

THE EQUATION $S(1.2)+S(2.3)+\cdots+S(n(n+1))=S(n(n+1)(n+2)/3)$

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Abstract. For any positive integer a , let $S(a)$ be the Smarandache function of a . In this paper we prove that the title equation has only the solution $n=1$.

Key words: Smarandache function, diophantine equation

Let \mathbf{N} be the set of all positive integers. For any positive integer a , let $S(a)$ be the Smarandache function of a . Recently, Bencze [1] proposed the following problem:

Problem Solve the equation

$$(1) \quad S(1 \cdot 2) + S(2 \cdot 3) + \cdots + S(n(n+1)) = S\left(\frac{1}{3}n(n+1)(n+2)\right), n \in \mathbf{N}.$$

In this paper we completely solve the above-mentioned problem as follows.

Theorem The equation (1) has only the solution $n=1$.

Proof By the definition of the Smarandache function (see [2]), we have $S(1)=1$, $S(2)=2$ and

$$(2) \quad S(a) \geq 3, a \geq 3.$$

Since $S(1.2)=S(1.2.3/3)=S(2)$, the equation (1) has a solution $n=1$.

Let n be a solution of (1) with $n > 1$. Then, by (2), we get

$$(3) \quad S(1 \cdot 2) + S(2 \cdot 3) + \cdots + S(n(n+1)) \geq 2 + 3(n-1) = 3n - 1.$$

Therefore, by (1) and (3), we obtain

$$(4) \quad S\left(\frac{1}{3}n(n+1)(n+2)\right) \geq 3n-1.$$

On the other hand, since $(n+2)! = 1 \cdot 2 \cdots n(n+1)(n+2)$, we get

$$(5) \quad \frac{1}{3}n(n+1)(n+2) \mid (n+2)!.$$

We see from (5) that

$$(6) \quad S\left(\frac{1}{3}n(n+1)(n+2)\right) \leq n+2.$$

The combination of (4) and (6) yields

$$(7) \quad n+2 \geq 3n-1,$$

whence we get $n \leq 3/2 < 2$. Since $n \geq 2$, it is impossible. Thus, (1) has no solutions n with $n > 1$. The theorem is proved.

References

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- [2] F. Smarandache, A function in number theory, Ann. Univ. Timisoara XVIII, 1980.

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