}}$ (B.22)

or,

$rG^{ch}_{\mu\mu}(r) = A^2_{poloidal}x\varepsilon_0 c^2 = (Bx)^2 x\varepsilon_0 c^2[J] \quad , A\left[\frac{J}{Am}\right]$

, where $r = x = 2.e-16[m]$, and the "poloidal" nonabelian electric field it could be expressed also as $A(r) = \frac{\mu_0}{4\pi}\frac{m\times\vec{r}}{r^3}$

or of the *Landau gauge* $A(x) = Bx\left[\frac{N}{A}\right]; B = 2.6e15\left[\frac{J}{Am^2}\right] from(46)$,

Therefore, (B.22) becomes:

$2/3\,rG_{\mu\mu}(r;M)(2\pi)^{3/2} \cong M\frac{e^{-Mr}}{(Mr)^{1/2}}$

At the distance $r \equiv \sqrt{(x_\mu - y_\mu)^2} \cong 2\lambda \cong 0.2\,fm$, results by trials:

$$6.56e-10[J] \cong M \frac{1}{(Mr)^{1/2}} \exp\left(-\frac{Mr}{\hbar c}\right) =$$

$$M \frac{1}{0.51} \exp\left(-\frac{3.7e-10 \times 2.e-16}{1.e-34 \times 2.997e8}\right) \approx 1.8M \to M \cong 3.6e-10[J] \Rightarrow 2.25[GeV]$$

, and $r_M = \frac{1.e-34 \times 2.997e8}{3.6e-10} \approx 0.1 fm \approx \lambda$, just as it was supposed before.

where the Yukawa-type damping factor e^{-Mr} expresses the short-range interaction in the coordinate space.

In the lattice calculation [6], the mass M of the vector field $A_\mu(x)$ is estimated from the slope in the logarithmic plot of $\frac{r^{3/2}}{\sqrt{M}} G_{\mu\mu}(r;M)$ as the function of r ,

$$\ln\left\{\frac{r^{3/2}}{\sqrt{M}} G_{\mu\mu}(r;M)\right\} \cong -Mr + const. \qquad \text{(B.23)}$$

Next, we calculate the scalar-type propagator $G_{\mu\mu}(r)$ as the function of the fourdimensional distance $r = \sqrt{(x_\mu - y_\mu)^2}$ in the MA gauge with the U(1)*3*-Landau gauge.

In this gauge, $U_\mu(s)$ is determined without the ambiguity on local gauge transformation. Here, we study the scalar-type propagator of the diagonal (neutral) gluon as $G_\mu^{Abel}(r) = \left\langle A_\mu^3(x) A_\mu^3(y)\right\rangle$ and that of the off-diagonal (charged) gluon as

$$G_{\mu\mu}^{ch}(r) \equiv \left\langle A_\mu^+ A_\mu^-(y)\right\rangle = \frac{1}{2}\left\{\left\langle A_\mu^1(x) A_\mu^1(y)\right\rangle + \left\langle A_\mu^2(x) + A_\mu^2(y)\right\rangle\right\}$$

with the charged gluons $A_\mu^\pm = \frac{1}{\sqrt{2}}\left\{A_\mu^1(x) \pm iA_\mu^2(x)\right\}$. In $G_{\mu\mu}^{ch}(r)$, the imaginary part, $-\frac{i}{2}\left\{A_\mu^1(x)A_\mu^2(y) - A_\mu^2(x)A_\mu^1(y)\right\}$, disappears automatically due to the symmetry $\left\langle A_\mu^1(x)A_\mu^2(y)\right\rangle = \left\langle A_\mu^2(x)A_\mu^1(y)\right\rangle$

and $\left(i\partial_\mu \pm eA_\mu^3\right)A_\mu^\pm = 0$

As shown in Fig.1 of ref. [22, the diagonal-gluon propagator $G_{\mu\mu}^{Abel}(r)$ and the charged-gluon propagator $G_{\mu\mu}^{ch}(r)$ manifestly differ in the MA gauge. The off-diagonal (charged) gluon $A_\mu^\pm$ seems to propagate only within the short range as $r \le 0.4 fm$, while the diagonal gluon A_μ^3 propagates over the long distance. Thus, we find the `infrared abelian dominance' for the gluon propagator in the MA gauge.

