PROPERTIES OF SMARANDACHE STAR TRIANGLE

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ABSTRCT: In [1] we define SMARANDACHE FACTOR PARTITION FUNCTION , as follows: Let $\alpha_1, \alpha_2, \alpha_3, \ldots \alpha_r$ be a set of r natural numbers and $p_1, p_2, p_3, \ldots p_r$ be arbitrarily chosen distinct primes then $F(\alpha_1, \alpha_2, \alpha_3, \ldots \alpha_r)$ called the Smarandache Factor Partition of $(\alpha_1, \alpha_2, \alpha_3, \ldots \alpha_r)$ is defined as the number of ways in which the number

$$N = p_1^{\alpha 1}\, p_2^{\alpha 2}\, p_3^{\alpha 3} \ldots p_r^{\alpha r}$$ could be expressed as the

product of its' divisors. For simplicity , we denote $F(\alpha_1, \alpha_2, \alpha_3, \ldots \alpha_r) = F'(N)$,where

$$N = p_1^{\alpha_1}\, p_2^{\alpha_2}\, p_3^{\alpha_3} \ldots p_r^{\alpha_r} \ldots p_n^{\alpha_n}$$

and p_r is the r^{th} prime. $p_1 = 2$, $p_2 = 3$ etc.

Also for the case

$$\alpha_1 = \alpha_2 = \alpha_3 = \ldots = \alpha_r = \ldots = \alpha_n = 1$$

Let us denote

$$F(\underbrace{1, 1, 1, 1, 1 \ldots}_{\leftarrow\ n\text{ - ones}\ \rightarrow}) = F(1\#n)$$

In [2] we define **The Generalized Smarandache Star Function** as follows:

Smarandache Star Function

(1) $$F'^{*}(N) = \sum_{d/N} F'(d_r) \qquad \text{where } d_r \mid N$$

(2) $$F'^{**}(N) = \sum_{d_r/N} F'^{*}(d_r)$$

d_r ranges over all the divisors of N.

If N is a square free number with n prime factors , let us denote

$$F'^{**}(N) = F^{**}(1\#n)$$

Smarandache Generalised Star Function

(3) $$F'^{n*}(N) = \sum_{d_r/N} F'^{(n-1)*}(d_r) \qquad n > 1$$

and d_r ranges over all the divisors of N.

For simplicity we denote

$$F'(Np_1p_2...p_n) = F'(N@1\#n) \quad \text{, where}$$

$$(N, p_i) = 1 \text{ for } i = 1 \text{ to } n \text{ and each } p_i \text{ is a prime.}$$

F'(N@1#n) is nothing but the Smarandache factor partition of (a number N multiplied by n primes which are coprime to N).

In [2] I had derived a general result on the Smarandache Generalised Star Function. In the present note we define SMARANDACHE STAR TRIANGLE' (SST) and derive some properties of SST.

DISCUSSION:

DEFINITION : 'SMARANDACHE STAR TRIANGLE' (SST)

As established in [2]

$$a_{(n,m)} = (1/m!) \sum_{k=1}^{m} (-1)^{m-k} . {}^{m}C_k . k^n \text{ ---------- } (1)$$

we have $a_{(n,n)} = a_{(n,1)} = 1$ and $a_{(n,m)} = 0$ for $m > n$. Now if one arranges these elements as follows

$a_{(1,1)}$

$a_{(2,1)}$ $a_{(2,2)}$

$a_{(3,1)}$ $a_{(3,2)}$ $a_{(3,3)}$

.

.

.

$a_{(n,1)}$ $a_{(n,2)}$ ... $a_{(n,n-1)}$ $a_{(n,n)}$

we get the following triangle which we call as the '**SMARANDACHE STAR TRIANGLE**' in which $a_{(r,m)}$ is the m^{th} element of the r^{th} row and is given by (A) above. It is to be noted here that the elements are the Stirling numbers of the first kind.

1

1 1

1 3 1

1 7 6 1

1 15 25 10 1

. . .

Some propoerties of the SST.

(1) The elements of the first column and the last element of each row is unity.

(2) The elements of the second column are $2^{n-1} - 1$, where n is the row number.

(3) Sum of all the elements of the n^{th} row is the n^{th} Bell.

PROOF:

From theorem(3.1) of Ref; [2] we have

$$F'(N@1\#n) = F'(Np_1p_2...p_n) = \sum_{m=0}^{n} a_{(n,m)} F'^{m}*(N)$$

if $N = 1$ we get $F'^{m}*(1) = F'^{(m-1)}*(1) = F'^{(m-2)}*(1) = ... = F'(1) = 1$

hence

$$F'(p_1p_2...p_n) = \sum_{r=0}^{n} a_{(n,m)}$$

(**4**)The elements of a row can be obtained by the following reduction formula

$$\mathbf{a_{(n+1,m+1)} = a_{(n,m)} + (m+1) . a_{(n+1,m+1)}}$$

instead of having to use the formula (4.5).

