THE LIMIT OF THE SMARANDACHE DIVISOR SEQUENCES

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Abstract. In this paper we prove that the limit $T(n)$ of the Smarandache divisor sequence exists if and only if n is odd.

Key words. Smarandache divisor sequence, limit, existence.

For any positive integers n and x, let the set

(1) $$A(n)=\{x \mid d(x)=n\},$$

where $d(x)$ is the number of distinct divisors of x. Further, let

(2) $$T(n) = \sum \frac{1}{x},$$

where the summation sign Σ denote the sum through over all elements x of $A(n)$. In [2], Murthy showed that $T(n)$ exists if n=1 or n is an odd prime, but $T(2)$ does not exist. Simultaneous, Murthy asked that whether $T(n)$ exist for n=4,6 etc. In this paper we completely solve the mentioned problem. We prove a general result as follows.

Theorem. $T(n)$ exists if and only if n is odd.

Proof. For any positive integer a with a>1, let

(3) $$a=p_1^{r_1}p_2^{r_2}\dots p_k^{r_k}$$

be the factorization of a. By [1, Theorem 27], we have

(4) $$d(a)=(r_1+1)(r_2+1)\dots(r_k+1).$$

If n is even, then from (1) and (4) we see that

$A(n)$ contains all positive integers x with the form

(5) $$x=pq^{n/2-1},$$

where p , q are distinct primes . Therefore , we get from (2) and (5) that

(6) $$T(n)>\frac{1}{2^{n/2-1}}\Sigma^*\frac{1}{p}=\frac{1}{2^{n/2-1}}[\,T(2)-2\,],$$

where the summation sigh Σ^* denote the sum through over all odd primes p . Since $T(2)$ does not exist , we find from (6) that $T(n)$ does not exist if n is even .

Let

(7) $$n=d_1d_2\cdots d_t$$

be a multiplicative parition of n , where $d_1,d_2,\ldots,d_t$ are divisors of n with $1<d_1\leqslant d_2\leqslant\cdots\leqslant d_t$. Further , let

(8)$T(d_1d_2\cdots d_t)=\{x\mid x=p_1^{d1-1}p_2^{d2-1}\cdots p_t^{dt-1},p_1,p_2,\cdots,p_t$ are distinct primes$\}$.

By (1),(4),(7) and (8) , we get

(9) $$T(n)=\Sigma^{**}T(d_1d_2\ldots d_t),$$

where the summation sign Σ^{**} denote the sum through over all distinct multiplicative partitions of n . For any positive integer m, let

(10) $$R(m)=\sum_{k=1}^{\infty}\frac{1}{k^m}$$

be the Riemann function . If n is odd , then from (7) we see that $d_1\geqslant 3$. Therefore , by (4),(8),(9) and (10) , we obtain

(11) $$\begin{aligned}T(n)&<\Sigma^{**}\left(\prod_{i=1}^{t}\left(\frac{1}{2^{d_i-1}}\Sigma^*\frac{1}{p^{d_i-1}}\right)\right)\\&<\Sigma^{**}\left(\prod_{i=1}^{t}R(d_i-1)\right)\leqslant\Sigma^{**}\left(\prod_{i=1}^{t}R(2)\right)\\&=\Sigma^{**}\left[R(2)\right]^t.\end{aligned}$$

Since the number of multiplicative partitions of n is finite and $R(2)=\pi^2/6$, we see from (11) that $T(n)$ exists if n is odd Thus, the theorem is proved.

References

[1] G.H. Hardy and E.M. Wright, An Introduction to the Theory of Numbers, Oxford University press, Oxford, 1937.

[2] A.Murthy, Some new Smarandache sequences, fumctions and partitions, Smarandache Notions J. 11(2000), 179-185.

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