The Galactic Black Hole

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Abstract

Many galaxies have a concentration of mass at their center. In what follows, the mass is attributed to a neutral gas of electrons and positrons. It is found that electron degeneracy pressure supports the smaller masses against gravity. The larger masses are supported by ideal gas and radiation pressure. Physical properties are calculated for the range 450 to 45 billion solar masses.

1. Introduction

It is now well-established that many galaxies harbor a concentration of mass at their center. It is thought that similar concentrations exist in the nuclei of active galaxies and quasars. The question arises as to the physical nature of these masses. In the following model, electrons and positrons form a neutral gas which is confined by gravitation. Such objects would have formed during the Big Bang, when positrons were plentiful.

The paper begins with a calculation of electron degeneracy pressure. When applied to the electron-positron gas, it is found that the radius decreases with increasing mass, as is the case with white dwarf stars [1]. This process continues until the Schwarzschild radius $R_s = 2GM/c^2$ is reached. This occurs at about $10^6\,\mathrm{M}_\odot$. It is postulated that the radius can be no smaller than R_s. With further additions of mass, R_s increases. In these supermassive objects, degeneracy pressure gives way to ideal gas and radiation pressure.

2. The electron-positron gas

Under the assumption of spherical symmetry, the gravitational field equation and the equation of hydrostatic equilibrium are given by

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d\psi}{dr}\right) = 4\pi G\rho \tag{1}$$

and

$$\frac{dP}{dr} = -\rho\,\frac{d\psi}{dr} \tag{2}$$

With the additional assumption of constant density, they yield the pressure formula

$$P = \frac{3GM^2}{8\pi R^4}\left(1 - \frac{r^2}{R^2}\right) \tag{3}$$

At $r = 0$, the pressure is

$$P_c = \frac{3GM^2}{8\pi R^4} \tag{4}$$

This equation will be used to determine the physical conditions at the center.

2.1 Electron degeneracy pressure

For a completely degenerate ($T = 0$) electron gas, the Fermi energy is

$$\epsilon_F = (3\pi^2)^{2/3} \frac{\hbar^2}{2m} \left(\frac{N^-}{V}\right)^{2/3} \tag{5}$$

where N^- is the number of electrons [2, 3]. The condition of neutrality is

$$N^- = N^+ = \frac{1}{2}N \tag{6}$$

where N is the number of leptons. Therefore, the Fermi energy for both electrons and positrons is

$$\epsilon_F = \left(\frac{3\pi^2}{2}\right)^{2/3} \frac{\hbar^2}{2m} \left(\frac{N}{V}\right)^{2/3} \tag{7}$$

The pressure of each gas is

$$P^- = P^+ = \left(\frac{3\pi^2}{2}\right)^{2/3} \frac{\hbar^2}{10m} \left(\frac{N}{V}\right)^{5/3} \tag{8}$$

so that the total degeneracy pressure is

$$P_F = \left(\frac{3\pi^2}{2}\right)^{2/3} \frac{\hbar^2}{5m} \left(\frac{N}{V}\right)^{5/3} \tag{9}$$

Substitution into (4) gives

$$\left(\frac{3\pi^2}{2}\right)^{2/3} \frac{\hbar^2}{5m} \left(\frac{N}{V}\right)^{5/3} = \frac{3GM^2}{8\pi R^4} \tag{10}$$

Set $M = Nm$ and $V = 4\pi R^3/3$ then rearrange to find

$$R = \frac{2}{5}\left(\frac{9\pi}{8}\right)^{2/3} \frac{\hbar^2}{Gm^3} N^{-1/3} \tag{11}$$

Clearly, the radius decreases as N increases. Solutions for these intermediate masses are as follows:

Table 1: Intermediate mass					
N	n	M	$R > R_s$	P_c	ϵ_F
	(cm^{-3})	(M$_\odot$)	(cm)	(Pa)	(eV)
10^{63}	$3(10^{22})$	450	$2(10^{13})$	$4(10^{9})$	2.14
10^{64}	$3(10^{24})$	$4.5(10^{3})$	$9(10^{12})$	$8(10^{12})$	45.8
10^{65}	$3(10^{26})$	$4.5(10^{4})$	$4(10^{12})$	$2(10^{16})$	990
10^{66}	$3(10^{28})$	$4.5(10^{5})$	$2(10^{12})$	$4(10^{19})$	$2.1(10^{4})$
$8(10^{66})$	$2(10^{30})$	$3.5(10^{6})$	(10^{12})	$3(10^{22})$	$3.2(10^{5})$

In the final line, $R = R_s$.