(5) If $N = p$ in theorem (3.1) Ref;[2] we get $F'^{m}*(p) = m + 1$. Hence

$$F'(pp_1p_2...p_n) = \sum_{m=1}^{n} a_{(n,m)} F'^{m}*(N)$$

or

$$B_{n+1} = \sum_{m=1}^{n} (m+1)\ a_{(n,m)}$$

(6) Elements of second leading diagonal are triangular numbers in their natural order.

(7) If p is a prime, p divides all the elements of the p^{th} row except the I^{st} and the last, which are unity. This has been established in the following theorem.

THEOREM(1.1):

$$\mathbf{a_{(p,r)} \equiv 0 \ (\bmod p) \text{ if } p \text{ is a prime and } 1 < r < p}$$

Proof:

$$a_{(p,r)} = (1/r!) \sum_{k=1}^{m} (-1)^{r-k} \cdot {}^{r}C_k \cdot k^p$$

Also

$$a_{(p,r)} = (1/(r-1)!) \sum_{k=0}^{r-1} (-1)^{r-1-k} \cdot {}^{r-1}C_k \cdot (k+1)^{p-1}$$

$$a_{(p,r)} = (1/(r-1)!) \sum_{k=0}^{r-1} [(-1)^{r-1-k} \cdot {}^{r-1}C_k \cdot \{(k+1)^{p-1} - 1\}] +$$

$$(1/(r-1)!) \sum_{k=0}^{r-1} (-1)^{r-1-k} \cdot {}^{r-1}C_k$$

applying Fermat's little theorem, we get

$$a_{(p,r)} = \text{a multiple of } p + 0$$

$$\Rightarrow \qquad \mathbf{a_{(p,r)} \equiv 0 \ (\bmod p)}$$

COROLLARY: (1.1)

$$F(1\#p) \equiv 2 \ (\bmod p)$$

$$a_{(p,1)} = a_{(p,p)} = 1$$

$$F(1\#p) = \sum_{k=0}^{p} a_{(p,k)} = \sum_{k=2}^{p-1} a_{(p,k)} + 2$$

$$F(1\#p) \equiv 2 \pmod p$$

(8) The coefficient of the r^{th} term $b_{(n,r)}$ in the expansion of x^n as

$$x^n = b_{(n,1)}\, x + b_{(n,2)}\, x(x-1) + b_{(n,3)}\, x(x-1)(x-2) + \ldots + b_{(n,r)}\, {}^{x}P_r + \ldots + b_{(n,n)}\, {}^{x}P_n$$

is equal to $a_{(n,r)}$.

THEOREM(1.2): B_{3n+2} is even else B_k is odd.

From theorem (2.5) in REF. [1] we have

$F'(Nq_1q_2) = F'^*(N) + F'^{**}(N)$ where q_1 and q_2 are prime.

and $(N,q_1) = (N,q_2) = 1$

let $N = p_1p_2p_3\ldots p_n$ then one can write

$$F'(p_1p_2p_3\ldots p_nq_1q_2) = F'^*(p_1p_2p_3\ldots p_n) + F'^{**}(p_1p_2p_3\ldots p_n)$$

or $F(1\#(n+2)) = F(1\#(n+1)) + F^{**}(1\#n)$

but

$$F^{**}(1\#n) = \sum_{r=0}^{n} {}^{n}C_r\, 2^{n-r}\, F(1\#r)$$

$$F^{**}(1\#n) = \sum_{r=0}^{n-1} \{ {}^{n}C_r\, 2^{n-r}\, F(1\#r)\} + F(1\#n)$$

the first term is an even number say $= E$,This gives us

$F(1\#(n+2)) - F(1\#(n+1)) - F(1\#n) = E$, an even number. ---(1.1)

Case- I: $F(1\#n)$ is even and $F(1\#(n+1))$ is also even $\Rightarrow$

F(1#(n+2)) is even.

Case -II: F(1#n) is even and F(1#(n+1)) is odd $\Rightarrow$ F(1#(n+2)) is odd.

again by (1.1) we get

F(1#(n+3)) - F(1#(n+2)) - F(1#(n+1)) = E , $\Rightarrow$ F(1#(n+3)) is even. Finally we get

F(1#n) is even $\Leftrightarrow$ F(1#(n+3)) is even

we know that F(1#2) = 2 $\Rightarrow$ F(1#2), F(1#5) , F(1#8) , . . .are even

$\Rightarrow$ B_{3n+2} **is even else** B_k **is odd**

This completes the proof.

REFERENCES:

[1] "Amarnath Murthy" , 'Generalization Of Partition Function, Introducing 'Smarandache Factor Partition', SNJ, Vol. 11, No. 1-2-3, 2000.

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[3] " The Florentine Smarandache " Special Collection, Archives of American Mathematics, Centre for American History, University of Texax at Austin, USA.

[4] 'Smarandache Notion Journal' Vol. 10 ,No. 1-2-3, Spring 1999. Number Theory Association of the UNIVERSITY OF CRAIOVA .