THE POWERS IN THE SMARANDACHE CUBIC PRODUCT SEQUENCES

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Abstract. Let P and Q denote the Smarandache cubic product sequences of the first kind and the second kind respectively. In this paper we prove that P contains only one power 9 and Q does not contain any power.

Key words. Smarandache cubic product sequence, power.

For any positive integer n, Let $C(n)$ be the n-th cubic. Further, let

(1) $$P(n)=\prod_{k=1}^{n} C(k)+1$$

and

(2) $$Q(n)=\prod_{k=1}^{n} C(k)-1 .$$

Then the sequences $P=\{P(n)\}_{n=1}^{\infty}$ and $Q=\{Q(n)\}_{n=1}^{\infty}$ are called the Smarandache cubic product sequence of the first kind and the second kind respectively (see [5]). In this paper we consider the powers in P and Q .We prove the following result.

Theorem. The sequence P contains only one power $P(2)=3^2$. The sequence Q does not contain any power.

Proof. If $P(n)$ is a power, then from (1) we get

(3) $$(n!)^3+1=a^r,$$

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where a and r are positive integers satisfying $a>1$ and $r>1$, By (3), if $2 \mid r$, then the equation

(4) $$X^3+1=Y^2$$

has a positive integer solution $(X,Y)=(n!, a^{r/2})$. Using a well known result of Euler (see [3,p.302]), (4) has only one positive integer solution $(X,Y)=(2,3)$. It implies that P contain only one power $P(2)=3^2$ with $2 \mid r$. If $2 \nmid r$, then the equation

(5) $$X^3+1=Y^m, m>1 \ 2 \nmid m$$

has a positive integer solution $(X,Y,m)=(n!,a,r)$. However, by [4] , it is impossible. Thus, P contains only one power $P(2)=3^2$.

Similarly, by(2), if $Q(n)$ is a power, then we have

(6) $$(n!)^3-1=a^r,$$

where a and r are positive integers satifying $a>1$ and $r>1$, It implies that the equation.

(7) $$X^3-1=Y^m, m>1,$$

has a positive integer solution $(x,Y,m)=(n!,a,r)$. However, by the results of [2] and [4], it is impossible. Thus, the suquence Q does not contain any power . The theorem is proved.

References

[1] C.Ko, On the diophantine equation $x^2=y^n+1, 0\neq xy$, Sci. Sinica 14(1964),457-460.

[2] V.A.Lebesgue, Sur l'impossbilité, en nombres entiers, de l'équation $x^m=y^2+1$, Nouv, Ann. Math. (1), 9(1850),178-181.

[3] L.j.Mordell, Diophantine Equations, Academic Press,

London,1969.
[4] T.Nagell, Des équations indeterminées $x^2+x+1=y^n$ et $x^2+x+1=3y^n$, Norsk. Mat Forenings Skr. Ser. I,2(1921),12-14.
[5] F.Russo, **Some** results about four Smarandache U-product sequences, Smarandache Notions J.11(2000), 42-49.

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