A CONJECTURE CONCERNING THE RECIPROCAL PARTITION THEORY

Maohua Le

Abstract . In this paper we prove that there exist infinitely many disjoint sets of positive integers which the sum of whose reciprocals is equal to unity.

Key words . disjoint set of positive integers, sum of reiprocals, unity.

In [1] and [2] , Murthy proposed the following conjecture.

Conjecture . There are infinitely many disjoint sets of positive integers which the sum of whose reciprocals is equal to unity.

In this paper we completely verify the mentioned conjecture. For any positive integer n with $n \geqslant 3$, let $A(n)=\{a(n,1)\ ,\ a(n,2),...a(n,n)\}$ be a disjoint set of positive integers having n elements, where $a(n,k)$ $(k=1,2...,n)$ satisfy

(1) $$a(3,1)=2,\quad a(3,2)=3,\quad a(3,3)=6,$$

and

(2) $$a(n,k)=\begin{cases} 2, & \text{if } k=1, \\ 2a(n-1,k-1), & \text{if } k>1, \end{cases}$$

for $n>3$. We prove the following result.

Theorem . For any positive integer n with $n \geqslant 3$, $A(n)$ is a disjoint set of positive integers satisfying

(3) $$\frac{1}{a(n,1)}+\frac{1}{a(n,2)}+\cdots+\frac{1}{a(n,n)}=1 .$$

Proof. We see from (1) and (2) that $a(n,1)<a(n,2)<\cdots$

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Theorem . For any positive integer n with $n \geqslant 3$, $A(n)$ is a disjoint set of positive integers satisfying

(3) $$\frac{1}{a(n,1)} + \frac{1}{a(n,2)} + \cdots + \frac{1}{a(n,n)} = 1 .$$

Proof. We see from (1) and (2) that $a(n,1)<a(n,2)<\cdots$

$<a(n,n)$. It implies that $A(n)$ is a disjoint set of positive integers. By (1), we get

$$\frac{1}{a(3,1)}+\frac{1}{a(3,2)}+\frac{1}{a(3,3)}=\frac{1}{2}+\frac{1}{3}+\frac{1}{6}=1. \quad (4)$$

Hence, $A(n)$ satisfies (3) for $n=3$. Further, by (2) and (4), we obtain that if $n>3$, then

$$\frac{1}{a(n,1)}+\frac{1}{a(n,2)}+\cdots+\frac{1}{a(n,n)}=\frac{1}{2}+\left[\frac{1}{2a(n-1,1)}+\frac{1}{2a(n-1,2)}+\cdots+\frac{1}{2a(n-1,n-1)}\right]=\frac{1}{2}+\frac{1}{2}=1. \quad (5)$$

Therefore, by (5), $A(n)$ satisfies (3) for $n>3$. Thus, the theorem is proved.

References

[1] A. Murthy, Smarandache maximum reciprocal representation function, Smarandache Notions J. 11(2000), 305-307.

[2] A. Murthy, Open problems and conjectures on the factor/reciprocal partition theory, Smarandache Notions J. 11(2000), 308-311.

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