Now, let us examine the hypothesis on the mass generation of charged gluons in the MA gauge. To this end, we investigate the logarithm plot of $r^{3/2}G_{\mu\mu}^{ch}(r)$ as the

function of r, since the massive vector boson with the mass M behaves like
$G^{ch}_{\mu\mu}(r) \approx \frac{\exp(-Mr)}{r^{3/2}}$ as shown in (B.22). In Fig.2 of ref. [22], the charged gluon correlation $\ln\{r^{3/2}G^{ch}_{\mu\mu}(r)\}[GeV]^{1/2}$
decreases linearly in the region of $0.2 \le r \le 1 fm$. Hence, $G^{ch}_{\mu\mu}(r)$ behaves as the Yukawa-type function

$$G^{ch}_{\mu\mu}(r) \approx \frac{\exp(-M_{ch}r)}{r^{3/2}}, \quad (B.24)$$

in the MA gauge. From the slope of the straight line in Fig.2 ref. [6], it was estimated the effective mass M_{ch} of the charged gluon as

$$M_{ch} \approx 4.7 fm^{-1} \cong 0.94 GeV \quad (B.25)$$

Following [6], this charged-gluon effective mass M_{ch} is considered to be induced by the MA gauge fixing. Due to the effective mass $M_{ch} \cong 1GeV$, the charged gluon propagation is restricted within about $M^{-1}_{ch} = 0.2 fm$. Then, in the infrared region as $r >> 0.2 fm$, the charged gluons $A^{\pm}_{\mu\mu}$ cannot contribute, and only the diagonal gluon A^{3}_{μ} can contribute to the long-range physics in the MA gauge. In conclusion, the effective-mass generation of the charged gluon in the MA gauge is considered as the physical origin of the abelian dominance in the infrared region.
Then, the origin of the infrared abelian dominance has been physically explained as the generation of the charged gluon mass M_{ch} induced by the MA gauge fixing. On the other hand, in the MA gauge, the charged gluon effects become negligible and the system can be described only by the diagonal gluon component at the long distance as $r >> M^{-1}_{ch} \cong 0.2 fm$. For the short distance as $r \le M^{-1}_{ch} \cong 0.2 fm$, the effect of charged gluons appears, and hence all the gluon components have to be considered even in the MA gauge.
As is said in [6], "to see the difficulty on the nonabelian property of QCD, let us consider the simple electro-magnetic system. In the ordinary electro-magnetism, we can individually consider the partial electro-magnetic field formed by each charge, and the total electro-magnetic field can be obtained by adding these individual solutions. Here, additivity of the solution plays the key role, and this additivity originates from the linearity of the field equation, $\partial_{\mu} F^{\mu\nu} = j^{\nu}$, in the electro-magnetism".
The nonabelian nature is one of the characteristic features of QCD. However, by taking the maximally abelian (MA) gauge in QCD, one can make the nonabelian (off-diagonal) ingredients of QCD inactive for the infrared QCD properties such as quark confinement and chiral-symmetry breaking. They [6], [22,23] call these phenomena as the infrared abelian dominance in the MA gauge.

The structure of the color-magnetic monopole

There remains large off-diagonal gluon component near the monopole center
Here, even in the MA gauge, where the off-diagonal gluon element is strongly suppressed, around monopoles, there remains large off-diagonal gluon component. This

off-diagonal-gluon-rich region around the monopole provides an "intrinsic size" and the structure of the monopole as shown in Fig.3(b) from ref. [6], like the 't Hooft-Polyakov monopole, at a large scale where this structure becomes invisible, QCD-monopoles can be regarded as point-like Dirac monopoles.
Here,

$m_B = \sqrt{3}gv$ and $m_\chi = 2\sqrt{\lambda}\, v$ (B.26)

are the masses of the dual gauge field B_μ and the monopole field $[\chi_\alpha] = v(\alpha = 1,2,3)$, The label $\alpha = 1,2,3$ corresponds to the color-electric charge, red(R), blue(B) and green(G), as from [16].
We consider the case of $m_\chi > m_B$ corresponding to the type II superconductor. The color-electric field $E_{diag}(\rho)$ takes a large value only in a region of $\rho \le m_B^{-1}$, and therefore the cylindrical radius of the hadron flux tube is roughly given by m_B^{-1} . One finds the reduction of the QCD-monopole condensate $|\chi(\rho)|$ in the central region of $\rho \le m_\chi^{-1}$ in the flux tube. The QCD-monopole condensate is regarded as an almost constant value v, $|\chi(\rho)| \cong v$, for the infrared region $\rho \ge m_\chi^{-1}$. On the contrary, the QCD-monopole condensate almost disappear, $|\chi(\rho) \cong 0|$ for the ultraviolet region $\rho \le m_\chi^{-1}$ and therefore the ultraviolet cutoff appears in m_B.
These inverse masses , $m_\chi^{-1} = 0.12 fm$ and $m_B^{-1} = 0.39 fm$ are regarded as the coherent length of the monopole field and the penetration depth of the color-electric field, respectively.
The ratio of these two lengths gives the Ginzburg-Landau (GL) parameter,

$$\kappa = \frac{m_B^{-1}}{m_\chi^{-1}} = \frac{\sqrt{2\hat{\lambda}}}{\hat{g}} \qquad \text{(B.27)}$$