2.2 Ideal gas and radiation pressure

In objects with $R = R_s$, the ordered kinetic energy of the quantum gas is replaced by the disordered thermal energy of an ideal gas. The electromagnetic radiation is trapped by gravity, and the temperature rises dramatically. With the ideal gas law and the Stefan-Boltzmann law for radiation pressure, equation (4) becomes

$$P_g + P_r = \frac{N}{V}kT + \frac{\pi^2}{45}\frac{(kT)^4}{(\hbar c)^3} = \frac{3GM^2}{8\pi R^4} \tag{12}$$

Set $R = 2GM/c^2$ and rearrange to obtain

$$N^2\left(\frac{kT}{mc^2}\right)^4 = \frac{135}{128\pi^3}\left(\frac{\hbar c}{Gm^2}\right)^3\left(1 - \frac{4kT}{mc^2}\right) \tag{13}$$

Solutions are as follows:

Table 2: Supermassive					
N	n	M	$R = R_s$	P_c	kT
	(cm^{-3})	(M$_\odot$)	(cm)	(Pa)	(eV)
10^{67}	$9.7(10^{29})$	$4.5(10^{6})$	$1.35(10^{12})$	$2(10^{22})$	$1.2(10^{5})$
10^{68}	$9.7(10^{27})$	$4.5(10^{7})$	$1.35(10^{13})$	$2(10^{20})$	$6.7(10^{4})$
10^{69}	$9.7(10^{25})$	$4.5(10^{8})$	$1.35(10^{14})$	$2(10^{18})$	$2.4(10^{4})$
10^{70}	$9.7(10^{23})$	$4.5(10^{9})$	$1.35(10^{15})$	$2(10^{16})$	$8.0(10^{3})$
10^{71}	$9.7(10^{21})$	$4.5(10^{10})$	$1.35(10^{16})$	$2(10^{14})$	$2.5(10^{3})$

With increasing mass, the pressure, density, and temperature all diminish. The ratio of ideal gas to radiation pressure is given by

$$\frac{P_g}{P_r} = \frac{45}{\pi^2}\left(\frac{\hbar c}{kT}\right)^3 \frac{N}{V} \tag{14}$$

Because of the high density, ideal gas pressure dominates near the transition.

3. Concluding remarks

The gravitational force at the Schwarzschild surface scales as $1/R$. It becomes quite feeble in the largest masses. An ambient disturbance could remove matter from the surface. It would then become more compact and stable. In contrast, the lightest intermediate masses have a vanishingly small Fermi energy. Ambient conditions could disrupt the equilibrium, to the point that the mass disappears altogether. This may have occurred in those galaxies which have little or no central mass.

In the transition region, 10^6 to 10^7 $M_\odot$, the temperature appears to change abruptly. However, the observed masses seem to form a continuum. A more detailed treatment is needed, involving a mixture of partially degenerate, ideal gas, and radiative contributions.

This paper ignores many issues, such as the variation of density with pressure, the attenuation of waves below the plasma frequency, and the large gravitational redshift as radiation moves upward from the center. Nevertheless, the work demonstrates the feasibility of the electron-positron gas, showing that it is able to account for the galactic central mass over a very wide range of values.

References

[1] B. Carroll and D. Ostlie, *An Introduction to Modern Astrophysics* (Addison-Wesley, 2nd ed., 2007) chap. 16.
[2] L. Landau and E. Lifschitz, *Statistical Physics* Part 1 (Pergamon, 3rd ed., 1980) sect. 57.
[3] R. Pathria and P. Beale, *Statistical Mechanics* (Elsevier, 3rd ed., 2011) chap. 8.