ON SMARANDACHE GENERAL CONTINUED FRACTIONS

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Abstract. Let $A=\{a_n\}_{n=1}^{\infty}$ and $B=\{b_n\}_{n=1}^{\infty}$ be two Smarandache type sequences. In this paper we prove that if $a_{n+1} \geq b_n > 0$ and $b_{n+1} \geq b_n$ for any positive integer n, the continued fraction

$$(2) \qquad a_1 + \frac{b_1}{a_2 +} \; \frac{b_2}{a_3 +} \; \cdots$$

is convergent.

Let $A=\{a_n\}_{n=1}^{\infty}$ and $B=\{b_n\}_{n=1}^{\infty}$ be two Smarandache type sequences. Then the continued fraction

$$(1) \qquad a_1 + \cfrac{b_1}{a_2 + \cfrac{b_2}{a_3 + \cfrac{b_3}{\cdots}}}$$

is called a Smarandache general continued fraction associated with A and B (see [1]). By using Roger's symbol, the continued fraction (1) can be written as

$$(2) \qquad a_1 + \frac{b_1}{a_2 +} \; \frac{b_2}{a_3 +} \; \cdots .$$

Recently, Castillo [1] posed the following question:

Question. Is the continued fractions $1 + \frac{1}{12} \; \frac{21}{+\,123} \; \frac{321}{+\,1234} + \cdots$

convergent?

In this paper we prove a general result as follows.

Theorem. If $a_{n+1} \geq b_n > 0$ and $b_{n+1} \geq b_n$ for any positive integer n, then the continued fraction (2) is convergent.

Proof. It is a well known fact that (2) is equal to the simple continued fraction

$$a_1 + \frac{1}{c_1 +} \frac{1}{c_2 + \dots}, \tag{2}$$

where

$$c_{2t-1} = \frac{b_2 b_4 \dots b_{2t-2}}{b_1 b_3 \dots b_{2t-1}} a_{2t}, \tag{4}$$

$$c_{2t} = \frac{b_1 b_3 \dots b_{2t-1}}{b_2 b_4 \dots b_{2t}} a_{2t+1}, \qquad t = 1,2,\dots.$$

Since $a_{n+1} \geq b_n > 0$ and $b_{n+1} \geq b_n$ for any positive n, we see from (4) that $c_n \geq 1$ for any n. It implees that the simple continued fraction (3) is convergent. Thus, the Smarandache general continued fraction (2) is convergent too. The theorem is proved.

Reference

1. J.Castillo, Smarandache continued fractions, Smarandache Notions J.,Vol.9,No.1-2,40-42,1998.