In [6] are used: $\lambda = 2\hat{\lambda}$, $B_\mu^R = \sqrt{3}B_\mu$, $\chi^R = \chi$, $g \equiv \frac{2}{\sqrt{3}}\hat{g}$, $v \equiv \frac{1}{\sqrt{2}}\hat{v}$

$g = 2.9; e = 4\pi/g = 4.3; \lambda = 66; v = 0.098 GeV$, results $m_\chi = 1.6 GeV$, and $m_B = 0.5 GeV$
In the MA gauge, the off-diagonal gluon contribution can be neglected and monopole condensation occurs at the infrared scale of QCD. Therefore, the QCD vacuum in the MA gauge can be regarded as the dual superconductor described by the DGL theory, and quark confinement can be understood with the dual Meissner effect.
APPENDIX C
Alternative ways for monopole mass calculation:
The derivation of the Rashba Hamiltonian

The Rashba effect [36] is a direct result of inversion symmetry breaking in the direction perpendicular to the two-dimensional plane. Therefore, let us add to the Hamiltonian a term that breaks this symmetry in the form of an electric field

$H_E = -E_0 z$ (C.29)

Due to relativistic corrections an electron moving with velocity v in the electric field will experience an effective magnetic field B

$B = (v \times E)/c^2$ (C.30)

This magnetic field couples to the electron spin [36]

$H_{SO} = \frac{\mu_B}{2c^2}(v \times E)\cdot \sigma$ (C.31)

, where the factor 1/2 is a result of the Thomas precession.

The Rashba field E_R exists at the interface and creates the monopole current j_m near the interface. The width of the monopole current distribution, d, is comparable to the decay length of the magnetization at the interface. The monopole current induces the electric current j via Ampe`re's law at the interface.

Within this toy model, the Rashba Hamiltonian is given by

$H_R = \alpha_R(\sigma \times p_T)\cdot \hat{z}$ (C.32)

where $\alpha_R = \frac{\mu_B E_0}{2mc^2}$, $\sigma = 1$ from Pauli matrix, p_T transverse magnetic moment of the monopole (gluon around); E_0 -the electric field induced by an quarks pair $q\overline{q}$; m -the mass of the monopole. With $p_T \cdot c = 0.7[GeV]$ as calculated below, and with $E_0 = 8.33e24[N/C]$, result: $\alpha_R = 3.2e9$, and $H_R = 1.2e-09[J]$, respectively.

The magnetic moment of the electron is

$m_s = -\frac{g_s \mu_B S}{\hbar}$ (C.33)

where

$\mu_B = 9.27 \times 10^{-24}[JT^{-1}]$, μ_B is the Bohr magneton, $S = \hbar/2$ is electron spin, and the g-factor g_s is 2 according to Dirac's theory, but due to quantum electrodynamic effects it is slightly larger in reality: 2.002, for a muon $g = 2$.

The Bohr magneton is defined in Si units by

$\mu_B = \frac{e\hbar}{2m_e}$ (C.34)

The vector potential of magnetic field produced by magnetic moment m_{Mo} is

$A(r) = \frac{\mu_0}{4\pi}\frac{m_{Mo} \times \vec{r}}{r^3}$ (C.35)

and magnetic flux density is

$B(r) = \nabla \times A = \frac{\mu_0}{4\pi}\left(\frac{3\vec{r}(m_{Mo}\cdot \vec{r})}{r^5} - \frac{m_{Mo}}{r^3}\right) \cong A/r$ (C.36)

$j_e = -\frac{c}{4\pi}\frac{dB}{dr}(4\pi c\varepsilon_0) = -\frac{c^2 \varepsilon_0 A}{r^2}$

(C.37)

We can calculate the observable spin magnetic moment (a vector), $\vec{\mu}_S$, for a sub-atomic particle with charge q, mass m, and spin angular momentum (also a vector), $\vec{s}$, via:

$\vec{\mu}_S = g\frac{q}{2m}\vec{s}$

Therefore, for a monopole $\mu_{Mo} = \frac{gQe\hbar}{2m_{Mo}}$, $Q = 68.5$, $\vec{s} = \hbar$, $g \cong 2$ (C.38)

And $m_{Mo} = -\frac{\mu_{Mo}S}{\hbar} \cong \mu_{Mo}$ (C.39)

Numerically, results: $\mu_{Mo} = 2.7e-25[J/T]$, $A = 0.6[N/A]$,
$B = A/r = 2.86e15[J/Am^2]$, (C.40)
if we consider $r = 0.2 fm$ results
$H_{So} = 3.9e-10[J] \rightarrow M_{monopole} = 2.45[GeV]$, and the electric current is $j_e = 1.2e7[A/fm^2]$ which is comparable with the magnetic current $j_\varphi = 1.15e7[A/fm^2]$, as from eq. (35.2).
The Rashba interaction contributes to the DC monopole current at the interface [36].

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