THE MODULE PERIODICITY OF SMARANDACHE CONCATENATED ODD SEQUENCE

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Abstract

In this paper we prove that the residue sequence of Smarandache concatenated odd sequence mod 3 is periodical.

Let p be a prime. For any integer a, let $<a>_p$ denote the least nonnegative residue of a mod p. Furter, for an integer sequence

$A = \{a(n)\}_{n=1}^{\infty}$, the sequence $\{<a(n)>_p\}_{n=1}^{\infty}$ is called the residue sequence of A mod p, and denoted by $<A>_p$.

In [1], Marimutha defined the Smarandache concatenated odd

sequence $S=\{s(n)\}_{n=1}^{\infty}$, where

(1) s(1)=1, s(2)=13, s(3)=135 , s(4)=1357, ...

In this paper we discuss the periodicity of $<S>_p$. Clearly, if p=2 or 5, then the residue sequence $<S>_p$ is periodical. We now prove the following result:

Theorem. If p=3, then $<S>_p$ is periodical.

Prof. For ahy positive integer k, we have $10^k \equiv 1 \pmod 3$. Hence, we see from (1) that

(2) $s(n) \equiv 1+3+5+...+(2n-1) = n^2 \pmod 3$.

Since

(3) $$<n^2>_3 = \begin{cases} 0, \text{ if } n \equiv 0 \pmod 3; \\ 1, \text{ if } n \equiv 1 \text{ or } 2 \pmod 3, \end{cases}$$

we find from (2) and (3) that

$$(4) \qquad \langle s(n)\rangle_3 = \begin{cases} 0, & \text{if } n \equiv 0 \pmod 3; \\ 1, & \text{if } n \equiv 1 \text{ or } 2 \pmod 3, \end{cases}$$

Thus, by (4), the sequence $\langle S\rangle_3 = \{\langle s(n)_3\rangle\}_{n=1}^{\infty}$ is periodical.
The theorem is proved.

Finally, we pose the following

Question. Is the residue sequence $\langle S\rangle_p$ periodical for every odd prime p?

Reference:

1. H. Marimutha, "Smarandache concatenate type sequences", Bulletin of Pure and Applied Sciences, 16E (1997), No. 2, 225 - 226.