ON SMARANDACHE SIMPLE CONTINUED FRACTIONS

Charles Ashbacher[1]

Charles Ashbacher Technologies, Box 294
119 Northwood Drive, Hiawatha, IA 52233, USA

and

Maohua Le[1]

Department of Mathematics, Zhanjiang Normal College
Zhanjiang, Guangdong, P.R.China.

Abstract. Let $A=\{a_n\}_{n=1}^{\infty}$ be a Smarandache type sequence. In this paper we show that if A is a positive integer sequence, then the simple continued fraction $[a_1, a_2, \ldots]$ is convergent.

Let $A=\{a_n\}_{n=1}^{\infty}$ be a Smarandache type sequence. Then The simple continued frction

$$(1) \qquad a_1 + \cfrac{1}{a_2 + \cfrac{1}{a_3 + \cfrac{1}{\cdots\cdots}}}$$

is called the Smarandache simple continued fraction associated A (See [1]). By the usually symbol (see [2, Notion 10.1]), the continued frction (1) can be written as $[a_1, a_2, a_3, \ldots]$. Recently, Castillo [1] posed the following guestion:

Question. Is the continued fraction (1) convergent? In particular, is the continued fraction [1, 12, 123, ...] convergent?

In this paper we give a positive answer as follows.

Theorem. If A is a positive integer sequence, then the

[1]Editor's Note (M.L.Perez): This article has been done by each of the above authors independently.

continued fraction (1) is convergent.

Proof. If A is a positive integer sequence, then (1) is a usually simple continued fraction and its quotient are positive integers. Therefore, by [2,Theorem165], it is convergent. The Theorem is proved.

On applying [2, Theorems 165 and 176], we get a further result immediately.

Theorem 2. If A is an infinite positive integer sequence, then (1) is equal to an irrational number α. Further, if A is not periodic, then α is not an algebraic number of degree two.

References

1. J.Castillo, Smarandache continued fractions, Smarandache Notions J., to appear. Vol.9,No.1-2,40-42,1998.
2. G.H.Hardy and E.M.Wright, An Introduction to the Theory of Numbers, Oxford Univ. Press, Oxford, 1938.